Lesson 1-6

Absolute Value and Distance

Vocabulary

absolute value

origin

BIG IDEA The absolute value of a number and the distance between two numbers on a number line are closely related.

Cameron and Maria took a vacation to White's City, New Mexico, near the Guadalupe Mountains and the famous Carlsbad Caverns. Being avid hikers, they wanted to go mountain climbing and explore the caves. They found the following information on the Internet.

Guadalupe Peak is the highest point in Texas.

Source: National Park Service

White's City is at an elevation of 5,740 feet (above sea level). Guadalupe Peak in Guadalupe Mountains National Park, 35 miles away, is at 8,749 feet. The entrance to Carlsbad Caverns National Park is 7 miles away. The entrance to Carlsbad Caverns is at an elevation of 4,400 feet, and the King's Palace Cavern is 900 feet below the entrance.

STOP **See Quiz Yourself 1 at the right.**

Mental Math

a. How many words can you type in 10 minutes if you type 52 words per minute?

b. How many words can you type in 20 minutes if you type 52 words per minute?

c. How many words can you type in t minutes if you type 52 words per minute?

QUIZ YOURSELF 1

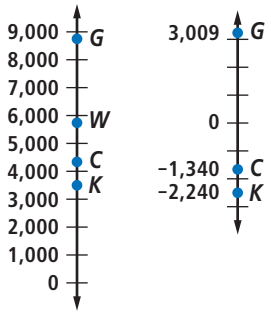
What is the elevation of King's Palace Cavern?

Measuring Elevations from White's City

Because they were starting from White's City (W), Cameron and Maria wanted to know how much they would be going up and down from 5,740 feet. They subtracted the elevation of White's City from the

elevation of each destination. The difference is a positive number if they were going up and a negative number if they were going down.

Destination	Elevation (ft)	Difference from White's City (ft)
Guadalupe Peak (*G*)	8,749	8,749 − 5,740 = 3,009
Carlsbad Caverns entrance (*C*)	4,400	4,400 − 5,740 = −1,340
King's Palace Cavern (*K*)	3,500	3,500 − 5,740 = −2,240



These elevations are graphed on the left vertical number line at the right.

The difference from White's City's elevation to the other places is a *deviation* in altitude. A deviation can be positive or negative. The deviations are on the right vertical number line. However, sometimes we just want to know how much difference there is, and not the direction of the deviation. Then we take the *absolute value* of the deviation.

The symbol for absolute value is two vertical lines: | |.

If $x > 0$, then $|x| = x$.

If $x < 0$, then $|x| = -x$.

Since $3{,}009 > 0$, $|3{,}009| = 3{,}009$.

Since $-1{,}340 < 0$, $|-1{,}340| = -(-1{,}340) = 1{,}340$.

Then we say that the *absolute difference* in elevations between White's City and the entrance to Carlsbad Caverns is 1,340 feet.

QUIZ YOURSELF 2

a. What is $|-2{,}240|$?

b. What is the absolute difference in the elevations of White's City and the King's Palace Cavern?

STOP **See Quiz Yourself 2 at the right.**

Absolute Value and Distance

From the right vertical number line drawn above, you can see the geometric interpretation of absolute value.

Absolute Value

The **absolute value** of a number is its distance from 0.

Because the absolute value of a number is a distance, it is never negative. Using the definition of absolute value, $-x$ looks like a negative number. But that is only the definition when x is negative, so $-x$ stands for a positive number. For example, when $x = 830$, $|x| = |830| = 830$ and when $x = -830$, $|x| = |-830| = -(-830) = 830$.

QUIZ YOURSELF 3

Evaluate each expression.

a. $|3 - 24|$

b. $|3| - |24|$

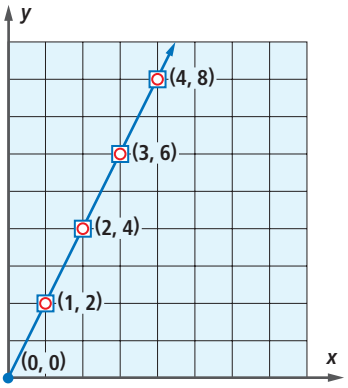
c. $|-3| - |-24| - |18|$

STOP **See Quiz Yourself 3 at the right.**

Expressions with Absolute Value

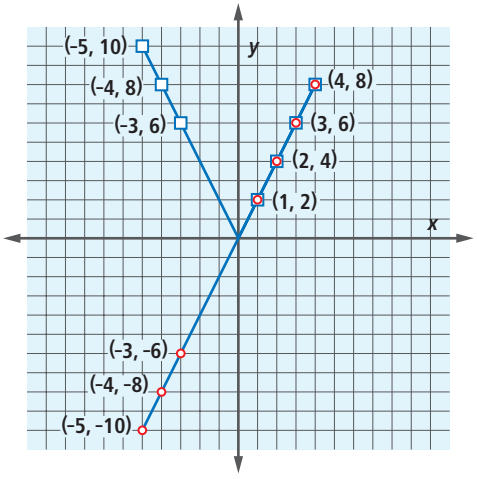
In this chapter you have seen many tables where the values are small, positive integers. However, using only these numbers can give you misleading information. For example, consider the expressions $2x$ and $|2x|$. The table at the right seems to show that $2x$ and $|2x|$ are equivalent. When x is a positive number, both expressions double it. So both $2x$ and $|2x|$ are positive. On the graph below the table, points for $2x$ are shown with open circles (○) and points for $|2x|$ are marked with squares (□). They are the same points.

x	$2x$	$\lvert 2x \rvert$
0	0	0
1	2	2
2	4	4
3	6	6
4	8	8



But this is misleading! No negative numbers were used in the table above. As soon as negative numbers are included, you can see that the expressions $2x$ and $|2x|$ have different values. In fact, for negative numbers the values are opposites. For negative values of x, $2x$ is also negative, so taking the absolute value changes its sign. This can be seen on the graph below by looking to the left of the point $(0, 0)$, called the **origin.** For each negative value of x, the point for $2x$ (marked ○) and the point for $|2x|$ (marked □) are on opposite sides of the x-axis.

x	$2x$	$\lvert 2x \rvert$
–5	–10	10
–4	–8	8
–3	–6	6



Activity

The graphing calculator or computer symbol for absolute value of a number x is **abs(x)**. In Lesson 1–5, you used technology to compare expressions. Now we use technology to compare $|x + 4|$ and $|x| + 4$.

Enter **Y1=abs(x+4)** and **Y2=abs(x)+4** in your graphing calculator with window $-10 \le x \le 10$ and $-10 \le y \le 10$. Sketch the graphs in your window. Do the expressions appear to be equivalent?

STOP **See Quiz Yourself 4 at the right.**

▶ QUIZ YOURSELF 4

Test whether $|x^2 - 6|$ and $-(x^2 - 6)$ are equivalent. Explain your reasoning.

Questions

COVERING THE IDEAS

1. The deepest cavern in Carlsbad Caverns is 1,567 feet below its entrance, making it the deepest cave in the United States.
   a. What is the elevation of that deepest cavern?
   b. What is the difference in its elevation from White's City?
   c. What is the absolute difference in its elevation from White's City?

Carlsbad Caverns in New Mexico has more than 100 caves.

Source: National Park Service

2. Suppose you begin the day at an elevation of E feet. After some hiking, you are at an elevation of H feet.
   a. What is the deviation of where you are now from where you began?
   b. What is the absolute deviation of where you are now from where you began?
   c. When will the answers to Parts a and b be different?

3. If $x = -40$ and $y = 35$, find each expression.
   a. $|x - y|$
   b. $|y - x|$
   c. $|x + y|$
   d. $|x| + |y|$

In 4–7, evaluate each expression.

4. $\left|\frac{3}{5}\right| + \left|\frac{3}{-5}\right|$
5. $|-1 - (-2)|$
6. $\text{abs}(2) - \text{abs}(-20)$
7. $\frac{\text{abs}(x)}{\text{abs}(-x)}$ when $x = 4.8673$
8. Graph $y = |x + 5|$.
9. Find a value of t for which $3t$ and $|-3t|$ are not equal.

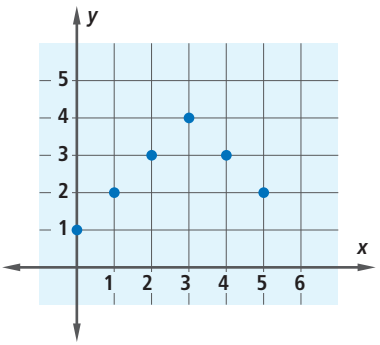
APPLYING THE MATHEMATICS

10. Test whether $|x + 4|$ and $|x| + 4$ are equivalent.
11. Graph $y = |x - 3| - 5$ on your calculator.
    a. Sketch the graph in a standard window.
    b. Sketch the graph in the window $-20 \le x \le 25$ with an x-scale of 5, and $-6 \le y \le 2$ with a y-scale of 1.
    c. Give Xmin, Xmax, Ymin, and Ymax for a window where only the left side of the angle is visible.

In 12–15, find all values of the variable that satisfy the equation.

12. $|x| = 15$
13. $|w - 5| = 0$
14. $|z| = -8.8$
15. $|A| = |A - 1|$

16. Use the graph at the right to answer the following questions. **(Lesson 1-4)**



a. Complete the following table of values.

x	1	2	3	4	5
y	?	?	?	?	?

b. What is the value of the expression when x equals 4?

c. What is the x-value when the value of the expression is 4?

d. Multiple Choice Which of the following equations best describes the graph?

A $y = |x - 2|$

B $y = |x - 3| + 4$

C $y = -1 \cdot |x - 3| + 4$

D $y = x + 1$

REVIEW

17. **a.** Graph $y = x^3 - 3x^2 + 3x - 1$ using the standard window.

b. What are the coordinates of the x-intercept? **(Lesson 1-5)**

18. Make a table of values and a graph for $y = -x^2 + 4$ using the window $-5 \le x \le 5$, and $-30 \le y \le 30$. On your table, increase x by 2 for each row. **(Lesson 1-5)**

19. Express the set $\{x: x \text{ is an integer}, 5 \le x \le 10\}$ in roster form. **(Lesson 1-4)**

20. **a.** Give a counterexample to show that the following pattern is not true for all real numbers x and y. **(Lesson 1-3)**

$$x - y = y - x$$

b. State the result of Part a using the word *commutative*. **(Lesson 1-2)**

21. Evaluate the following expressions for the given value of x. **(Lesson 1-1)**

a. $-(-x^2) + x \cdot x \div (-x)$ for $x = 4$

b. $\left(\frac{2}{1 - (-x)}\right)^{x+1}$ for $x = 1$

EXPLORATION

22. Describe all integer values of x that satisfy $|x| < |x + 1|$.

QUIZ YOURSELF ANSWERS

1. 3,500 ft
2. a. 2,240
   b. 2,240 ft
3. a. 21
   b. -21
   c. -39
4. The expressions are not equivalent. For example, when $x = 3$: $|3^2 - 6| = |9 - 6| = 3$, and $-(3^2 - 6) = -(9 - 6) = -3$.