

Lesson

6-8

Standard Form of the Equation of a Line

► **BIG IDEA** Every line has an equation of the form $Ax + By = C$.

In this chapter, lines have been used to describe situations involving a constant increase or decrease. Lines have also been used to model data in a scatterplot. The slope-intercept form $y = mx + b$ arises naturally from these applications. However, other situations can also lead to linear relations.

Linear Combination Situations

Lourdes had ordered party favors from an online store. When the 30 neon bouncing balls and the 12 glow-in-the-dark necklaces came, however, she had forgotten the price of each item. The bill said that the total cost was \$24. What was the cost of one neon ball? What was the cost of one glow-in-the-dark necklace?

We can describe this situation as “The cost of the neon balls plus the cost of the necklaces is \$24.” Let x = the cost of one neon ball and y = the cost of one glow-in-the-dark necklace. Then $30x + 12y = 24$.

$$\begin{array}{ccc}
 & \nearrow & 30x + 12y = 24 & \nwarrow \\
 \text{cost of 30 neon balls} & & \text{cost of 12 necklaces} & \text{total cost}
 \end{array}$$

This equation can be quickly graphed by finding its x - and y -intercepts.

To find the x -intercept, find x when $y = 0$.

$$\begin{aligned}
 30x + 12 \cdot 0 &= 24 \\
 30x &= 24 \\
 x &= 0.80
 \end{aligned}$$

The point is $(0.80, 0)$.

To find the y -intercept, find y when $x = 0$.

$$\begin{aligned}
 30 \cdot 0 + 12y &= 24 \\
 12y &= 24 \\
 y &= 2
 \end{aligned}$$

The point is $(0, 2)$.

Plot the two intercepts and draw the line. Negative numbers do not make sense for x or y , so we ignore the part of the line in Quadrants II or IV.

Vocabulary

linear combination
standard form of an equation for a line
oblique

Mental Math

Find the average speed of a car going

- 300 miles in 6 hours.
- 50 mph for an hour, then 70 mph for an hour.
- 55 mph for 2 hours, 45 mph for an hour, then 60 mph for 2 hours.

Each pair of possible prices (cost of one ball, cost of one necklace) corresponds to a point. As shown at the right, the point $(0.50, 0.75)$ lies on the graph. If a ball costs \$0.50 and a necklace costs \$0.75, then 30 balls and 12 necklaces cost \$24. Other possible (x, y) pairs can be found algebraically by first changing the equation of the line into slope-intercept form.

$$\begin{aligned} 30x + 12y &= 24 \\ 12y &= -30x + 24 \\ y &= -2.5x + 2 \end{aligned}$$

This is a formula that gives y , the cost of a necklace, in terms of x , the cost of a neon ball. Suppose a neon ball costs \$0.20. Then the cost of a necklace could be found by $y = -2.5(0.20) + 2 = -0.50 + 2 = \1.50 .

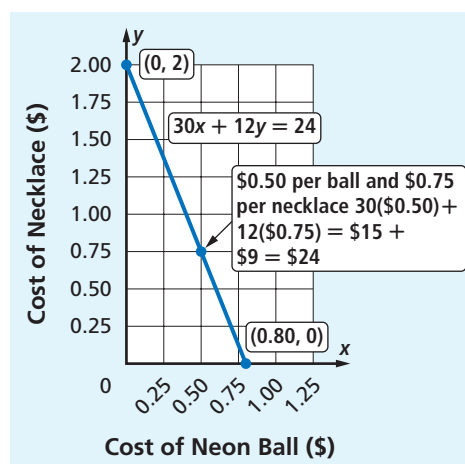
STOP QY1

An expression of the form $Ax + By$, where A and B are fixed numbers, is called a **linear combination** of x and y . The name *linear combination* is appropriate because when $Ax + By$ has a constant value, the graph of all ordered pairs (x, y) lies on a line.

The Standard Form of an Equation for a Line

The equation $3x - 4y = 24$ has the form $Ax + By = C$, where $A = 3$, $B = -4$, and $C = 24$. The variables x and y are on one side of the equation and the constant term C is on the other. The equation $Ax + By = C$, where A , B , and C are constants, is the **standard form of an equation for a line**. Linear combination situations naturally lead to equations of lines in standard form.

To graph a line whose equation is in standard form, you do not need to rewrite the equation in slope-intercept form. Instead, you can find the intercepts and draw the line that contains the intercepts.



► QY1

Use $y = -2.5x + 2$ to find another possible pair of costs for the neon balls and necklaces that Lourdes ordered.

Example 1

Graph $3x - 4y = 24$.

Solution

Find the x -intercept. Let $y = 0$.

$$\begin{aligned} 3x - 4(0) &= 24 \\ x &= 8 \end{aligned}$$

The x -intercept is 8, so the point $(8, 0)$ is on the line.

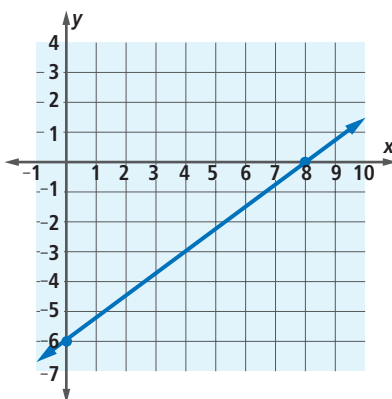
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Find the y -intercept. Let $x = 0$.

$$\begin{aligned} 3(0) - 4y &= 24 \\ y &= -6 \end{aligned}$$

The y -intercept is -6 , so the point $(0, -6)$ is on the line.

Plot $(8, 0)$ and $(0, -6)$ and draw the line through them, as shown.



Check Find a point satisfying the equation, and check that it is on the graph of the line. The point $(4, -3)$ is on the graph so it satisfies $3x - 4y = 24$.

STOP QY2

Rewriting Equations in Slope-Intercept and Standard Form

The idea of equivalent equations is useful in dealing with lines. In Example 1, the standard form equation $3x - 4y = 24$ is convenient to use to find the intercepts. But the slope-intercept equation is useful if you are graphing with a calculator, or if you need to know the slope. You should be able to quickly change an equation of a line into either of these forms. In standard form, the equation is usually written with A , B , and C as integers.

Example 2

Rewrite $y = \frac{4}{7}x + \frac{9}{7}$ in standard form with integer coefficients. Find the values of A , B , and C .

Solution

$$7y = 7 \cdot \frac{4}{7}x + 7 \cdot \frac{9}{7} \quad \text{Multiply each side of the given equation by 7 to clear the fractions.}$$

$$7y = 4x + 9 \quad \text{Simplify.}$$

$$-4x + 7y = 9 \quad \text{Add } -4x \text{ to both sides so the } x \text{ and } y \text{ terms are both on the left side of the equation. Write the } x \text{ term first.}$$

This is written in standard form with $A = -4$, $B = 7$, and $C = 9$. Some people prefer that A be positive. Then multiply both sides of the equation by -1 to obtain $4x - 7y = -9$.

QY2

Give the equation in standard form that fits the following situation and draw a graph. A fruit grower ships peaches and berries to grocery stores. A carton of peaches weighs 5 pounds and a carton of berries weighs 4 pounds. The shipment to one store weighed 90 pounds. Let x = the number of cartons of peaches and y = the number of cartons of berries.

Example 3

Rewrite $y = -0.75x$ in standard form with integer values of A , B , and C .

Solution

$$y = -0.75x$$

$$100y = -75x$$

$$75x + 100y = 0$$

Multiply each side by 100 to clear the decimal.

Add $-75x$ to both sides so the x and y are both on the left side. Write the x term first.

Here $A = 75$, $B = 100$, and $C = 0$.

In the Activity below, you will practice changing the form of equations. An equivalent form of the equation of a line describes the same set of points.

Activity

Five equations are given below. Fill in the table so that each equation is written in both slope-intercept form and in standard form.

	Equation	Slope-Intercept Form	Standard Form
1.	$20x - 5y = 35$?	?
2.	$y - 5 = 8(x - 1)$?	?
3.	$2x + 4(y + x) = 5y + 15$?	?
4.	$y = \frac{2}{3}x + 11$?	?
5.	$14x + 4y = 20$?	?

The lines graphed so far in this lesson are **oblique**, meaning they are neither horizontal nor vertical. In Lesson 4-2, you saw very short equations for horizontal and vertical lines. If a line is vertical, then its equation has the form $x = h$. If a line is horizontal, it has an equation $y = k$. These short equations are in standard form, but only one variable shows because one coefficient is zero. For example, $x = 4$ is equivalent to $x + 0y = 4$, where $A = 1$, $B = 0$, $C = 4$. The equation $y = -3$ is equivalent to $0x + y = -3$, where $A = 0$, $B = 1$, $C = -3$.

Thus, *every line* has an equation in the standard form $Ax + By = C$.

Questions**COVERING THE IDEAS**

1. What is the form $Ax + By = C$ called?

2. Refer to the following situation. The Hawkins family bought 3 sandwiches and 4 salads. They spent \$36. Let x = the cost of each sandwich and y = the cost of each salad.
- Write an equation to describe the possible combinations of costs for the sandwiches and salads.
 - Graph the equation from Part a.
 - If the salads cost \$6.30 each, how much did each sandwich cost?
 - Give the coordinates of the point on the graph corresponding to your answer to Part c.
 - Give another pair of possible costs for the sandwich and salad.

In 3 and 4, the equation is in standard form. Give the values of A , B , and C .

3. $5x - 3y = 9$

4. $8x + y = 2.4$

In 5 and 6, an equation for a line is given.

- Find the x - and y -intercepts of the graph of the line.
- Graph the line.

5. $2x + 5y = 20$

6. $3x - 2y = 12$

APPLYING THE MATHEMATICS

7. Refer to the following situation. Cheryl scored a total of 27 points in her basketball game last night. None of the points came from free throws. All of her points came from 2- or 3-point shots.
- Find three different combinations of 2- and 3-point shots that Cheryl may have had last night.
 - Write an equation in standard form to describe all the different possible combinations of 2-point shots and 3-point shots Cheryl may have made.
 - What is the greatest number of 3-point shots she may have made?
 - What is the greatest number of 2-point shots she may have made?
 - Graph the solutions to the equation in Part b.

In 8–10, an equation in slope-intercept form is given.

- Find an equivalent equation in standard form with integer coefficients.
- Give the values of A , B , and C .

8. $y = -\frac{4}{5}x + 10$

9. $y = \frac{9}{8}x$

10. $y = -x - 12$



Basketball is the most popular high school sport for girls with 452,929 participants in 2006.

Source: National Federation of State High School Associations

In 11 and 12, an equation in standard form is given. Find the equivalent equation in slope-intercept form.

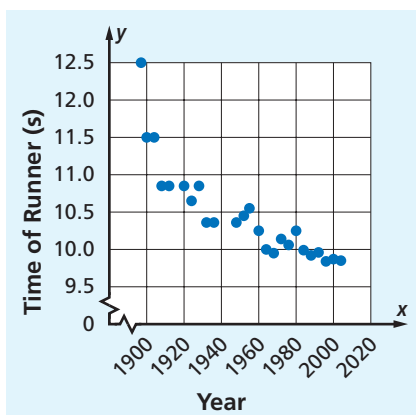
11. $4x - 7y = 308$

12. $x + y = 0$

13. On many multiple-choice tests, 1 point is given for each correct answer and 0.25 point is taken away for every wrong answer. (This is to discourage guessing.) Answers that are left blank do not affect the score. Gloria scored 62 on a test with 100 questions. Let C be the number of questions Gloria correctly answered and let W be the number of wrong answers she had.
- Give three possible pairs of values of C and W .
 - Write an equation that describes all possible solutions.
 - Graph all possible solutions.
14. Suppose $Ax + By = C$, with $B \neq 0$.
- Solve this equation for y .
 - Identify the slope and the y -intercept of this line.

REVIEW

15. The time of the winning runner in the men's 100-meter race has decreased since the first Olympics in 1896. The scatterplot below shows the time of the winning race in seconds in each Olympic race from 1900 through 2004. (Lesson 6-7)



Source: International Olympic Committee

- Trace the graph and fit a line to the data.
 - Find an equation for the line in Part a by approximating the coordinates of two points it goes through.
 - Use your equation to predict the time of the winning runner in the year 2012. Why might this prediction be incorrect?
16. Find an equation for the line through the points $(-3, 4)$ and $(6, 1)$. (Lesson 6-6)

17. The water in a 6-foot-deep pool is 8 inches deep. A hose is being used to fill the pool at the rate of 4 inches per hour. Let x be the number of hours passed and y be the depth (in inches) of the water in the pool. (Lessons 6-5, 6-2)
- Write an equation for a line which relates x and y .
 - Give the slope of the line.
 - Use your answer to Part a to find how long it will take to fill the 6-foot pool.
18. A robot is moving along a floor that has a coordinate grid. The robot starts at the point $(10, -4)$ and moves along a line with slope 3. Does the robot pass through the given point? Justify your answer. (Lessons 6-5, 6-2)
- $(12, 0)$
 - $(17, 17)$
19. What is the probability that you will randomly select a letter that is *not* U in the word ALBUQUERQUE? (Lesson 5-6)

In 20 and 21, tell whether $(0, 0)$ is a solution to the sentence. (Lesson 3-7)

20. $x - 2y < 3$

21. $5x > 4 + \frac{2}{3}y$

EXPLORATION

22. Explain why an equation for the line with x -intercept a and y -intercept b is $\frac{x}{a} + \frac{y}{b} = 1$. (This is called the *intercept form* of the equation for a line.)
23. a. On a graphing calculator, graph the lines with the following equations.
- $$3x - 2y = 6$$
- $$3x - 2y = 12$$
- $$3x - 2y = 18$$
- b. What happens to the graph of $3x - 2y = C$ as C gets larger?
- c. Try values of C that are smaller, including 0 and negative values. What can you say about the graphs of $3x - 2y = C$ in these cases?

QY ANSWERS

1. Answers vary. Sample answer: cost of a neon ball = \$0.30, cost of a necklace = \$1.25.
 $30(0.30) + 12(1.25) = 24$.
2. $5x + 4y = 90$

