# Lesson 1-1 Evaluating Expressions

## Vocabulary

variable

algebraic expression

evaluating the expression

**BIG IDEA** The order of operations is used to evaluate expressions with variables and expressions with numbers.

## Algebraic Expressions

Adults are roughly twice as tall as they were when they were 3 years old, as shown in the photograph at the right. This suggests a simple rule of thumb for predicting the adult height of a 3-year-old child: multiply his or her current height by 2. In the language of algebra, if $c$ represents the height of a 3-year-old child, then $2 \cdot c$, or $2c$ for short, is the child's predicted height as an adult. The letter $c$ is a *variable* and $2c$ is an *algebraic expression*. A **variable** is a letter or other symbol that can be replaced by any number (or other object) from a set. When numbers and variables are combined using the operations of arithmetic, the result is called an **algebraic expression.** Some examples of algebraic expressions are $7y$, $5x - 9$, and $\frac{4n^3p + 16}{z}$. Finding the numerical value of an expression is called **evaluating the expression.** To evaluate an algebraic expression, substitute numbers for its variables. The order of operations for algebraic expressions is the same as in numerical expressions.

### Mental Math

**a.** For how long did a gym class play softball if they played from 10:45 A.M. to 11:35 A.M.?

**b.** For how long did a cross country team run if they ran from 11:10 A.M. to 1:40 P.M.?

**c.** If a 1 hour, 45 minutes long gym class starts at 11:30 A.M., at what time does it end?

The father is approximately 2 times the height of his daughter.

### Order of Operations in Evaluating Expressions

1. Perform operations within parentheses or other grouping symbols.
2. Within grouping symbols, or if there are no grouping symbols:
   a. Evaluate all powers from left to right.
   b. Next multiply and divide from left to right.
   c. Then add and subtract from left to right.

## Activity

Suppose $a$ and $b$ are whole numbers with $a + b = 100$. What is the largest possible value of $a + ab - b$?

**Example 1**

**a.** If $a = 5$, find $3a^2$.

**b.** If $a = 5$, calculate $(3a)^2$.

**Solution**

**a.** Substitute 5 for *a*. There are no grouping symbols, so the power is the first operation to perform. Then multiply.

$3 \cdot 5^2 = 3 \cdot 25 = 75$

**b.** Substitute 5 for *a*. Perform the operation within the parentheses first. Then square the product.

$(3 \cdot 5)^2 = 15^2 = 225$

## Subtraction and Division Expressions

Subtracting a number yields the same result as adding the opposite of that number. For example, substitute 12 for *x* in the following equation.

| | |
|---|---|
| $35 - 4x = 35 - 4 \cdot 12$ | Substitute 12 for *x*. |
| $= 35 - 48$ | Multiply first. |
| $= 35 + -48$ | Rewrite using addition. –48 is the opposite of 48. |
| $= -13$ | Add. |

For some people, rewriting 35 – 48, as 35 + –48 makes the computation easier.

When subtracting a negative number, rewriting the expression is helpful.

| | |
|---|---|
| $-30 - -6 = -30 + 6$ | 6 is the opposite of –6. |
| $= -24$ | |

The relationship between addition and subtraction is true for all real numbers. It is known as the *algebraic definition of subtraction*.

**Algebraic Definition of Subtraction**

For all real numbers *a* and *b*, $a - b = a + -b$.

There is a similar relationship between multiplication and division. Dividing a number by *b* is the same as multiplying by the reciprocal of *b*, or $\frac{1}{b}$. For example, $32 \div 5 = 32 \cdot \frac{1}{5} = 6.4$.

**Algebraic Definition of Division**

For all real numbers *a* and *b* with $b \neq 0$, $a \div b = a \cdot \frac{1}{b}$.

Remember that there are several symbols for division. The operation $a \div b$ can also be written as $\frac{a}{b}$ or $a/b$. In all cases, $b$ (the denominator or the divisor) cannot be zero. For example:

$$5 \div 12 = \frac{5}{12} = 5/12 = 5 \cdot \frac{1}{12}$$

$$24 \div 6 = \frac{24}{6} = 24/6 = 24 \cdot \frac{1}{6}$$

$$\frac{1}{4} \div 3 = \frac{\frac{1}{4}}{3} = (1/4)/3 = \frac{1}{4} \cdot \frac{1}{3}$$

STOP **See Quiz Yourself at the right.**

*Quiz Yourself (QY) questions are designed to help you follow the reading. You should try to answer each Quiz Yourself question before reading on. The answer to the Quiz Yourself is found at the end of the lesson.*

**QUIZ YOURSELF**

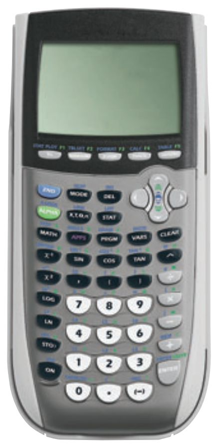
Complete the pattern as shown in the three examples above and at the left.

$$a \div b = \frac{a}{b}$$
$$= \underline{\ \ ?\ \ }$$
$$= \underline{\ \ ?\ \ }$$

## Evaluating with Technology

Scientific and graphing calculators use the algebraic order of operations. However, they can differ in the keys they use for squares, exponents, and square roots. Some possible key sequences are shown below. The order of entry is similar to the order you use to write the symbols by hand.

| Key | Example |
|---|---|
| squaring key | For $5^2$ enter 5 [$x^2$] to get 25. |
| exponent key | For $5^3$ enter 5 [^] 3 to get 125. |
| square root key | For $\sqrt{1{,}156}$ enter [√] 1156 to get 34. |



With most calculators, you must input fractions on one line. Therefore, you must include grouping symbols to show calculations within fractions. For example, to evaluate $\frac{24}{8-2}$, you must enter $24/(8-2)$ to get the correct answer 4. If you enter $24/8 - 2$, the calculator will follow the order of operations doing the division $24 \div 8$ first and then calculating $3 - 2$ to get 1 for the answer.

**GUIDED**

### Example 2

*A Guided Example is an example in which some, but not all, of the work is shown. You should try to complete the example before reading on. Answers to Guided Examples are in the Selected Answers section at the back of the book.*

**Use a calculator to evaluate $\left(\frac{6.8 - w}{n + w}\right)^3$ when $n = 21$ and $w = 0.5$.**

**Solution**

**Step 1** Write the expression. $\left(\frac{?}{?}\right)^3$

**Step 2** Substitute given values for the variables. $\left(\frac{6.8 - ?}{? + ?}\right)^3$

**Step 3** Enter into a calculator. Here is a start.

((6.8 – _?_)/_?_

**Step 4** Round the result to the nearest thousandth.

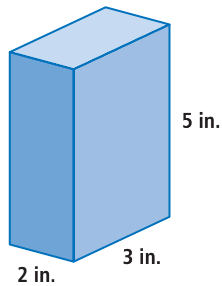
In Example 2 above, notice that *w* appears in the expression two times. If the same variable appears more than once in an expression, the same number must be substituted for it every time the variable appears.

## Evaluating Expressions in Formulas

Sometimes you will need to evaluate an expression as part of working with a formula. For example, the formula $V = \ell wh$ is used to find the volume $V$ of a rectangular solid with length $\ell$, width $w$, and height $h$ as shown in Example 3 below.

### Example 3

**Find the volume of the box shown at the right.**




**Solution** Substitute the given dimensions for $\ell$, $w$, and $h$ in the expression. Let $\ell = 2$ in., $w = 3$ in., and $h = 5$ in., and evaluate the expression $\ell \cdot w \cdot h$.

Volume $= \ell \cdot w \cdot h = 2 \cdot 3 \cdot 5 = 6 \cdot 5 = 30$

So, the volume is 30 $\text{in}^3$.

## Three Important Properties

In Example 3, we followed the order of operations and multiplied $2 \cdot 3 \cdot 5$ from left to right. However, you could first multiply 3 and 5 and then multiply the product by 2 to get the same answer, 30. This illustrates that $(\ell \cdot w) \cdot h = \ell \cdot (w \cdot h)$. This important general property, true for all real numbers, is called the *Associative Property of Multiplication*. Notice that the product of numbers $a$ and $b$ can be written as $ab$, $a \cdot b$, $a(b)$, or $(ab)$. The product is usually not written as $a \times b$ in algebra because $x$ is such a common letter for a variable.

**Associative Property of Multiplication**

For any real numbers $a$, $b$, and $c$, $(ab)c = a(bc)$.

A similar property is true for addition. You can regroup numbers being added without affecting the sum. For example, the sum $(67 + 98) + 2$ is easier to calculate in your head if the numbers being calculated are regrouped.

$$\begin{aligned}(67 + 98) + 2 &= 67 + (98 + 2)\\ &= 67 + 100\\ &= 167\end{aligned}$$

This is an example of the *Associative Property of Addition*.

**Associative Property of Addition**

For any real numbers $a$, $b$, and $c$, $(a + b) + c = a + (b + c)$.

Another property of algebra, called the *Transitive Property of Equality*, has been used throughout this lesson in evaluating expressions. It allows us to look at calculations like $20 - 46 = 20 + -46$ and $20 + -46 = -26$, and deduce that $20 - 46 = -26$.

**Transitive Property of Equality**

For any real numbers $a$, $b$, and $c$, if $a = b$ and $b = c$, then $a = c$.

## Questions

### COVERING THE IDEAS

*These questions cover the content of the lesson. If you cannot answer a Covering the Ideas question, you should go back to the reading for help in obtaining an answer.*

**In 1–6, evaluate the expression using the order of operations.**

1. $36 - 3 \cdot 4$
2. $36 \div 3 \cdot 4 - (-2)$
3. $(4 + 5)^2$
4. $4^2 + 5^2$
5. $10 - 70 - 5^{(2+1)} + 4(20)$
6. $-16 + 3(4 - 5) \div \frac{9 - 3}{17 - 13}$

**In 7 and 8, use the order of operations to evaluate the expression for the given values of *x*.**

7. $2(4x - 5) + 2$
   a. $x = 5$
   b. $x = -1$
8. $2^x - (1 + x)$
   a. $x = 1$
   b. $x = 3$

9. Evaluate each expression when $t = 10$.
   a. $5t^2$  b. $(5t)^2$

10. Evaluate $(-1 + r)^3$ when $r = \frac{1}{3}$. Write your answer as a fraction.

11. Evaluate $\frac{a + 2b}{5}$ when $a = 11.6$ and $b = 9.2$.

**In 12–14, use the Associative Properties of Addition and Multiplication to classify each statement as true or false. Then check your answer by doing the arithmetic.**

12. $5 \cdot (2 \cdot 7) = (5 \cdot 2) \cdot 7$

13. $-2 \cdot \left(\frac{1}{2} + 6\right) = \left(-2 + \frac{1}{2}\right) \cdot 6$

14. $13.5 + (-2 + 3 + -4) = (13.5 + -2 + 3) + -4$

## APPLYING THE MATHEMATICS

*These questions extend the content of the lesson. You should study the examples and explanations if you cannot answer the question. For some questions, you can check your answers with the ones in the back of this book.*

15. Joshua found that $\frac{63}{225} = \frac{7}{25}$. Sabrina realized that $\frac{7}{25} = \frac{28}{100}$.
   a. What can be deduced from Joshua's and Sabrina's results using the Transitive Property of Equality?
   b. $\frac{28}{100} = 28\%$. Is it true that $\frac{63}{225} = 28\%$? Explain your answer.

In 1926, Robert H. Goddard launched the first liquid-fueled rocket and laid the foundation for a technology that would eventually take humans to the moon.

Source: NASA

**In 16 and 17, which expression is *not* equal to the others?**

16. $27 - 3$  $-27 + 3$  $27 + -3$  $-3 + 27$

17. $-9 \div 4$  $-9 \cdot \frac{1}{4}$  $\frac{1}{-9} \cdot 4$  $\frac{-9}{4}$

18. When an object is shot from the ground into the air, the formula $h = -16t^2 + vt$ gives the height $h$ in feet of the object $t$ seconds later, where $v$ is the velocity of the object in feet per second when it first leaves the ground. If a toy rocket is launched with a velocity of 80 feet per second, find its height 2 seconds later.

19. Use the rule $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ to multiply the fractions $\frac{-2}{11}$ and $\frac{-3}{5}$. Write your answer as a single fraction.

20. Elias and Marissa are the same height. Samuel and Elias are equally tall.
   a. What conclusion can be made based on the Transitive Property of Equality?
   b. Write another real-world situation that uses the same property.

## REVIEW

*Every lesson contains review questions to practice ideas you have studied earlier.*

**In 21–23, compute in your head. (Previous Course)**

**21.** $-4 \cdot \$1.25$ **22.** $1{,}000 \cdot 11.4$ **23.** $3\frac{1}{2} \cdot 20$

**24.** Put these numbers in order from least to greatest. **(Previous Course)**

$12 \quad 5.3 \quad 2 \quad -\frac{2}{7} \quad -7 \quad 5.39 \quad -\frac{1}{6} \quad \pi$

**25.** Nikki's Bike Shop gives customers a free helmet with the purchase of any new bike from the store. Last week, 19 new bikes were purchased from Nikki's Bike Shop. If the cost of each helmet was $32.50, what was the total cost of all the helmets that the store gave away last week? **(Previous Course)**

**In 26–29, compute without a calculator. (Previous Course)**

**26.** $5 + -9 - 22$ **27.** $4 + 11 - -7$

**28.** $\frac{3}{4} + \frac{2}{3} - \frac{1}{6}$ **29.** $-3.52 - 11.4 + 30$

**Wearing a bicycle helmet while riding helps reduce injuries by up to 88%.**

Source: Bicycle Helmet Safety Institute

## EXPLORATION

*These questions ask you to explore topics related to the lesson. Sometimes you will need to use references found in a library or on the Internet.*

**30.** Suppose $a$, $b$, $c$, and $d$ are different positive integers whose sum is 100, and $a - c = 5$. What is the greatest possible value of $ab - cd$?

**QUIZ YOURSELF ANSWER**

$a/b$, $a \cdot \frac{1}{b}$