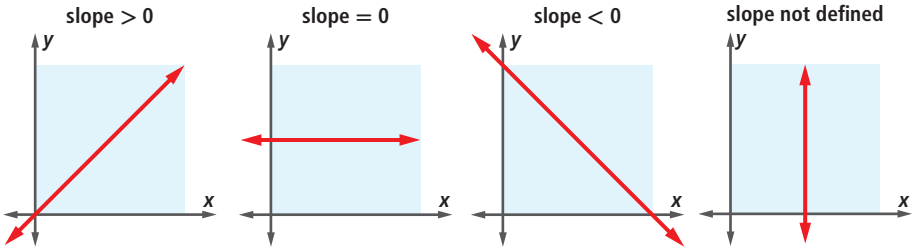
Chapter 6

Summary and Vocabulary

- The rate of change between two points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.
- When points all lie on the same line, the rate of change between them is constant and is called the slope of the line. The slope tells how much the line rises or falls for every move of one unit to the right. When the slope is positive, the line goes up and to the right. When the slope is negative, the line goes down and to the right. When the slope is 0, the line is horizontal. The slope of vertical lines is undefined.



- Constant-increase or constant-decrease situations lead naturally to linear equations of the form $y = mx + b$. The graph of the set of points (x, y) satisfying this equation is a line with slope m and y-intercept b. Linear-combination situations lead naturally to linear equations in the standard form $Ax + By = C$. When the $=$ sign in equations of either form is replaced by $<$ or $>$, the graph of the resulting linear inequality is a half-plane, the set of points on one side of a line. If the inequality is $\leq$ or $\geq$, the boundary line is included in the graph.
- A line is determined by any point on it and its slope or by two points on the line. Its equation can be found from this information. If data are roughly linear, they can be modeled by lines. One model, the line of best fit or least-squares line, is easily found using a calculator or computer.

Theorems and Properties

Slope-Intercept Equation of a Line (p. 350)
Point-Slope Equation of a Line (p. 357)

Vocabulary

6-1
rate of change
rate unit

6-2
slope

6-4
y-intercept
slope-intercept form
direct variation

6-5
point-slope form
x-intercept

6-7
line of best fit
linear regression
least squares line

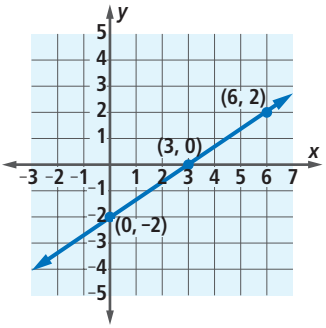
6-8
linear combination
standard form of an equation for a line
oblique

6-9
boundary line
half-planes
linear inequalities

Chapter 6 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

1. Refer to the line graphed below.



   a. Find its y-intercept.

   b. Find its x-intercept.

   c. Find its slope.

2. Do the points (8, 2), (18, –6), and (12, –1) lie on the same line? Justify your answer.

3. Find an equation of the line with slope $\frac{8}{9}$ and y-intercept –12.

4. The temperature at 8 A.M. is 58°F and is expected to rise 3°F each hour until 4 P.M. Write an equation in slope-intercept form to find the temperature T for each hour h from 8 A.M. to 4 P.M.

5. Describe the slope of every vertical line.

6. Rewrite the equation $8 - 3x = y$ in standard form $Ax + By = C$ and give the values of A, B, and C.

7. If the slope of a line is $-\frac{1}{2}$, how does the y-coordinate change as you go one unit to the right?

8. Find the slope and y-intercept of the line with equation $4x - 8y = 2$.

9. Lenny has $6,400 to spend at a music store buying trumpets and trombones for a jazz band. Suppose the cost of each trumpet p is $300 and the cost of each trombone b is $400.

   a. Write an inequality that describes all possible values of p and b.

   b. Graph these values.

In 10 and 11, use the following data of average yearly attendance at Pennsylvania State University football games.

Year	Average Attendance
1999	96,500
2000	95,543
2001	107,576
2002	107,239
2003	105,629
2004	103,111

Source: NCAA

10. Between which two consecutive years was there the greatest decrease in average yearly attendance?

11. What is the rate of change in average yearly attendance from 1999 to 2004?

In 12 and 13, graph the sentence.

12. $y = \frac{3}{8}x - \frac{5}{8}$

13. $y \leq 6 - 4x$

14. After driving for 22 miles, the gas tank in Masao's car had 14 gallons of gas. After 132 miles, the gas tank had 9 gallons of gas. Find a linear equation relating the amount of gas y in Masao's gas tank to the miles he has driven x.

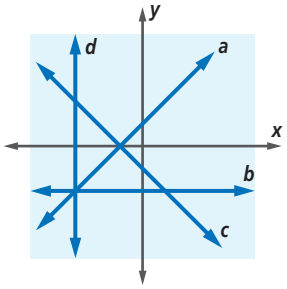
15. Find another point on the line through the point $(-4, 0)$ with slope $\frac{5}{4}$.

In 16 and 17, consider the data table and scatterplot that show the annual salary of employees of a company and the number of years they have worked for the company.

Years of Employment	Average Annual Salary (in thousands)
2	$35
3	$40
4	$39
5	$45
6	$60
7	$55
8	$62
9	$64
10	$68
12	$63
13	$75
15	$78
16	$80
17	$88
19	$82

16. a. Find the coordinates of two points on a line that you estimate would fit these data.
    b. Write an equation for your line in slope-intercept form.
    c. Use the equation from Part b to estimate the annual salary of an employee who has worked at the company for 25 years.
17. Find an equation for the line of best fit for the data.

In 18 and 19, use the graph of lines *a*, *b*, *c*, and *d* below.



18. a. Name the line that has a negative slope.
    b. Name the line that has a slope of 0.
19. a. Which line could represent money in an account over time where money is added to the account at a constant rate?
    b. Name the line that has an x-intercept but does not have a y-intercept.
20. Felipe has only x ten-dollar bills and y five-dollar bills in his wallet. Graph all possible values of x and y if the total amount of money in his wallet is less than $100.
21. Patrick left his apartment and headed down the building stairs. He descended at a constant rate of 3 flights per minute. After 4 minutes, he was on the 10th floor.
    a. Write an equation in point-slope form relating the floor Patrick was on, y, to the number of minutes, x, it has been since he left.
    b. Use your equation to find the floor of Patrick's apartment.

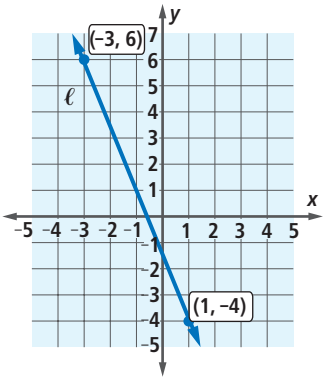
Chapter 6 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

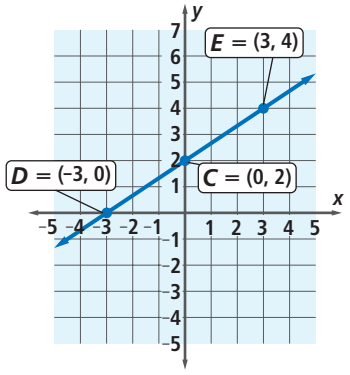
SKILLS Procedures used to get answers

OBJECTIVE A **Find the slope of the line through two given points. (Lesson 6-2)**

1. Calculate the slope of the line containing (2, 9) and (7, 15).
2. Calculate the slope of the line through (–5, 1) and (–8, –8).
3. Find the slope of line ℓ below.



4. Using two different pairs of points, show that the slope of line m below is $\frac{2}{3}$.



5. The points (3, –1) and (5, y) are on a line with slope 2. What is y?
6. The points (x, –6) and (2, 12) are on a line with slope $-\frac{3}{4}$. What is x?

OBJECTIVE B **Find an equation for a line given either its slope and any point or two points on it. (Lessons 6-4, 6-5, 6-6)**

7. Give an equation for the line with slope $2\frac{3}{5}$ and y-intercept 7.
8. What is an equation for the line with slope s and y-intercept t?

In 9–11, find an equation for the line through the given point with slope m.

9. (–5, 20), $m = -0.5$
10. (18, –16), $m = 0$
11. (30, 0.25), $m = 3$
12. What is an equation for the line through (19, –11) with undefined slope?

In 13–16, find an equation for the line through the two given points.

13. (3, 4), (1, 1)
14. (–16, –18), (0, 0)
15. (–3, –8), (–3, 8)
16. (1, 1), (2, 1)

OBJECTIVE C **Write an equation for a line in standard form or slope-intercept form, and using either form find its slope and y-intercept. (Lessons 6-4, 6-8)**

In 17 and 18, write the equation in the form $Ax + By = C$. Then give the values of A, B, and C.

17. $x + 14 = 2y$
18. $y = \frac{7}{16}x + 14$

In 19 and 20, rewrite the equation in slope-intercept form.

19. $8x + 4y = 10$
20. $30x - 90y = 270$

In 21–24, find the slope and *y*-intercept of the line.

21. $y = 12x - 6.2$

22. $3x - 6y = 2$

23. $x + y = 0$

24. $x = 3y + 500$

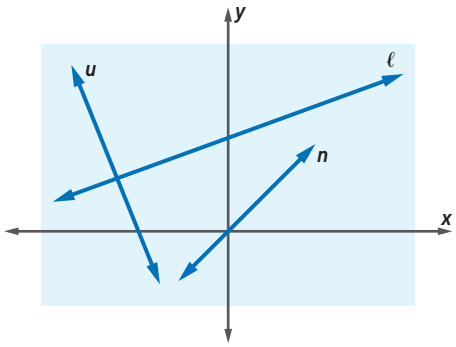
PROPERTIES The principles behind the mathematics

OBJECTIVE D Use the definition and properties of slope. (Lessons 6-2, 6-3)

25. **Fill in the Blanks** The slope determined by two points is the change in the ___?___ coordinates divided by the ___?___ in the *x*-coordinates.

26. **Fill in the Blanks** Slope is the amount of change in the ___?___ of the graph for every change of one unit to the ___?___.

In 27 and 28, five lines are shown below, including the *x*-axis and *y*-axis.



27. Which line or lines have negative slope?

28. Which line or lines have positive slope?

29. What is the slope of the line with equation $y = 5$?

30. What is true about the slope of the line with equation $x = 2$?

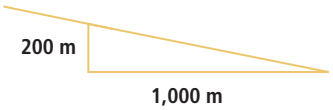
31. How can you use slope to show that three points do not all lie on the same line?

USES Applications of mathematics in real-world situations

OBJECTIVE E Calculate rates of change from real data and describe their real-world meanings. (Lessons 6-1, 6-3)

32. Consider the following ordered pairs of hours worked and pay received: (12, \$93.60), (15, \$117.00), (17, \$132.60).
    a. Calculate the slope of the line through these points.
    b. What does the slope represent in this situation?

33. The picture below represents a ski slope. What is the slope of this ski slope?



In 34–36, use the table below that shows the number of subscribers to a magazine.

Year	Subscribers
1985	1,145
1990	1,358
1995	1,601
2000	1,886
2005	2,023

34. Find the rate of change of the number of subscribers from 1985 to 1990.

35. Find the rate of change of the number of subscribers from 2000 to 2005.

36. a. According to these data, in which ten-year period did the number of subscribers increase the fastest?
    b. What is this rate of change?

In 37–39, use the table below of mean Fahrenheit temperatures each month in Fairbanks, Alaska, based on records from 1971–2000.

Month	Month Number	Mean (°F)
Jan.	1	–10
Feb.	2	–4
Mar.	3	11
Apr.	4	32
May	5	49
Jun.	6	60
Jul.	7	62
Aug.	8	56
Sept.	9	45
Oct.	10	24
Nov.	11	2
Dec.	12	–6

Source: National Climatic Data Center

37. Find the average rate of change of temperature per month from July to December.

38. Between which two months is the rate of change of temperature per month the greatest?

39. Between which two months is the rate of change of temperature per month the least (it will be negative)?

OBJECTIVE F **Use equations for lines to describe real situations. (Lessons 6-4, 6-5, 6-6, 6-8)**

In 40 and 41, each situation can be represented by a straight line. Give the slope and *y*-intercept of the line describing this situation.

40. Julie rents a truck. She pays an initial fee of \$29.95 and then \$0.49 per mile driven. Let y be the cost of driving x miles.

41. Nestor is given \$1,000 to spend on a vacation. He decides to spend at most \$100 a day. Let y be the minimal amount Nestor has left after x days.

42. The 28th Summer Olympic games were in 2004. The 27th summer Olympic games were 4 years earlier. Let y be the year of the nth summer Olympic games. Give a linear equation which relates n and y.

In 43 and 44, each situation leads to an equation of the form $y = mx + b$. Find that equation.

43. A stack of 25 small paper cups is 8 in. high. Each additional cup adds $\frac{1}{4}$ inch to the stack. Let y be the height of the stack when there are x cups.

44. A plane loses altitude at the rate of 5 m/sec. It begins at an altitude of 8,000 m. Let y be its altitude after x seconds.

45. Each month, about 50 new people move into a town. After 5 months, the town has 25,600 people. Write an equation relating the number of months m to the number of people p in the town.

46. On March 2, 1962, Wilt Chamberlain scored 100 points in a professional basketball game. At this time, only free throws worth one point and baskets worth two points were possible. (There were no 3-point shots.) Let F be the number of free throws and B be the number of baskets a team might make to score a total of 100 points. What equation do F and B satisfy?

47. Roberto baby-sat for \$7 an hour and worked in a store for \$8 an hour. He earned a total of \$820. Write an equation that describes the possible hours B of baby-sitting and hours S of store work that he could have worked at those jobs.

OBJECTIVE G Given data whose graph is approximately linear, find a linear equation to fit the graph and make predictions about data values. (Lesson 6-7)

48. Beef production in the United States has tended to increase since 1970. Here is the production in selected years.

Year	Beef Production (millions of pounds)
1970	21,684
1980	21,643
1990	22,743
1995	25,525
2000	27,338
2003	26,339

Source: U.S. Dept. of Agriculture

a. Graph the data and draw a line of fit.
b. Find an equation for your line.
c. Use the equation to predict the amount of U.S. beef production in 2010.
d. Calculate the line of best fit for these data. Use that line to predict the amount of U.S. beef production in 2010.

49. The table below shows the number of fish, crustaceans, and mollusks caught in the Philippines from 1997 to 2002.

Year	Weight (thousands of tons)
1997	327
1998	313
1999	353
2000	394
2001	435
2002	443

Source: Food and Agricultural Organization of the United Nations

a. Graph the data and eyeball a line to fit the data.
b. Find the slope of the fitted line.
c. Explain what the slope tells you about the trend in the data.
d. Find an equation for the eyeballed line.
e. Find an equation for the line of best fit for these data.
f. By how much do your line and the line of best fit differ in their predictions for the number of tons that would be caught in 2010?

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE H Graph a line given its equation, or given a point and its slope. (Lessons 6-3, 6-4, 6-8)

In 50–53, graph the line with the given equation.

50. $y = 3x + 5$
51. $y = -\frac{1}{2}x + 8$
52. $7x - 5y = 70$
53. $x + 2y = -6$

In 54–57, graph the line satisfying the given condition.

54. passes through (0, 1) with a slope of 0.4
55. passes through (4, 7) with a slope of –17
56. slope 8 and y-intercept –8
57. slope $-2\frac{1}{2}$ and y-intercept 10

OBJECTIVE I Graph linear inequalities. (Lesson 6-9)

58. Choose the correct words. The graph of a linear inequality with a $<$ sign is a (line, plane, half-plane) that (does, does not) contain its boundary line.
59. If you have only n nickels and q quarters and a total of less than \$1.00, graph all possible values of n and q.

In 60–65, graph the inequality on the coordinate plane.

60. $x \leq 12$
61. $y \geq 4.5$
62. $y \geq 2x + 1$
63. $y < -x + 5$
64. $2x - 5y < 10$
65. $x + 4y \geq 0$