Lesson
1-2 Describing Patterns

Vocabulary

pattern
instance
define a variable
term
factor

BIG IDEA Patterns in tables are often described by expressions with variables.

Using Tables to Look at Patterns

Ian wants to save money to buy a new bike. He already has $25, and he decides he can save $15 each week. If the bike costs $220, will Ian have saved enough to buy the bike after 12 weeks?

To answer this question it may help to look at a table.

Number of Weeks (w)	Calculation	Pattern	Money Saved
0	25	$25 + 15 \cdot 0$	$25
1	$25 + 15$	$25 + 15 \cdot 1$	$40
2	$25 + 15 + 15$	$25 + 15 \cdot 2$	$55
3	$25 + 15 + 15 + 15$	$25 + 15 \cdot 3$	$70
4	$25 + 15 + 15 + 15 + 15$	$25 + 15 \cdot 4$	$85
5	$25 + 15 + 15 + 15 + 15 + 15$	$25 + 15 \cdot 5$	$100
6	$25 + 15 + 15 + 15 + 15 + 15 + 15$	$25 + 15 \cdot 6$	$115

The table shows the amount Ian has saved after 0, 1, 2, 3, 4, 5, and 6 weeks. The key to the table is the *pattern* column. In that column, the repeated adding of 15 is rewritten as multiplication. A **pattern** is a general idea for which there are many **instances.** The last row of the table shows one instance, stating that after 6 weeks, Ian has saved $25 + 15 \cdot 6$, or $115. The pattern lets you write an algebraic expression to describe the amount of money Ian has after *any* number of weeks. If the variable w is used to represent the number of weeks that have passed, an expression that describes the pattern is $25 + 15 \cdot w$, as shown in the table on page 14.

Mental Math

a. During a 25%-off sale, what is the cost of a pair of shoes that normally costs $80?

b. How much do you save on that pair of shoes?

Number of Weeks	Calculation	Pattern	Money Saved
w	$25 + \underbrace{15 + 15 + \ldots + 15}_{w \text{ addends}}$	$25 + 15 \cdot w$	$25 + 15w$

The expression $25 + 15w$ can be used to find out how much money Ian will have after 12 weeks. Replace w with 12 and evaluate $25 + 15 \cdot 12$. In 12 weeks, Ian will have $25 + 15(12)$, or \$205. Since the bike costs \$220, he will not have enough money after 12 weeks.

STOP **See Quiz Yourself at the right.**

▶ QUIZ YOURSELF

Assuming the pattern continues, will Ian have enough money after 14 weeks? How much money will he have saved up?

Notice that an important step is to define the variable used in the algebraic expression. To **define a variable** means to describe the quantity the variable represents. This is an important step. Defining a variable is often signaled by the word *let,* as in "Let $x = \ldots$".

GUIDED

Example 1

At the start of the school year, a school's library had a total of 3,600 individual magazines that it had collected over time. Each month 22 new magazines are added to the collection.

a. Complete the table to show the number of magazines in the library each month.

b. Define a variable and write an algebraic expression for the number of magazines.

Solution

a.

Months Since Start of School Year	Calculation	Pattern	Magazines in Library
0	?	$3{,}600 + 22(0)$	3,600
1	?	?	?
2	$3{,}600 + 22 + 22$	?	3,644
3	?	$3{,}600 + 22($? $)$	?

b. Let $m =$ ___?___.
The number of magazines after m months is ___?___.

Patterns Having Two or More Variables

Some patterns have two or more variables and can also be represented by tables and expressions.

Example 2

A family consisting of 2 adults and 3 children was planning a vacation. They looked in a tour book to find the cost of some activities.

a. Complete the table to show the family's cost for each activity.

Activity	Adult Ticket	Child Ticket	Family's Cost
Movie	\$15.00	\$8.00	$2 \cdot 15 + 3 \cdot 8$
Ferryboat	\$22.00	\$10.00	?
Water Park	\$10.00	\$4.50	?

b. How much does each activity cost the family?

c. Let a = the adult ticket price for an activity and let c = the child ticket price. Write an algebraic expression for the family's total cost for this activity.

Approximately 73 million people visited North American water parks during the summer 2004 season.

Source: World Waterpark Association

Solution

a. Multiply the cost of an adult ticket by the number of adults (2), and the cost of a child ticket by the number of children (3).

Activity	Adult Ticket	Child Ticket	Family's Cost
Movie	\$15.00	\$8.00	$2 \cdot 15 + 3 \cdot 8$
Ferryboat	\$22.00	\$10.00	$2 \cdot 22 + 3 \cdot 10$
Water Park	\$10.00	\$4.50	$2 \cdot 10 + 3 \cdot 4.50$

b. Movie: $2 \cdot 15 + 3 \cdot 8 = 30 + 24 = \$54$
Ferryboat: $2 \cdot 22 + 3 \cdot 10 = 44 + 30 = \$74$
Water Park: $2 \cdot 10 + 3 \cdot 4.50 = 20 + 13.50 = \$33.50$

c. The total cost is $2 \cdot a + 3 \cdot c$, or $2a + 3c$.

The Commutative Properties of Addition and Multiplication

Numbers or expressions that are added are called **terms.** Numbers or expressions that are multiplied are called **factors.** In the expression $2a + 3c$ of Example 2, there are two terms: $2a$ and $3c$. 2 and a are factors of $2a$, and 3 and c are factors of $3c$.

The expression $2a + 3c$ stands for "twice the cost of an adult ticket added to three times the cost of a child ticket." The order of operations indicates to do both multiplications before adding. But you can switch the terms being added because the total cost is the same whether you buy adult tickets first or child tickets first. The expression $3c + 2a$ gives the same values as $2a + 3c$.

It is also the case that you can switch the order of the factors. $2 \cdot a$ gives the same values as $a \cdot 2$. These examples are examples of the *commutative properties,* The word *commutative* comes from the French word *commutatif,* which means "switchable."

Commutative Property of Addition

For all real numbers a and b, $a + b = b + a$.

Commutative Property of Multiplication

For all real numbers a and b, $a \cdot b = b \cdot a$.

Examples of the commutative properties can also be numerical. They are true for all real numbers. For example, when $a = 72$ and $b = 10$, $72 \cdot 10 = 10 \cdot 72$.

Questions

COVERING THE IDEAS

In 1–4, give two instances of each pattern.

1. $y - y = 0$
2. $x + x = 2x$
3. $x \cdot x = x^2$
4. $\frac{x}{y} = x \cdot \frac{1}{y}$

In 5 and 6, describe the given pattern using one variable.

5. $(3 + 9) - 2 = 1 + 9$
   $(3 + 4) - 2 = 1 + 4$
   $(3 + 90) - 2 = 1 + 90$
6. $15 + 2 \cdot 15 = 3 \cdot 15$
   $\frac{1}{3} + 2 \cdot \frac{1}{3} = 3 \cdot \frac{1}{3}$
   $47.1 + 2 \cdot 47.1 = 3 \cdot 47.1$

7. Pearl is starting a lawn mowing business. She plans to spend $1,200 on advertising during the first few weeks that she is in business. Each week she spends $45 to place an ad in a newspaper.
   a. Give the amount of money she will have left to spend on advertising after 1, 2, and 3 weeks of advertising.
   b. Write an algebraic expression for the amount she will have left to spend after w weeks of advertising.

8. A group consisting of 1 adult and 5 children went to a special Kid's Day baseball game that offered special prices for children. Some prices are shown in the table below.

JACKSON FIELD
Home of the Barracudas

Item	Adult Price	Child Price	Group's Cost
Ticket	$17.00	$10.00	?
Hot Dog	$5.00	$3.50	?
Drink	$3.00	$2.00	?

   a. Fill in the group's cost for each item.
   b. Write an algebraic expression that would describe the group's cost for *any* item in terms of the adult price P and the child price p.

9. **Multiple Choice** Which expression below has three terms?

   A $4ab + 25$ **B** pqr **C** $2n + 6y + 15$ **D** $3x$

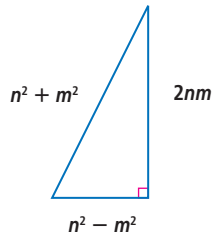
APPLYING THE MATHEMATICS

10. The following describes a pattern. A number is multiplied by 12, the product is divided by 2, and then 3 is subtracted from the result.
   a. Give three instances of the pattern.
   b. Write an algebraic expression to describe the pattern if the original number is n.

11. Istu is 5 years older than his sister Christine.
   a. Copy the table at the right and fill in Christine's age.
   b. Let i = Istu's age. Write an expression for Christine's age.
   c. Flor is 3 years older than Christine. Use your answer to Part b to write an expression for Flor's age.

Istu's Age	Christine's Age
9	?
16	?
25	?
89	?

12. Use the following information. When m is positive and n is greater than m, there exists a right triangle with side lengths given by the expressions $n^2 - m^2$, $2nm$, and $n^2 + m^2$, as shown in the diagram at the right.
   a. Let $n = 2$ and $m = 1$. Find the lengths of the three sides of the right triangle.
   b. Let $n = 3$ and $m = 2$. Find the lengths of the three sides of the right triangle.



13. A pizza is cut into pieces by making each cut pass through the center.

1 cut

2 cuts

3 cuts

a. Describe the number of pieces made for 1, 2, 3, 4, and 5 cuts.

b. Use two variables to describe the pattern.

14. a. Evaluate each expression.

$\frac{2}{3} \cdot \frac{3}{2}$ $\frac{23}{11} \cdot \frac{11}{23}$ $\frac{-7}{5} \cdot \frac{5}{-7}$ $\frac{0.03}{6} \cdot \frac{6}{0.03}$

b. Describe the pattern.

c. Do you think the pattern is true for all numbers? Why or why not?

15. Suppose $x = 58$ and $y = 31$.

a. Evaluate $xy - yx$.

b. Suppose the values of x and y are changed. Will the answer to Part a change? Why or why not?

REVIEW

In 16–17, evaluate when $x = 2$, $y = 8$, and $z = 4$. (Lesson 1-1)

16. $x^3 + \frac{y}{z}$

17. $y + z + \frac{x+z}{x} - z(z-1)^4$

18. The planet Neptune is approximately a sphere with radius (r) of 25,000 kilometers. Use the formula $V = \frac{4}{3}\pi r^3$ to estimate the volume V of Neptune in scientific notation. **(Lesson 1-1, Previous Course)**

19. Evaluate each of the following without a calculator. **(Previous Course)**

a. $3 \cdot -5$

b. $-70 \cdot -6$

20. Round π to the nearest thousandth. **(Previous Course)**

21. Often, 20% of a restaurant bill is left for a tip. If a bill is \$34.76, what would be a 20% tip rounded to the nearest dollar? **(Previous Course)**

22. ***Skill Sequence*** Find the sum. **(Previous course)**

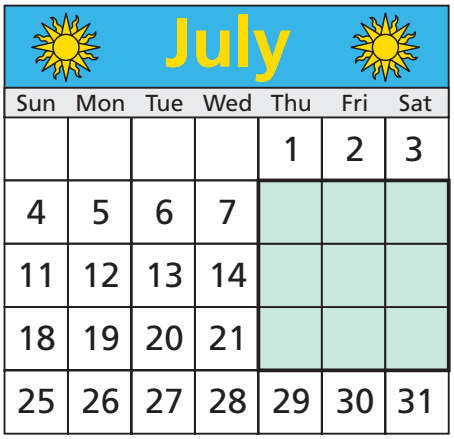
a. $\frac{3}{5} + \frac{4}{5}$

b. $\frac{3}{5} + \frac{4}{15}$

c. $\frac{3}{5} + \frac{4}{17}$

EXPLORATION

23. A monthly calendar contains many patterns.



July						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
				1	2	3
4	5	6	7			
11	12	13	14			
18	19	20	21			
25	26	27	28	29	30	31

a. Consider a 3×3 square such as the one drawn on the calendar. Copy the square and insert the nine dates. Then add the numbers along the diagonals. What is the relationship between the sums? Try this again with a different 3×3 square. Does it always seem to work?

b. In a 3×3 square portion of the calendar, if the middle date is expressed as N, then the date above would be $N - 7$ because it is 7 days earlier. Copy the chart at the right and fill in the other blanks.

?	$N - 7$	?
?	N	?
?	?	?

c. Show how your result from Part b can be used to explain your conclusion in Part a.

QUIZ YOURSELF ANSWER

yes; $235