Lesson 4-1

Solving Percent Problems Using Equations

Vocabulary

percent

BIG IDEA Many types of questions involving percents can be answered by solving an equation of the form $ax = b$.

The word **percent** (often written as the two words per cent) comes from the Latin words "per centum," meaning "per 100." So 7% literally means 7 per 100, the ratio $\frac{7}{100}$, or the decimal 0.07. The symbol % for percent is only a little more than 100 years old.

Mental Math

Evaluate.

a. -2^2

b. $-(-2^2)^2$

c. $-((-2^2)^2)^2$

Putting Percent Problems into the Form $p \cdot q = r$

Recall that to find a percent p of a given quantity q you simply multiply q by p. For example, to find 12% of 85, multiply $0.12 \cdot 85 = 10.2$. To find $5\frac{1}{4}$% of \$3,000, calculate $0.0525 \cdot 3{,}000 = \$157.50$. This gives a straightforward method for solving many percent problems. Just translate the words into an equation of the form $p \cdot q = r$, where p is the percent in decimal or fraction form, q is the initial quantity, and r is the resulting quantity.

Example 1

7% of what number is 91?

Solution

7% of what number is 91?

$0.07 \cdot q = 91$ Change 7% to 0.07.

$\frac{0.07q}{0.07} = \frac{91}{0.07}$ Divide each side by 0.07.

$q = 1{,}300$ Simplify.

Check 7% of $1{,}300 = 0.07 \cdot 1{,}300 = 91$. It checks.

Example 2

What percent of 160 is 38.4?

Solution

What percent of 160 is 38.4?

$$\underset{\downarrow}{p} \quad \underset{\downarrow}{\cdot} \; 160 \underset{\downarrow}{=} 38.4$$

Translate into an equation.

$$\frac{p \cdot 160}{160} = \frac{38.4}{160}$$

Divide each side by 160.

$$p = 0.24$$

Simplify. This is the solution to the equation.

$$p = 24\%$$

Rewrite the solution as a percent.

Check 24% of 160 = 0.24 · 160 = 38.4. It checks.

Percent Add-Ons and Discounts

Percents are very common in business, science, statistics, and even everyday shopping. In the next example, the Distributive Property gives a useful approach to a common situation.

Example 3

Ian, Cassady, Quincy, and Dylan ate at a local restaurant. The meal cost $36.50 without a tip. The bill states, "An 18% gratuity (tip) will be added for parties of 4 or more people." Find the total cost of the meal and tip.

An average of one out of five meals consumed by Americans—4.2 meals per week—is prepared in a commercial setting.

Source: Meal Consumption Behavior

Solution 1 Let M be the cost of the meal. We know M = $36.50, but we ignore that for a while. Note that total cost = cost of meal + cost of tip.

$$\text{total cost} = 100\% \cdot M + 18\% \cdot M$$ 18% of $36.50

$$= 118\% \cdot M$$ Distributive Property

$$= 1.18 \cdot 36.50$$ Rewrite 118% as 1.18.

$$= 43.07$$ Multiply.

So the total cost was $43.07.

Solution 2 Find the amount of the tip and then add it to the $36.50 meal cost.

$$\text{tip} = 18\% \cdot 36.50$$ 18% of $36.50

$$= 0.18(36.50)$$ 18% = 0.18

$$= 6.57$$ Simplify.

$$\text{total cost} = 36.50 + 6.57 = 43.07$$

Again, the total cost was $43.07.

STOP QY

The use of the Distributive Property in Solution 1 of Example 3 enables the total cost to be found with just one calculation. This idea is used in Example 4 to quickly calculate the price for an item on sale.

▶ QY

If a meal cost $21.16 with a 15% tip, what was the cost before the tip?

GUIDED

Example 4

A dishwasher normally costs $320, but it is on sale for 15% off. What is the sale price?

Solution Let P be the regular price. Then the discount is 15% • P, or $0.15P$.

Sale price = regular price − discount

= ? − ? Translate into an equation.

= ? − ? Multiplicative Identity Property

= ? Combine like terms.

We know $P = \$320$. So the sale price is ? (?) = $? .

Another way to think about the sale price is that when 15% of the price is removed, 100% − 15%, or 85% of the price remains.

Markups and Discounts

If an item is discounted x%, you pay (100 − x)% of the original or listed price.

If an item is marked up or taxed x%, you pay (100 + x)% of the original or listed price.

Questions

COVERING THE IDEAS

1. 123% of 780 is what number?
2. 40% of what number is 440?
3. What percent of 4.7 is 0.94?
4. What number is 62% of 980?
5. Suppose a shirt is on sale for 10% off its original price of $23.50.
   a. What percent of the original price does the customer pay?
   b. How much does the customer pay before tax?

6. Suppose a dinner costs D dollars and you wish to give a 20% tip.
   a. What is the amount of the tip?
   b. What is the total cost of the meal with tip?
7. A table is being sold at "40% off." If the price of the table before the sale was T dollars, what is the sale price?
8. An electronics store owner buys a television from the manufacturer, and then adds 45% of that cost to get the price the customer pays. If a TV sells for $499, what was the price the store owner paid?
9. The total cost of a digital camera including an 8.5% sales tax is $215. How much tax was paid on this purchase?

Digital-camera sales surpassed $4 billion in 2004.
Source: *Twice*

10. The Cupertinos bought a new car. The total amount they paid was $28,250.75 including the 8% sales tax. What was the price before the sales tax was added?
11. Consider the sales receipt at the right. Determine the sales-tax rate as a percent.

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– CASH RECEIPT –
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AMOUNT	15.85
TAX	1.15
TOTAL:	
CASH:	
CHANGE:	

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12. Clearwater High School expects a 4% increase in enrollment next year. There are 1,850 students enrolled this year.
    a. How many students will the school gain?
    b. What is the expected enrollment next year?
13. According to the 2000 census, 73% of the 221 million U.S. residents age 15 and older had been married at least once. How many U.S. residents over 15 had *never* been married?

APPLYING THE MATHEMATICS

In 14 and 15, use the following information. Sucrose, or common table sugar, is composed of carbon, hydrogen, and oxygen. Suppose an experiment calls for 68.4 grams of sucrose.

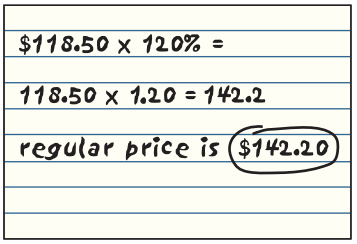
14. If 4.2% of the weight of sucrose is carbon, how many grams of carbon are in the 68.4 grams?
15. If 35.2 grams of the 68.4 grams are oxygen, what percent of the weight of sucrose is oxygen?

16. On a mathematics test there were eight A's, twelve B's, ten C's, two D's, and zero F's. What percent of the students earned A's?

17. Consider the following situation. Jorge works at a clothing store in the local mall. As an employee, he receives a 20% discount on clothes. He spent $118.50 at the store.

    a. What was the regular price of the clothes Jorge bought before the discount was figured?

    b. Suppose a student answers Part a in the manner shown below. Show why the student's answer is not correct.



18. A DVD player originally cost $150. It is on sale for $72.50. What is the percent of the discount, rounded to the nearest tenth of a percent?

In 2002, about 40 million households in the United States had DVD players.

Source: Consumer Electronics Association

REVIEW

In 19–21, solve and check the equation. (Lesson 3-8)

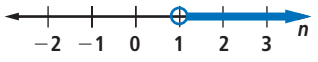
19. $\frac{k}{7} + 18 = \frac{19}{2}$ 20. $\frac{u}{12} - \frac{u}{2} = 35$ 21. $\frac{a+3}{4} + 3 = \frac{1}{2}$

22. Penelope Nichols spends half her monthly income on housing and food, and budgets the other half as follows: $\frac{1}{4}$ on clothes, $\frac{1}{3}$ on entertainment, and $\frac{1}{4}$ on transportation. She saves the remaining $40. What is her monthly income? (Lesson 3-8)

23. Rebecca works at a clothing store that pays her $7.25 per hour plus 9% of the cost of the clothes that she sells. If she works for 8 hours, what is the cost of the clothing she must sell if she wants to earn at least $87? (Lesson 3-7)

24. **Multiple Choice** Which graph below shows the solutions of $6 < -4n + 10$? (Lesson 3-7)

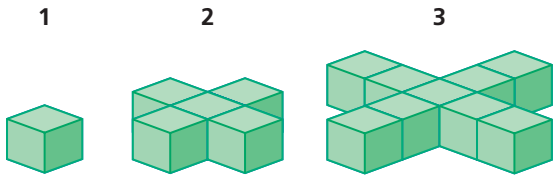
A

B 

C

D

25. Consider the sequence made with cubes below. **(Lessons 3-2, 1-4)**



a. How many cubes will be needed to make the 4th term?

b. Write an equation for the number c of cubes that are in the nth term.

c. Using your equation from Part b, find the term number that contains 33 cubes.

26. Trina timed her commute to school every morning for two weeks. Her times, in minutes, were 15, 22, 17, 12, 14, 16, 20, 21, 21, and 19. Compute the m.a.d. of Trina's commute. **(Lesson 1-7)**

27. Evaluate $(w + 4.7)(2.6 - w)(w + 7.1)$ when $w = -4.7$. **(Lesson 1-1)**

28. A recipe calls for $\frac{1}{4}$ tablespoon of vanilla. If Sally wants to make $\frac{2}{3}$ of the recipe, how much vanilla will she use? **(Previous Course)**

EXPLORATION

29. Jerome noticed that 40% of 50 is equal to 50% of 40 and concluded a% of b is always equal to b% of a. Explain why this works for any positive values of a and b.

QY ANSWER

$18.40