Lesson **1-3**

Equivalent Expressions

Vocabulary

sequence

term

equivalent expressions

counterexample

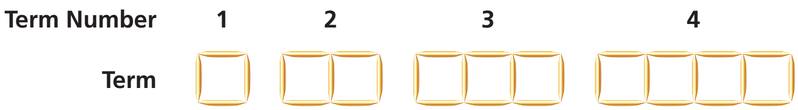
BIG IDEA If different expressions describe the same patterns, then the expressions are equivalent.

Finding Equivalent Expressions

The picture below shows a *sequence* of designs made from toothpicks. A **sequence** is a collection of numbers or objects in a specific order. The objects are called the **terms** of the sequence.

Activity 1

Refer to the toothpick sequence below.



1. Use toothpicks to find how many are used to make the pattern for terms 1 through 10?
2. In words, explain how to use the term number to find the number of toothpicks needed to make that pattern.
3. Use n to represent the term number. Write an algebraic expression for the number of toothpicks needed for the nth term.
4. Use your algebraic expression to complete the table below.

Term Number	Calculation	Number of Toothpicks
11	?	?
12	?	?
15	?	?
100	?	?

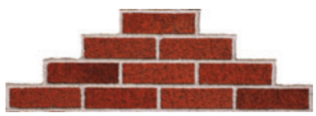
5. Compare your expression with others in your class. Are the expressions the same or different? Write down all the expressions that you think are correct.

Mental Math

a. If international stamps cost 75¢ each, how much does a pack of 20 stamps cost?

b. If the price of the stamps increases by 2¢ per stamp, by how much will a pack of 20 stamps increase?

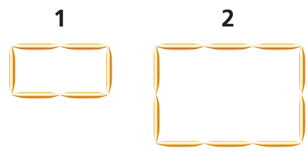
c. If a pack of 20 stamps cost $2.00 less two years ago, how much less did one stamp cost?



This sequence of designs is made from bricks.

Activity 2

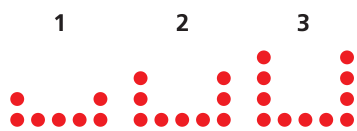
Now consider a different toothpick sequence. The picture at the right shows the number of toothpicks required to construct a sequence of rectangles. The first and second terms are shown. Each rectangle is one toothpick wider and one toothpick taller than the previous rectangle.



1. How many toothpicks are needed for the 1st term?
2. How many toothpicks are needed for the 2nd term?
3. Build the 3rd term in the sequence. How many toothpicks are needed?
4. In words, describe a method to find the number of toothpicks it takes to make the *n*th term.
5. Write an algebraic expression to find the number of toothpicks it takes to make the *n*th term.
6. Fill in the table below.

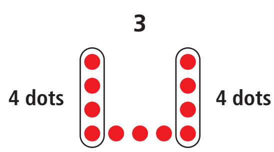
Term Number	Calculation	Number of Toothpicks
4	?	?
5	?	?
6	?	?
13	?	?
58	?	?

In the activities, you looked at sequences of toothpicks and the number of toothpicks needed to make them. For each sequence, you used the term number *n* to determine the number of toothpicks needed for the *n*th term. You and your classmates may have found that there can be more than one expression to describe the *n*th term.



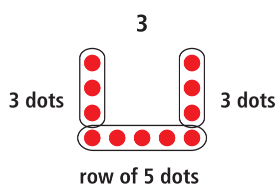
Consider the sequence of dots at the right. Students were asked to describe the number of dots needed to make the *n*th term.

Alf explained that in each term he saw 2 columns of dots and 3 dots between the columns. He wrote the following.



left column		middle		right column
$(n + 1)$	$+$	3	$+$	$(n + 1)$

Beth saw a row of 5 dots at the bottom and 2 columns above the row. She wrote the following.



bottom row		two columns
5	$+$	$2n$

Are both Alf and Beth correct? One way to tell is by substituting a value for n and testing whether the two expressions equal the same value. For instance, let $n = 11$.

$$\text{Alf's expression: } (n + 1) + 3 + (n + 1) = (11 + 1) + 3 + (11 + 1)$$
$$= 12 + 3 + 12$$
$$= 27$$

$$\text{Beth's expression: } 5 + 2n = 5 + 2(11)$$
$$= 5 + 22$$
$$= 27$$

However, it is risky to test only one instance. Sometimes a pattern works for a few numbers and fails for all others. For example, suppose you are not sure whether $2w$ and w^2 have the same meaning.

STOP **See Quiz Yourself 1 at the right.**

The Quiz Yourself shows that $2w$ and w^2 are not equal for all values of w. When you evaluate an expression for several values of the variable, a table of values helps to organize your work.

QUIZ YOURSELF 1

a. Show that $2w$ and w^2 have the same value when $w = 2$.

b. Show that $2w$ and w^2 have different values when $w = 5$.

GUIDED

Example 1

Compare Alf's and Beth's expressions for several instances by filling in the tables below.

Alf

n	$(n + 1) + 3 + (n + 1)$
1	?
2	?
3	?
10	?
20	?
35	?

Beth

n	$5 + 2n$
1	?
2	?
3	?
10	?
20	?
35	?

Solution Comparing the second columns of the two tables, you should have found that when the expressions $(n + 1) + 3 + (n + 1)$ and $5 + 2n$ are evaluated for these values of n, the same result is obtained. Therefore, $(n + 1) + 3 + (n + 1)$ and $5 + 2n$ appear to be *equivalent.*

Equivalent expressions are expressions that have the same value for *every* number that can be substituted for the variable(s). If two expressions produce different results when evaluated for the same number, then the expressions are not equivalent.

Example 2

Suppose Azami and Haley both wrote expressions to describe the same pattern. Azami wrote $k - 8 + 5$ and Haley wrote $k - 13$. Are the expressions equivalent?

Solution Pick a number to substitute for *k*. Let $k = 10$.

Azami's expression: $k - 8 + 5 = 10 - 8 + 5 = 7$

Haley's expression: $k - 13 = 10 - 13 = -3$

When $k = 10$ the expressions have different values, so the expressions are not equivalent.

To show that $k - 8 + 5$ and $k - 13$ are not always equal, you only need to show one instance in which the expressions have different values. The situation in which $k = 10$ is a *counterexample.* A **counterexample** is an instance which shows that a general statement is not always true. It is not true that for all values of k, $k - 8 + 5 = k - 13$.

STOP **See Quiz Yourself 2 at the right.**

QUIZ YOURSELF 2

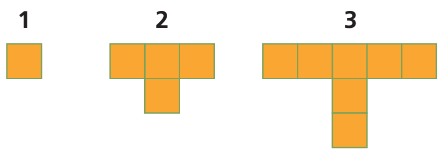
Which two of these expressions seem to be equivalent? How do you know?

$(n + 3)^2$
$n^2 + 9$
$n^2 + 6n + 9$

Questions

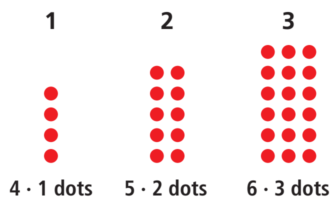
COVERING THE IDEAS

1. Consider the sequence created with square tiles shown below.



   a. Evaluate the expression $3n - 2$ for various values of n to show that it describes the number of squares in the nth term for the 1st, 2nd, and 3rd terms.
   b. Use the expression to find the number of tiles in the 100th term.

2. Consider the sequence of dots below.



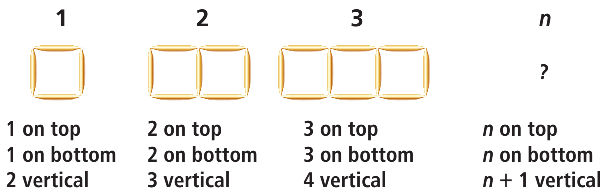
   a. Draw the 4th and 5th terms. For each, write the multiplication expression for the number of dots.
   b. In words, describe how to find the number of dots in the nth term if you know the term number.
   c. Write an expression for the number of dots in the nth term.

3. Two different expressions were used to generate each table of values below. Use the tables to decide whether the expressions seem to be equivalent. Explain how you know.

x	$3x - 17$
10	?
9	?
8	?
7	?
6	?
5	?

x	$x - 6 - (11 - 2x)$
10	?
9	?
8	?
7	?
6	?
5	?

4. Consider the sequence of toothpicks shown at the right. A student describes the number of toothpicks required to make the nth pattern in the following way: I split the toothpicks up into three kinds: (1) top toothpicks; (2) bottom toothpicks; and (3) vertical toothpicks.



In the nth figure, there were n toothpicks on top, n on the bottom, and $n + 1$ vertical ones. So, in the nth figure, there are $n + n + (n + 1)$ toothpicks.

a. Substitute for n to verify that the expression $n + n + (n + 1)$ describes the number of toothpicks needed for the first three terms.

b. Use the expression to find the number of toothpicks in the 100th term.

In 5–7, two expressions are given.

a. Using a table, evaluate each expression for four different values of the variable.

b. Based on your results, do the two expressions appear to be equivalent?

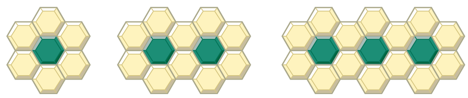
5. $25 + (x - 5)(x + 5)$ and x^2

6. $\frac{8n - 4}{4}$ and $8n - 1$

7. $x^2 - 4x - 3$ and $(x - 3)(x + 1)$

APPLYING THE MATHEMATICS

8. Tile patterns are often used in bathroom flooring. Consider this sequence created with green and yellow hexagonal tiles.



a. Write an expression describing the number of green hexagonal tiles in the nth term of the sequence.

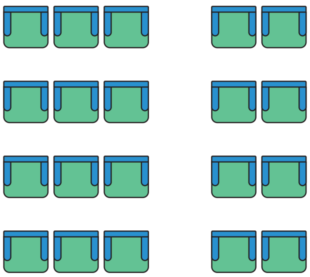
b. Write an expression describing the number of yellow hexagonal tiles in the nth term of the sequence.

c. Use your answers from Parts a and b to write an expression describing the total number of hexagonal tiles in the nth term of the sequence.

d. How many hexagonal tiles are in the 100th term?

9. Give a counterexample to show that the equation $(a - 5) + b = a - (5 + b)$ is not true for all real numbers a and b.

10. An airplane manufacturer sells small planes that have seats arranged so there is a center aisle. In the each row, there are 3 seats on the left side of the aisle and 2 seats on the right side of the aisle. If there are r rows of seats, find two different expressions for the total number of seats in a plane.



REVIEW

11. Describe the general pattern using one variable. **(Lesson 1-2)**

$$8^2 - 8 = 8(8 - 1)$$
$$30^2 - 30 = 30(30 - 1)$$
$$6.5^2 - 6.5 = 6.5(6.5 - 1)$$

12. Each morning, Crystal buys a cup of coffee for \$2.25. She uses a table to record her total coffee expenditures. **(Lesson 1-2)**

Day	Calculation	Cost
1	\$2.25	\$2.25
2	\$2.25 + \$2.25	\$4.50
3	\$2.25 + \$2.25 + \$2.25	\$6.75
4	?	?
5	?	?

a. Complete the table.

b. After one year how much will Crystal have spent on coffee?

c. After d days, how much will Crystal have spent on coffee?

Newer airplanes have seats that are made with a lightweight carbon fiber-reinforced frame that helps reduce weight and fuel costs.

Source: www.boeing.com

13. Give two instances of each pattern. **(Lesson 1-2)**
 a. $x^2 \cdot x = x^3$ **b.** $3g - g - g = g$ **c.** $n(3 + 8) = 3n + 8n$

14. **a.** Evaluate $\frac{1}{10}x^2$ when $x = 200$.
 b. Evaluate $\left(\frac{1}{10}x\right)^2$ when $x = 200$.
 c. Find a value of x so that the value of $\frac{1}{10}x^2$ is the same as the value of $\left(\frac{1}{10}x\right)^2$. **(Lesson 1-1)**

In 15–17, fill in the blank with =, <, or >. (Lesson 1-1)

15. $(-25)(-16)$ __?__ $-25 + -16$

16. $3^3 + 3^3$ __?__ 3^6

17. $(8 \cdot 6) \cdot 3 + 1$ __?__ $8 \cdot (6 \cdot 3) + 1$

18. Raul's Video Store has certain DVDs on sale for \$9.95, but all others are priced at \$14.95. Suppose a customer wishes to purchase 7 DVDs, 2 of which are on sale. What is the total cost? **(Lesson 1-1)**

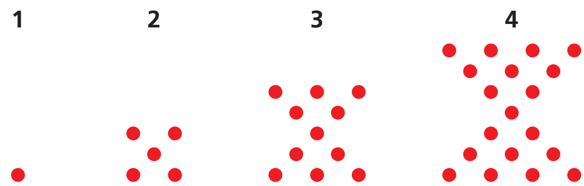
19. In 2006, the city of Los Angeles charged a sales tax of 8.25%. If you bought a pair of jeans in Los Angeles that cost \$35 before tax, what was the total cost after tax? **(Previous Course)**

A DVD has about 26 times the storage capacity of a CD.

Source: about.com

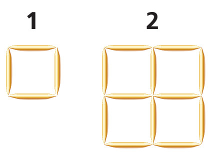
EXPLORATION

20. In this sequence of dots, two rows are added to each term to get the next term.



 a. Find the number of dots in the 5th and 6th terms.
 b. Find the number of dots in the 20th term.
 c. Try to find an expression for the number of dots in the nth term.

21. Two terms in a sequence of squares are shown.



 a. Draw the 3rd and 4th terms.
 b. Give the number of toothpicks in the 1st, 2nd, 3rd, 4th, and 5th term.
 c. In words, describe a method to find the number of toothpicks it takes to make the nth term.
 d. Write an algebraic expression to find the number of toothpicks in the nth term.

QUIZ YOURSELF ANSWERS

1a. When $w = 2$, $2w = 2 \cdot 2 = 4$ and $w^2 = 2^2 = 32 \cdot 2 = 4$.

1b. When $w = 5$, $2w = 2 \cdot 5 = 10$ and $w^2 = 5^2 = 5 \cdot 5 = 25$.

2. $(n + 3)^2$ and $n^2 + 6n + 9$ are equivalent. $(n + 3)^2$ and $n^2 + 9$ are not equivalent since $(1 + 3)^2 = 4^2 = 16$, $1^2 + 9 = 1 + 9 = 10$. $n^2 + 9$ and $n^2 + 6n + 9$ are not equivalent since $1^2 + 9 = 10$, $1^2 + 6 \cdot 1 + 9 = 1 + 6 + 9 = 16$. Thus, we know that $(n + 3)^2$ and $n^2 + 6n + 9$ are equivalent.