Lesson
1-4 Picturing Expressions

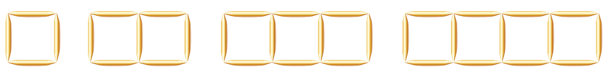
Vocabulary

scatterplot

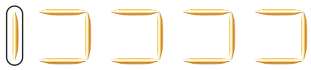
domain of the variable

BIG IDEA Graphing the ordered pairs (value of variable, value of expression) helps you understand the relationship between these values.

Activity 1 in Lesson 1-3 discussed this toothpick sequence.



Suppose a student looks at each instance as one vertical toothpick on the left and 3 more toothpicks being added for each additional square.



Then the student might write the expression $1 + 3n$ to describe the sequence.

Mental Math

a. If cantaloupes cost \$2.98 each, estimate how much 5 cantaloupes cost.

b. If you paid for the 5 cantaloupes with \$15, how much change would you expect?

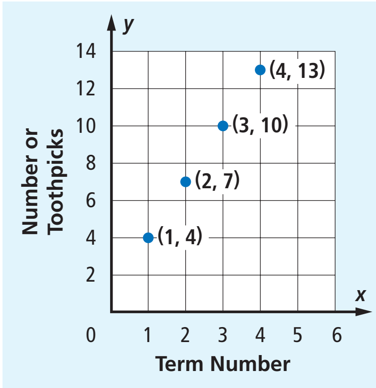
Scatterplots

The table shows values for the term number n and for $1 + 3n$, the number of toothpicks in the nth term. Each row can be written as an ordered pair. These pairs can then be graphed on the coordinate plane.

Term Number	Number of Toothpicks	(Term Number, Number of Toothpicks)
1	4	(1, 4)
2	7	(2, 7)
3	10	(3, 10)
4	13	(4, 13)
n	$1 + 3n$	$(n, 1 + 3n)$

Every point in the plane of the graph can be identified with *coordinates.* A graph like this, in which individual points are plotted, is called a **scatterplot,** as shown on the next page.

If the points on the graph had been connected with a line, then the numbers between 1, 2, 3, and 4 would be allowed for n, and therefore the numbers for $1 + 3n$ between 4, 7, 10, and 13. But this is not possible in the toothpick sequence because n represents the term number. There is a third term and a fourth term, but for $n = 3.5$ there is no toothpick term. In this situation, n must be a positive integer. All the values that may be meaningfully substituted for a variable make up the **domain of the variable.** For the toothpick sequence on page 27, the domain is the set of positive integers {1, 2, 3, 4, …}.



Connected Graphs

When writing expressions for real-world problems, you often must decide what domain makes sense for a situation.

Example 1

Suppose Rebecca drives to her grandmother's house, which is 500 miles away. Her average speed is 65 miles per hour. Fill in the table of values to calculate her remaining distance after each hour. Write an expression to represent her remaining distance, and then plot the points on a graph.

Hours of Driving	Remaining Distance (mi)
0	
1	
2	
3	
h	

California drivers consume about 11% of the fuel used in the United States.

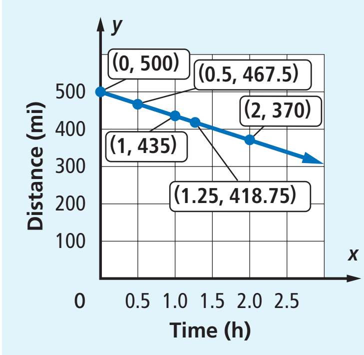
Source: U.S. Department of Transportation

Solution Let h represent the number of hours spent driving.

Hours of Driving	Calculation and Pattern	Remaining Distance (mi)
0	500	500
1	500 – 65	435
2	500 – 65 – 65 = 500 – 65(2)	370
3	500 – 65 – 65 – 65 = 500 – 65(3)	305
h	$500 - \underbrace{65 - \ldots - 65}_{h \text{ terms}} = 500 - 65h$	500 – 65h

So after h hours, Rebecca is $500 - 65h$ miles away from her grandmother's house. The graph is shown on the next page.

Connecting the points is appropriate because time and distance are both measures and do not need to be integers. Rebecca could drive for a half hour or an hour and fifteen minutes. Substituting these numbers into the expression $500 - 65h$ will result in a distance that is not a whole number. The points (0.5, 467.5) and (1.25, 418.75) are plotted here. By connecting the points on the graph, you are showing that all nonnegative real numbers make sense in this situation. Therefore, the domain of h is the set of nonnegative real numbers.



Common Domains of Variables

The following domains are frequently used in arithmetic and algebra.

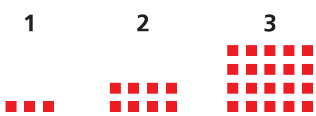
Name of Set	Description	Examples of Elements
whole numbers	{0, 1, 2, 3, …}	Five; $\frac{16}{2}$; 2,007; 7 million
integers	{…, −2, −1, 0, 1, 2, …} whole numbers and their opposites	$\frac{-21}{3}$, −17.00, negative one thousand
real numbers	the set of all numbers that can be represented as terminating or nonterminating decimals	5, 0, π, −0.0042, $-3\frac{1}{3}$, $0.\overline{13}$, $\sqrt{2}$, one hundred thousand
nonnegative real numbers	$\{x: x \geq 0\}$ the set consisting of 0 and all positive real numbers	2.56; 3,470; $\frac{1}{100}$; 0

In the above table, the sets of whole numbers and integers are described with a *roster,* or list of elements. Set-builder notation is used to describe the set of nonnegative real numbers. In $\{x: x \geq 0\}$, the symbol ":" is read "such that." It is followed by an expression that describes the set. The set $\{x: x \geq 0\}$ is read "the set of numbers x such that x is greater than or equal to 0." Some people write $\{x| \ x \geq 0\}$, using a single vertical bar to mean "such that."

Questions

COVERING THE IDEAS

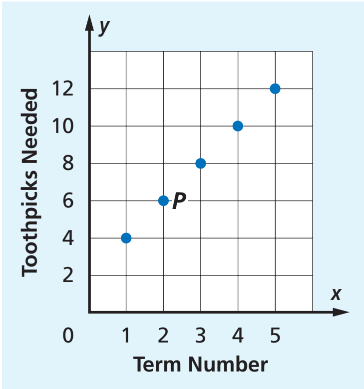
1. Each term in the following sequence is a rectangular array of squares. The nth term has n rows and $n + 2$ columns and therefore $n(n + 2)$ squares.



   a. Make a table of values for $n(n + 2)$ using $n = 1, 2, 3, 4,$ and 5.

   b. Make a scatterplot to graph the values from Part a.

2. The graph below shows the number of toothpicks used to make each term in a sequence.



a. Give the coordinates of *P*.

b. Describe the situation that corresponds to *P*, giving the term number and number of toothpicks needed.

c. Explain why the domain of n is the set of positive integers.

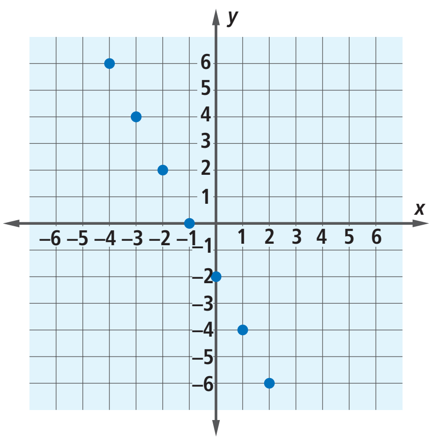
3. Make a table and scatterplot for $4n - 5$ using the following numbers for n: –3, –2, 0, 2, and 3.

4. Make a table and graph for values of the expression $n \cdot (-1)^n$ when $n = 1, 2, 3, 4,$ and 5.

In 5–7, values of a variable *x* and an expression are graphed. The coordinates of each point are integers.

5. Fill in the table based on the graph.

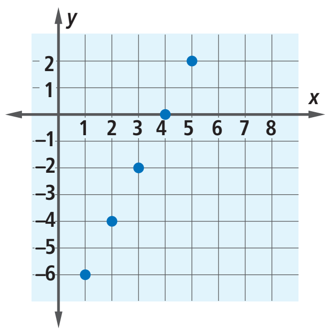
x	Value of Expression
–3	?
0	?
2	?
?	0
?	6
?	–4



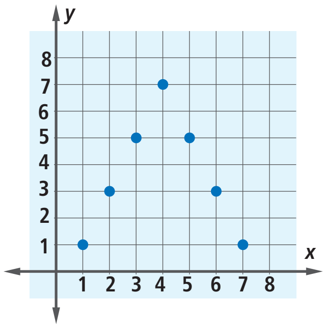
6. Use the graph at the right.

a. What is the value of the expression when *x* is 2?

b. What value(s) of *x* makes the value of the expression equal to 2?



7. **a.** If $x = 5$, what is the value of the expression graphed below?
   b. What value(s) of x makes the value of the expression equal to 5?



8. Express the set $\{x\colon x \text{ is an integer and } x \geq 20\}$ in roster form.

9. Express the set $\{-9, -10, -11, \ldots\}$ in set-builder notation.

APPLYING THE MATHEMATICS

In 10 and 11, consider these two situations.

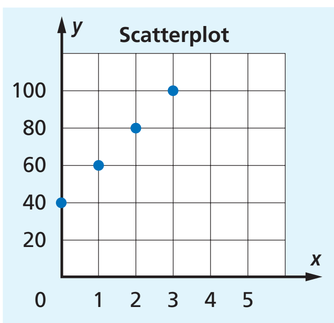
a. A landscaper sells black dirt for $20 per cubic yard, plus a $40 delivery fee. Let x = number of cubic yards ordered.

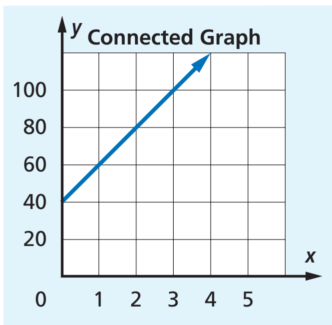
b. Arrangements for special dinners can be made at a restaurant. There is a $40 rental fee for a special room, plus the $20 per person cost for the food. Let x = number of people at the dinner.

Topsoil is sold in 40-pound bags or in bulk measured in cubic yards.

10. For which situation does it make sense to have $x = 2\frac{1}{2}$? In that situation, find the cost when $x = 2\frac{1}{2}$.

11. Match each graph below with the situation it represents.





In 12–14, choose the most reasonable domain for the variable.

a. set of whole numbers
b. set of real numbers
c. set of integers
d. set of positive real numbers

12. n = the number of people at a restaurant on Friday night

13. t = time it takes to do your homework

14. E = elevation of a place in the United States

REVIEW

15. Complete the table of values for each expression. Use the table to conclude whether the expressions seem to be equivalent. Explain your reasoning. **(Lesson 1-3)**

n	$2(n - 3)$
–5	?
–3	?
1	?
2	?
5	?

n	$2n - 3$
–5	?
–3	?
1	?
2	?
5	?

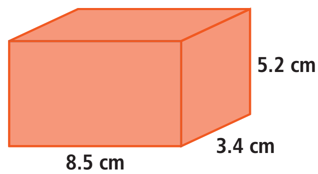
16. Find a counterexample to show that the equation $(n + 10) \cdot x = n + 10 \cdot x$ is not true for all real numbers n and x. **(Lesson 1-3)**

In 17 and 18, an equation describing a pattern is given.
a. Give three instances of the pattern. (Lesson 1-2)
b. Do you think the equation is true for all real numbers? Explain your reasoning. (Lesson 1-3)

17. $t - 2t + 3t = 2t$

18. $9 - (2x - 4) = -2x + 13$

19. Using the formula $V = \ell wh$, find the volume of the figure at the right. **(Lesson 1-1)**



20. Evaluate the expression $a^2 + b^2 + 2ab$ for the following values of a and b. **(Lesson 1-1)**

a. $a = 2, b = 3$ **b.** $a = -2, b = 3$ **c.** $a = -2, b = -3$

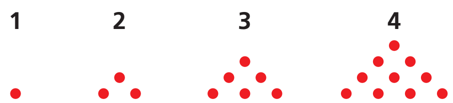
21. Suppose there are 273 students in a school. Of these, $\frac{3}{7}$ play on an athletic team, and of those that are on an athletic team, $\frac{2}{13}$ play on the soccer team. How many students play on the soccer team? **(Previous Course)**

In 2004–2005, there were a total of 670,691 high school soccer players, including both boys and girls.

Source: The National Federation of State High School Associations

EXPLORATION

22. Here is a pattern of dots.



a. Find an expression for the nth term in the pattern.

b. Make a scatterplot.