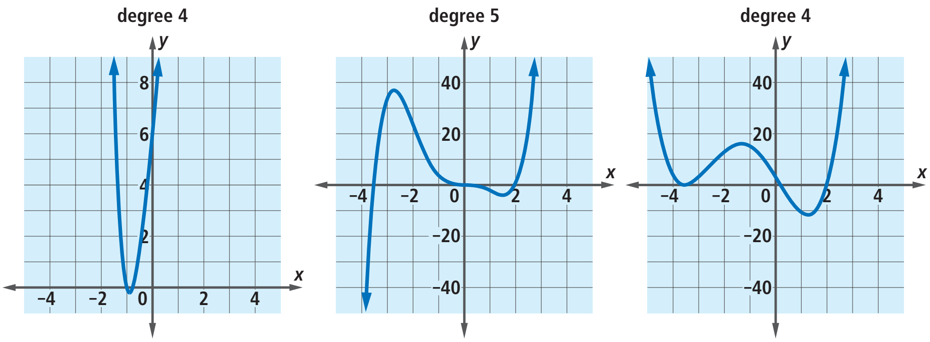
Chapter 11 Summary and Vocabulary

- A **polynomial in *x* of degree *n*** is an expression that can be written in the standard form

$$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0,$$

where $a_n \neq 0$. a_n is called the **leading coefficient**, and a_0 is the **constant term**. Polynomials written in factored form can be rewritten in standard form using the Extended Distributive Property. **Polynomial functions** include linear, quadratic, and functions of higher degree such as the ones graphed below.



- When a polynomial $P(x)$ of degree n is set equal to zero, the resulting equation has n **roots**, or **zeros**. The Fundamental Theorem of Algebra guarantees that $P(x) = 0$ has at least one complex root. The Factor Theorem states that if r is a root of $P(x) = 0$, then $P(x)$ can be factored as $P(x) = (x - r) \cdot Q(x)$. From this we can deduce that a polynomial function of degree n has exactly n complex roots, although some may be **multiple roots**.

- If the degree of a polynomial function is less then 5, formulas such as the Quadratic Formula can be used to find exact zeros. For polynomials of degree 5 or higher, such formulas do not exist. The Rational-Root Theorem provides a way to identify all the possible rational roots of a polynomial with integer coefficients. A CAS can be used to find exact or approximate roots, or to factor or expand polynomials.

Vocabulary

Lesson 11-1
degree of a polynomial
term of a polynomial
*polynomial in *x* of degree *n*
standard form of a polynomial
coefficients of a polynomial
leading coefficient
constant term, constant
polynomial function

Lesson 11-2
monomial, binomial, trinomial
degree of a polynomial in several variables

Lesson 11-3
factoring a polynomial
factored form of a polynomial
greatest common monomial factor
*prime polynomial, irreducible polynomial

Lesson 11-4
zero of a polynomial, root of a polynomial

Lesson 11-6
double root, root of multiplicity 2
multiplicity

Lesson 11-7
method of finite differences

- Polynomials arise directly from compound-interest situations, orbits, and questions about volume. They describe many numerical patterns. They can model many other real-world situations based on finite sets of data points. The degree of the model's polynomial can be found by the **method of finite differences**, and the coefficients of the polynomial can be found by solving a system of linear equations or polynomial regression.

Theorems and Properties

Extended Distributive Property (p. 739)
Difference of Squares Factoring Theorem (p. 747)
Binomial Square Factoring Theorem (p. 747)
Zero-Product Theorem (p. 752)
Factor Theorem (p. 753)
Rational-Root (or Rational-Zero) Theorem (p. 761)
Fundamental Theorem of Algebra (p. 766)
Number of Roots of a Polynomial Equation Theorem (p. 768)
Polynomial-Difference Theorem (p. 773)

Chapter 11 Self-Test

Take this test as you would take a test in class. Use the Selected Answers section in the back of the book to check your work.

1. Khalil has a 30-inch by 40-inch rectangular piece of cardboard. He forms a box by cutting out squares with sides of length x from each corner and folding up the sides. Write a polynomial formula for the volume $V(x)$ of the box.

2. **Multiple Choice** Which term appears in the expansion of $(a + b)(c + d + e)(f + g)$?

   A acf B bde

   C af^2 D ad^2g

3. Write $(z + 5)(z^2 - 3z + 1)$ in the standard form of a polynomial.

4. Give an example of a binomial with degree 4.

In 5 and 6, use these facts: When Francesca turned 16, she began saving money from her summer jobs. After the first summer, she saved \$680. After the second summer, she saved \$850. After the third, she saved \$1020, and after the following two summers she saved \$1105 and \$935, respectively. Francesca invested all this money at an annual percentage yield r and did not deposit or withdraw any other money.

5. If $x = (1 + r)$, write a polynomial in terms of x that gives the final amount of money in Francesca's account the summer after her 20th birthday.

6. How much money would she have the summer after her 20th birthday if she were able to invest all the money at an APY of 5.1%?

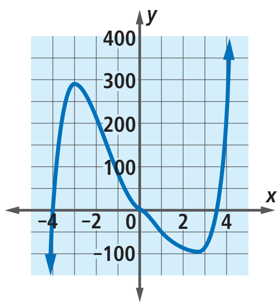
In 7 and 8, consider the polynomial function P where $P(x) = x^5 - 17x^2 + 4x^6 + 11$.

7. a. What is the degree of the polynomial?

   b. How many complex roots does the polynomial have?

8. State all rational roots of $P(x) = 0$ that are possible according to the Rational-Root Theorem.

In 9 and 10, consider the polynomial function with equation $y = x^5 - 11x^3 + 12x^2 - 35x - 12$. A graph of the function is given below.



9. a. How many real zeros does the function have? Assume no multiplicities are greater than 1.

   b. How many nonreal zeros does the function have?

10. a. List all possible rational zeros of the function according to the Rational-Root Theorem.

    b. Using the results of Part a and the graph, between what pairs of consecutive integers must irrational zeros be located?

In 11 and 12, consider the polynomial function g where $g(x) = (x - 2)^3(11x + 37)(x^2 - 7)$.

11. Find the zeros of g.

12. What is the multiplicity of each zero you found in Question 11?

13. Multiple Choice Suppose $x - r$ and $x - s$ are factors of a quadratic polynomial $P(x)$. Which of the following is *not* true for all x?

A $P(r) = 0$ B $k(x - r)(x - s) = P(x)$

C $P(s) = 0$ D $(x - r)(x - s) = 0$

14. Use the Rational-Root Theorem to prove that $\sqrt{21}$ is an irrational number.

In 15 and 16, consider the polynomial function r with $r(y) = y^5 - 5y^3 - 27y^2 + 135$.

15. a. Factor $r(y)$ over the rationals.

b. Factor $r(y)$ over the complex numbers.

16. Find all complex zeros of r.

17. Factor completely: $10uv^2w + 24vw$.

18. A polynomial function p has a root at $x = -2$ and a double root at $x = 6$. Write a possible equation for p and graph it.

19. A function f produces the table of values below.

n	1	2	3	4	5	6
$f(n)$	0	6	24	60	120	210

a. Can f be modeled by a polynomial?

b. If so, what is the smallest possible degree of the polynomial? If not, why not?

20. Find an equation for a polynomial function that describes the data points below.

t	–2	–1	0	4	2	3	4
r	12	4	0	0	4	12	24

Chapter 11 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Use the Extended Distributive Property to multiply polynomials. (Lesson 11-2)

In 1 and 2, write the polynomial in standard form.

1. $(x + 1)(x^2 - 2x + 3)$
2. $(a + 2)(a + 3)(a + 4)$

In 3 and 4, expand and combine like terms.

3. $(x - y)^2(x + y)$ 4. $(r + s + t)(r - s + t)$

OBJECTIVE B Factor polynomials. (Lesson 11-3)

Fill in the Blanks In 5 and 6, complete the factoring.

5. $15a^7b^{11} - 40a^{13}b^6 = \underline{\ ?\ }(3b^5 - \underline{\ ?\ })$
6. $z^2 + \underline{\ ?\ } + 81 = (z + \underline{\ ?\ })^2$

In 7–10, factor over the set of integers by hand and check with a CAS.

7. $x^2 - 6x + 9$ 8. $9m^2 - 48m + 64$
9. $p^4q^4 - 16$ 10. $x^2 + 5x + 6$

In 11–14, factor over the reals, if possible.

11. $a^2 + b^2$ 12. $y^2 + 7y + 10$
13. $37 + 3t - t^2$ 14. $z^2 - 17$

OBJECTIVE C Find zeros of polynomial functions by factoring. (Lessons 11-4, 11-6)

In 15–17, solve the equation and identify any multiple roots.

15. $0 = 6n(n + 3)(8n - 7)$
16. $0 = (t + 13)^3(2t - 3)^2$
17. $x^3 + 12x^2 + 36x = 0$

In 18 and 19, find the exact zeros of the polynomial function.

18. $f(x) = x(x - \pi)(2x + 1)$
19. $P(a) = a^4 - 25a^2$

OBJECTIVE D Determine an equation for a polynomial function from data points. (Lessons 11-7, 11-8)

20. Consider the polynomial function of smallest degree that models the data points below.

x	1	2	3	4	5	6
y	1	5	14	30	55	91

a. What is the degree?

b. **Multiple Choice** Which system of equations could be solved to find the coefficients of the polynomial?

i. $\begin{cases} 64a + 16b + 4c + d = 30 \\ 27a + 9b + 3c + d = 14 \\ 8a + 4b + 2c + d = 5 \\ a + b + c + d = 1 \end{cases}$

ii. $\begin{cases} 4x^3 + 4x^2 + 4x + 4 = 0 \\ 3x^3 + 3x^2 + 3x + 3 = 0 \\ 2x^3 + 2x^2 + 2x + 2 = 0 \\ x^3 + x^2 + x + 1 = 0 \end{cases}$

iii. $\begin{cases} a + b + c + d = 5 \\ a + 2b + c + d = 19 \\ a + 2b + 3c + d = 43 \\ a + 2b + 3c + 4d = 77 \end{cases}$

iv. none of these

c. Determine an equation for the polynomial function.

In 21–23, can the given relation be described by a polynomial function of degree ≤ 3? If so, find an equation for the function. If not, explain why not. Assume the first variable is the independent variable.

21.

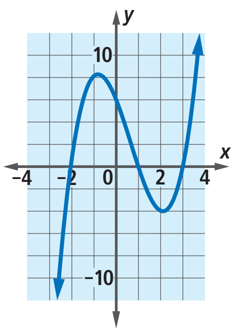
x	1	2	3	4	5	6
y	1	1	3	7	15	31

22.

x	1	2	3	4	5	6
y	56	58	62	74	100	146

23. the sequence defined by
$$\begin{cases} a_1 = 4 \\ a_n = 2a_{n-1} - 1, \text{ for integers } n \ge 2 \end{cases}$$

24. The graph of a function G below contains (–2, 0), (0, 6), (1, 0), and (3, 0). Suppose $G(x) = a_3x^3 + a_2x^2 + a_1x + a_0$.



a. What is the value of a_0?

b. Find the values of a_1, a_2, and a_3.

c. Give the value of $G(5)$.

PROPERTIES Principles behind the mathematics

OBJECTIVE E **Describe attributes of polynomials.** (Lessons 11-1, 11-2, 11-3)

In 25 and 26, state: a. the degree and b. the leading coefficient of the polynomial.

25. $18c^6 + 9c^4 + 2c + 15$

26. $3 + 2d - 2d^5 - 35d^9$

Multiple Choice In 27–29, choose the term that applies to the polynomial.

A monomial
B binomial
C trinomial
D none of these

27. $p^7 - 8$ 28. $17r^4t^5$ 29. $\frac{\pi}{h^2}$

30. Give an example of a trinomial with degree 7.

31. a. Is $x^2 - 42$ prime over the rationals? If not, factor it.

b. Is $x^2 - 42$ prime over the reals? If not, factor it.

OBJECTIVE F **Apply the Zero-Product Theorem, Factor Theorem, and Fundamental Theorem of Algebra.** (Lessons 11-4, 11-6)

In 32 and 33, explain why the Zero-Product Theorem cannot be used directly on the given equation.

32. $(x + 11)(x + 6) = 4$

33. $3(m + 2) - (2m - 6) = 0$

34. Suppose $g(t) = (t + 4)^3(t - 5)^a$. If g has degree 8, what is the value of a?

35. **True or False** If $(r - \pi)$ is a factor of a polynomial function V, then $V(\pi) = 0$.

36. **Multiple Choice** Suppose $p(x)$ is a polynomial, $p(r) = 0$, $p(s) = 0$, and $p(t) = 5$. Which of the following is *not* true?

A $p(s) \cdot p(r) = 0$

B $k(x - r)(x - s)(x - t) = p(x)$

C r and s are intercepts of the graph of $p(x)$.

D r and s are roots of the equation $p(x) = 0$.

37. Suppose a 7th-degree polynomial equation has 5 real roots and 2 irrational roots.

a. How many rational roots does the equation have?

b. How many nonreal complex roots does it have?

OBJECTIVE G **Apply the Rational-Root Theorem.** (Lesson 11-5)

38. True or False By the Rational-Root Theorem, $P(x) = 11x^2 - 5x + 3$ could have a rational root at $x = -\frac{11}{3}$.

39. a. List all possible rational roots of $R(x) = 2x^4 - 7x^3 + 5x^2 - 7x + 3$, according to the Rational-Root Theorem.

b. Find the rational zeros of R.

40. Use the Rational-Root Theorem to factor $z^4 - z^3 - 19z^2 - 11z + 30$ over the integers.

41. Consider $f(n) = 10n^5 - 3n^2 + n - 6$.

a. List all possible rational zeros of f, according to the Rational-Root Theorem.

b. Use a graph to explain why f has exactly one irrational zero.

42. Prove that $25^{\frac{1}{3}}$ is an irrational number.

USES Applications of mathematics in real-world situations

OBJECTIVE H **Use polynomials to model real-world situations.** (Lessons 11-1, 11-8)

43. The number G of games needed for n tic-tac-toe players to play every other player twice is given in the following table.

n	2	3	4	5	6
G	2	6	12	20	30

a. Find a polynomial function relating n and G.

b. Assume there are 24 students in your class. How many games will your class play if each student plays every other student twice?

44. Consider $75x^3 + 100x^2 + 150x + 250$.

a. Write a question involving money that could be answered by evaluating this expression.

b. Answer your question in Part a.

45. Gary's grandmother put \$250 in a savings account each year starting on Gary's tenth birthday. The account compounds at an APY of r.

a. Write a polynomial in x, where $x = 1 + r$, that represents the value of the account on Gary's fifteenth birthday.

b. If the account pays 6% interest annually, calculate how much money Gary would have on his fifteenth birthday.

c. If Gary had \$1700 on his fifteenth birthday, what rate of interest did he earn on his account?

46. Recall that when a beam of light in the air strikes the surface of water, it is refracted. Below are the earliest known data on the relation between i, the angle of incidence in degrees, and r, the angle of refraction in degrees, recorded by Ptolemy in the 2nd century CE.

i	10	20	30	40	50	60	70	80
r	8	15.5	22.5	29	35	40.5	45.5	50

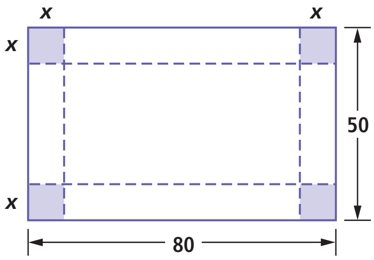
a. Can these data be modeled by a polynomial function?

b. If so, what is the degree of the function? If not, explain why not.

OBJECTIVE I **Use polynomials to describe geometric situations.** (Lesson 11-2)

47. A student forms a small open box out of a 3-inch by 5-inch index card by cutting squares of side length s out of the corners and folding up the sides. Write a polynomial for the volume $V(s)$ of the box.

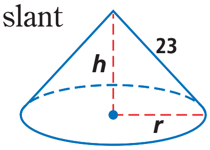
In 48 and 49, an open box is formed out of a 50-cm by 80-cm sheet of cardboard by removing squares of side length *x* cm from each corner and folding up the four sides.



48. Write an expression for the area of the bottom of the box.

49. Write a formula for the volume $V(x)$ of the box.

50. A right circular cone has a slant height $s = 23$.



a. Express its radius r in terms of its altitude h.

b. Use the results of Part a to express the volume V of this cone as a polynomial function in h.

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE J Graph polynomial functions. (Lessons 11-1, 11-4, 11-5)

In 51 and 52, an equation for a function is given. Graph the function. Then, use the graph to factor the polynomial.

51. $f(x) = 3x^3 - x^2 - 20x - 12$

52. $g(x) = x^3 - x^2 - 20x$

53. A polynomial function h with degree 4 has zeros at -1, 0, 1, and 3.

a. Write a possible equation for h in factored form.

b. Suppose that the leading coefficient of $h(x)$ is 3. Find an equation for h.

c. Graph your equation in Part b.

54. A polynomial function f of degree 3 has zeros at -4, -1, and 5.

a. Write an equation for one function satisfying these conditions.

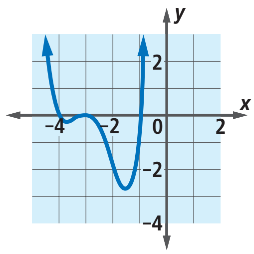
b. Graph the function in Part a.

c. Write the general form of an equation for f.

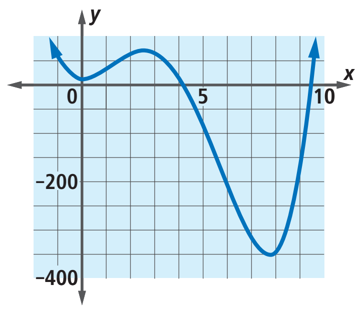
d. What do the graphs of all functions with equations of the form in Part c have in common with the graph in Part b?

OBJECTIVE K Estimate zeros of polynomial functions using graphs. (Lessons 11-4, 11-5)

55. A 4th-degree polynomial function with equation $y = g(x)$ and integer zeros is graphed at the right. List the four zeros.



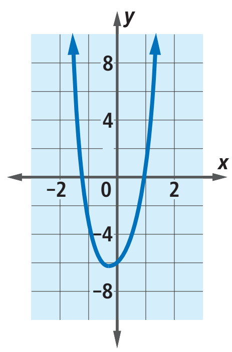
56. Refer to the graph of a function at the right. Name two pairs of consecutive integers between which a zero of f must occur.



57. Let $p(x) = 2x^4 + 3x^2 + 2x - 6$.

a. List all the possible rational roots of $p(x)$, according to the Rational-Root Theorem.

b. A graph of p is shown at the right. Based on the graph, which possible rational roots from Part a should you test? Explain.



c. Find all rational roots of $p(x)$.