Lesson 6-9 Graphing Linear Inequalities

Vocabulary

boundary line
half-planes
linear inequalities

BIG IDEA The two sides (half-planes) of the line with equation $Ax + By = C$ can be described by the inequalities $Ax + By < C$ and $Ax + By > C$.

Mental Math

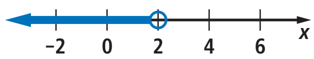
Find a single rule that describes each sequence. Then use your rule to find the next term in the sequence.

a. 2, 5, 8, 11, ...

b. 1, 2, 4, 8, 16, ...

c. 101, 103, 105, 107, 109, ...

In Chapter 3, you graphed solutions to inequalities on a number line. Recall that to graph an inequality such as $x < 2$ you first find the point where $x = 2$. Next, decide which part of the line contains the solution to the inequality. The sentence $x < 2$ states that we want values less than 2, so we shade the points to the left of 2. An open circle is placed on the boundary point 2, because 2 is not a solution to $x < 2$ (2 is not less than 2).



These ideas can be extended to graphs of inequalities in two dimensions. In this case, the boundary is a line instead of a point. We call this line the **boundary line.**

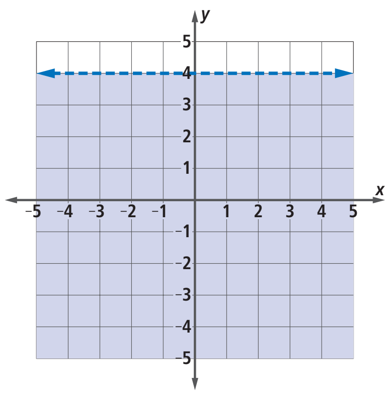
Inequalities Involving Horizontal or Vertical Lines

Example 1

Graph $y < 4$ on the coordinate plane.

Solution Graph the line $y = 4$. This horizontal line is the boundary line of the solution. The line is *dashed* to show that the points having a y-coordinate of 4 are not part of the solution set. The solution set consists of all points that have a y-coordinate less than 4. This is the region below the boundary line, so this region is shaded purple.

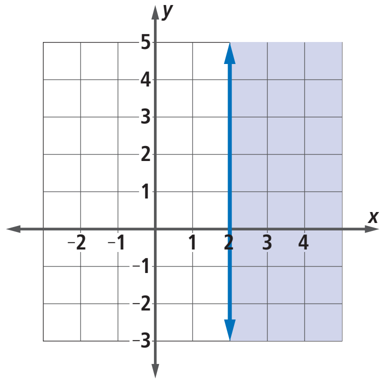
Check Pick a point in the purple shaded region. We choose (1, 1). Do the coordinates of this point satisfy the inequality $y < 4$? Yes, the y-coordinate is 1 and $1 < 4$.



The regions on either side of a line in a plane are called **half-planes.** The boundary line is the edge of the half-plane. In Example 1, the line $y = 4$ is the edge of the half-plane $y < 4$. If you were asked to graph $y \le 4$, then the boundary line $y = 4$ would be included and shown as a solid line.

Example 2

Write an inequality that describes the set of points in the shaded region.



Solution The boundary line is solid, which indicates that the edge $x = 2$ should be included. All points to the right of the line $x = 2$ are shaded purple, meaning every point in the half-plane has an x-coordinate greater than 2. So the sentence describing the region is $x \geq 2$. This region is the union of a half-plane and its edge.

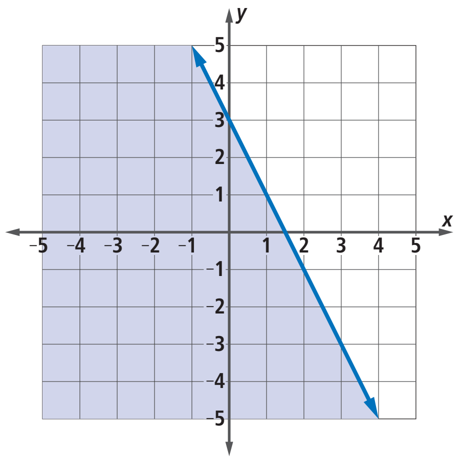
To distinguish $x > 2$ in the coordinate plane from $x > 2$ on a number line, we use set-builder notation. $\{(x, y): x \geq 2\}$ denotes a set of ordered pairs, so its graph is on the coordinate plane. $\{x: x \geq 2\}$ is a set of numbers, so its graph is on a number line.

Inequalities Involving Oblique Lines

Every line with an equation of the form $y = mx + b$ is the boundary line of the two half-planes described by $y < mx + b$ and $y > mx + b$.

Example 3

Draw the graph of $y \leq -2x + 3$.



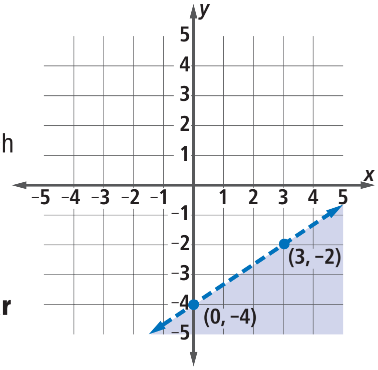
Solution Begin by graphing the boundary line $y = -2x + 3$. The $\leq$ sign indicates the points on the line should be included in the solutions, so make the line solid. We want the points whose y-coordinates are less than or equal to the y values that satisfy $y = -2x + 3$. Since y values decrease as x increases, shade the region below the line.

Check Pick a point in the shaded region. We choose (0, 0). Does it satisfy the inequality $y \leq -2x + 3$? Is $0 \leq 2(0) + 3$? Yes, 0 is less than 3, so the correct side of the line has been shaded.

If an inequality is of the form $y < mx + b$, then you shade below the line, and if the sentence is of the form $y > mx + b$, then you shade above the line. But when an inequality is in standard form $Ax + By < C$, you cannot use the inequality sign to determine which side of the line to shade. For this type of inequality, the method of testing a point is usually used. The point (0, 0) can be chosen if it is not on the boundary line. If (0, 0) is a solution to the inequality, then the half-plane that contains (0, 0) is shaded. If (0, 0) does not satisfy the inequality, shade the half-plane on the other side of the boundary line.

Example 4

Graph $2x - 3y > 12$.



Solution First graph the boundary line $2x - 3y = 12$. The line is dashed to show that the boundary line is not part of the solution. To determine which side of the line to shade, substitute (0, 0) into the original inequality. Is $2(0) - 3(0) > 12$? No, $0 \not> 12$. Since (0, 0) is in the upper half-plane and is *not* a solution, we shade the half plane that does not contain (0, 0).

Sentences equivalent to $Ax + By < C$ or $Ax + By \leq C$ are called **linear inequalities.** The preceding examples show that there are two steps to graphing linear inequalities.

Graphing Linear Inequalities

Step 1 Graph the corresponding linear equation. Make this boundary line dashed (for $<$ or $>$) or solid (for $\leq$ or $\geq$).

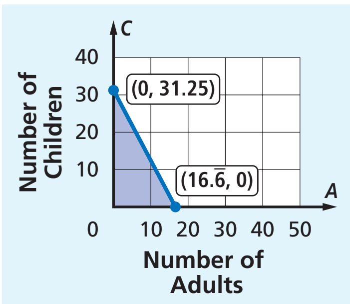
Step 2 Shade the half-plane that makes the inequality true. (You may have to test a point. If possible, use (0, 0).)

Sometimes the graph of all solutions is not an entire half plane.

Example 5

An elevator has a capacity of 2,500 pounds. If an average adult weighs 150 pounds and an average child weighs 80 pounds, how many adults and children can the elevator hold?

Solution Let A = the number of adults on the elevator. Let C = the number of children on the elevator. Then the total weight of the adults and children is $150 \cdot A + 80 \cdot C$ pounds. So the elevator can hold A adults and C children as long as $150A + 80C \leq 2{,}500$.



In 1852, Elisha Graves Otis invented the first safety brake for elevators.

Source: www.ideafinder.com

We can make either variable be first. We choose A to be the horizontal coordinate and C to be the vertical coordinate. Notice that the domain of both A and C is the set of nonnegative integers. So the graph will have no points in Quadrants II, III, or IV. To graph the boundary line $150A + 80C = 2{,}500$, we find its intercepts.

When $A = 0$, $C = \frac{2{,}500}{80} = 31.25$.

When $C = 0$, $A = \frac{2{,}500}{150} = 16.\overline{6}$.

(continued on next page)

Because the elevator can hold 0 adults and 0 children, (0, 0) is a solution. So the points will be those on or below the line $150A + 80C = 2{,}500$ in which both coordinates are nonnegative integers. Although the graph is shaded for all rational numbers in the region, integers are the only possible values for *A* and *C*.

STOP QY

▶ QY

If no children are on the elevator in Example 5, use the graph to find the maximum number of adults it can hold.

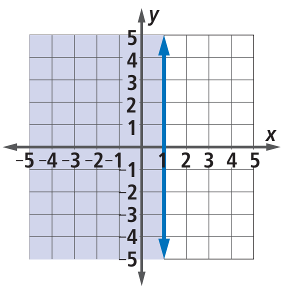
Questions

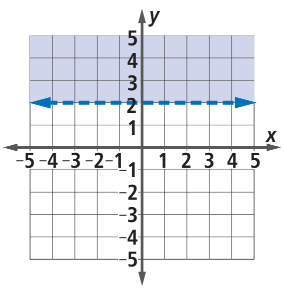
COVERING THE IDEAS

1. **a.** Graph $\{x: x > -7\}$
   b. Graph $\{(x, y): x > -7\}$

2. **a.** Graph $\{y: y < 0.5\}$
   b. Graph $\{(x, y): y < 0.5\}$

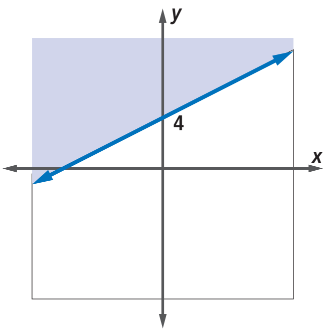
In 3 and 4, write an inequality describing the graph.

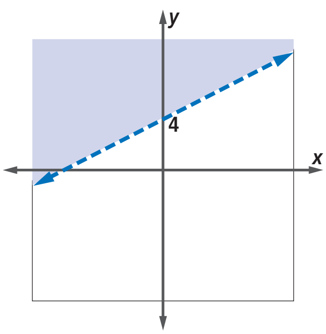
3. 

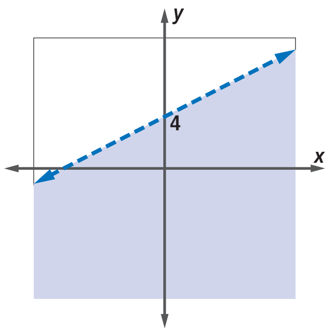
4. 

5. **a. Fill in the Blank** A line separates a plane into two distinct regions called ___?___.
   b. Fill in the Blank The line in Part a is called a ___?___ line.

6. Match the inequality with its graph.

a. 

b. 

c. 

i. $y < \frac{1}{2}x + 4$ ii. $y \le \frac{1}{2}x + 4$

iii. $y > \frac{1}{2}x + 4$ iv. $y \ge \frac{1}{2}x + 4$

7. What is the difference between the graphs of $y < -2x + 6$ and $y \le -2x + 6$?

8. **True or False** The ordered pair (1, 3) is a solution to $y < -2x + 6$.

In 9 and 10, graph all points (x, y) that satisfy the inequality.

9. $x + y < 3$

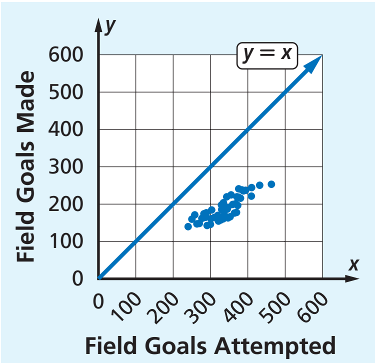
10. $y \geq 2x - 5$

11. A person in another country read Example 5 and felt the weights should be in kilograms rather than pounds. So here is a similar situation, but using kilograms. An elevator has a capacity of 1,100 kg. If an average adult weighs 65 kg and an average child weighs 35 kg, how many adults and children can the elevator hold? Answer this question with an appropriate graph.

APPLYING THE MATHEMATICS

12. The Strikers volleyball team is selling spirit items to raise money. Pom pons (p) cost \$7 each and "Go Team!" buttons (b) cost \$2.50 each. The team needs to make at least \$400. Graph the set of points (p, b) that satisfies these conditions.

13. The scatterplot at the right shows data from the 75 top-ranked players in NCAA Division I men's basketball in 2006. Each point shows the number of field goals a player attempted and the number that the player actually made.



Source: NCAA

a. Explain why there are no points above the line $y = x$.

b. Write an inequality that represents the half-plane bounded by the line $y = x$ and contains the data points.

c. Suppose a similar scatterplot of pairs (field goals attempted, field goals actually made) is made for a group of 8-year-old players. How would you expect the graph to look compared to the graph for the Division I players?

Approximately 14,578 high schools in the U.S. participate in girls' volleyball.

Source: National Federation of State High School Associations

14. Suppose a person has less than \$4.00 in nickels and dimes. Let n = the number of nickels and d = the number of dimes.

a. Write an inequality to describe this situation.

b. Give one example of a combination of nickels and dimes that satisfies the inequality.

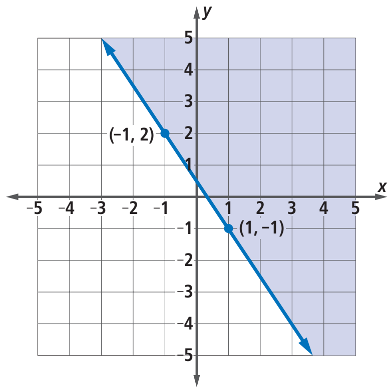
c. Graph the number of possible combinations of nickels and dimes.

15. Find a point that satisfies $y \leq x - 6$ but does not satisfy $y < x - 6$.

16. Refer to the graph at the right.

a. Find an equation of the boundary line.

b. Determine the inequality that is graphed.



REVIEW

17. Anna and Dion are selling cakes for a bake sale. Some cost \$8 and some cost \$12. They forgot to keep track of the number of each type of cake they sold, but when the bake sale was over they had collected \$288. Let x = the number of \$8 cakes, and y = the number of \$12 cakes. **(Lessons 6-8, 6-6)**

a. Write an equation in standard form describing the relationship between x and y.

b. Find the number of \$12 cakes sold if only \$12 cakes were sold.

c. Find the number of \$12 cakes sold if fifteen \$8 cakes were sold.

18. **a.** Rewrite $5x - 36 = 3y$ in standard form.

b. Give the values of A, B, and C from the standard form of an equation. **(Lesson 6-8)**

19. The weight of an object on the moon is one-sixth its weight on Earth. **(Lesson 3-4)**

a. Write an equation relating an object's weight y on the moon to its weight x on Earth.

b. How much will a 171-pound man carrying a 9-pound camera weigh on the moon?

c. If a backpack weighs 6.2 lb on the moon, how much does it weigh on Earth?

20. Evaluate each of the following. **(Lesson 1-1)**

a. 3^4 **b.** $(-4)^3$ **c.** $(-2)^8$

21. Rewrite each expression in decimal form. **(Previous Course)**

a. $1 \cdot 10^{-2}$ **b.** $6 \cdot 10^{-3}$ **c.** $3 \cdot 4 \cdot 10^{-6}$

EXPLORATION

22. In previous lessons you graphed equations like $y = |x - 3|$ using tables. Graph $y = |x - 3|$; then shade the appropriate region to represent the inequality $y > |x - 3|$. Check a point to verify that your shading is correct.

QY ANSWER

16 adults