Lesson

6-6 Equations for Lines through Two Points

▶ **BIG IDEA** An equation for the line through any two given points can be determined by first calculating the slope of the line.

A powerful approach to problem solving is to change the problem you are given into a simpler problem that you know how to solve. In Lesson 6-5, you learned how to find an equation of a line if you know its slope and one point on it. Now, consider a different problem. You are given two points and want to find the equation of the line that passes through them. With just a little work, you can turn this problem into a problem like one in the last lesson.

Example 1

Find an equation for the line that passes through (10, −4) and (18, 20).

Solution Use the coordinates of the given points to find the slope.

$$m = \frac{20 - -4}{18 - 10} = \frac{24}{8} = 3$$

Now you have more information than you need. You know two points and need to use only one of them to find an equation. Either point can be used. We choose to use (18, 20). Follow the algorithm from Lesson 6-5.

$y - k = m(x - h)$ Write the equation.
$y - 20 = 3(x - 18)$ Substitute $h = 18$, $k = 20$, and $m = 3$.
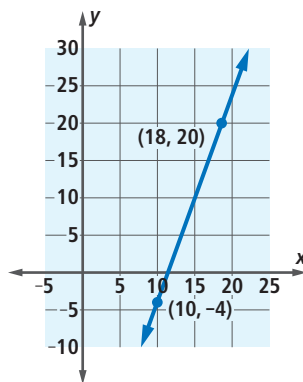
An equation of the line is $y - 20 = 3(x - 18)$.

Check Substitute the coordinates of the two points in the equation $y - 20 = 3(x - 18)$ to check if they produce true statements.

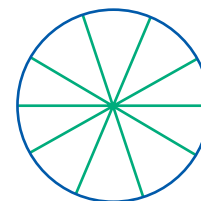Using (10, −4), does −4 − 20 = 3(10 − 18)?

$-24 = 30 - 54$ Yes, the equation checks.

Using (18, 20), does 20 − 20 = 3(18 − 18)?

$0 = 3 \cdot 0$ Yes, the equation checks.

Mental Math

Find the measure of the angle between spokes if the spokes are equally spaced around the circle and there are:

a. 3 spokes.
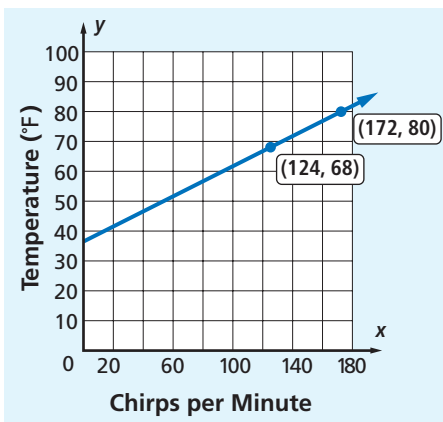
b. 10 spokes (pictured)

c. 15 spokes

d. n spokes

Relationships that can be described by an equation of a line occur in many places. Here is a relationship that may surprise you.

Example 2

Biologists have found that the number of chirps field crickets make per minute is related to the outdoor temperature. The relationship is very close to being linear. When field crickets chirp 124 times per minute, it is about 68°F. When they chirp 172 times per minute, it is about 80°F. Below is a graph of this situation.

Field crickets are the crickets everyone sees and hears in late summer and fall.

a. Find an equation for the line through the two points.

b. About how warm is it if an instrument records 150 chirps in a minute?

Solutions

a. Find the slope using the points (124, 68) and (172, 80).

$$\text{slope} = \frac{80 - 68}{172 - 124} = \frac{12}{48} = \frac{1}{4}$$

Substitute $\frac{1}{4}$ and the coordinates of (124, 68) into $y = mx + b$.

$$68 = \tfrac{1}{4}(124) + b$$

Solve for b.

$$68 = 31 + b$$
$$37 = b$$

Substitute for m and b.

An equation is $y = \frac{1}{4}x + 37$.

b. Substitute 150 for x in the equation $y = \frac{1}{4}x + 37$.

$$y = \tfrac{1}{4} \cdot 150 + 37$$
$$= 37.5 + 37$$
$$= 74.5$$

It is about 75°F when you hear 150 chirps in one minute.

STOP QY

> ▶ **QY**
>
> Check Example 2 by showing that (124, 68) is on the line with equation $y = \frac{1}{4}x + 37$.

The equation in Example 2 enables the temperature to be found for any number of chirps. By solving for x in terms of y, you could get a formula for the number of chirps expected at a given temperature. Formulas like these seldom work for values far from the data points. Field crickets tend not to chirp at all below 50°F, yet the formula $y = \frac{1}{4}x + 37$ predicts about 50 chirps per minute at 50°F.

GUIDED

Example 3

Suppose when using a calling card to call Iceland, you were charged $2.23 for a 10-minute call and $4.12 for a 20-minute call. Assume that this is a constant-increase situation.

a. Express the given information as two ordered pairs (minutes, cost).
b. Find an equation that expresses the cost y of the call in terms of x, the length of the call in minutes.
c. What does the y-intercept b represent in this situation?
d. What does the slope m represent in this situation?
e. How long could you talk for $25?

Solutions

a. Two points the line passes through are (__?__, __?__) and (__?__, __?__).
b. First, find the __?__, m.

$m = \dfrac{?\ -\ ?}{?\ -\ ?} = 0.189$

Substitute 0.189 and the coordinates of one of the points into $y = mx + b$.

__?__ $= 0.189(\underline{\ ?\ }) + b$

Solve for b.

Substitute the values found for m and b into the equation $y = mx + b$.

$y = \underline{\ ?\ }x + \underline{\ ?\ }$

c. The y-intercept represents the cost of calling for __?__ minutes. This is usually a charge that is automatic no matter how long the call lasts. In this case, the automatic charge is $0.34.
d. The slope represents the __?__.
e. Substitute $25 for __?__ in the equation $y = 0.189x + 0.34$ and solve for __?__. Solving this equation shows that you could talk __?__ minutes for $25.

The equation $y = 0.189x + 0.34$ in Example 3 gives the cost for talking any number of minutes. If the equation were solved for x in terms of $y,$ the result would be a formula for the number of minutes you could talk at a given cost.
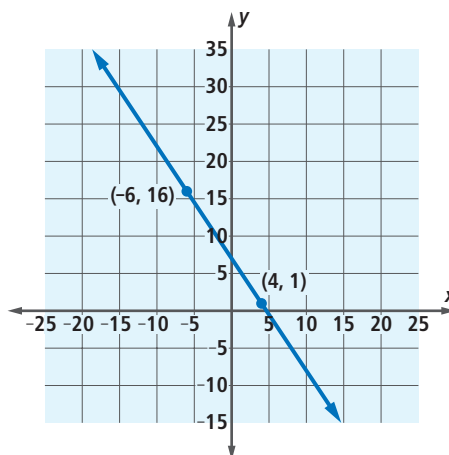
Questions

COVERING THE IDEAS

1. A katydid, or long-horned grasshopper, chirps about 70 times per minute at 70°F and 124 times per minute at 95°F.
   a. Find an equation relating x, the number of chirps per minute and y, the temperature in degrees Fahrenheit.
   b. Use your equation from Part a to estimate the temperature when a katydid cricket is chirping 90 times a minute.

2. Suppose a 15-minute call costs $5.26, and a 30-minute call costs $10.24. Find a formula relating time (in minutes) and cost (in dollars).

3. Find an equation for the line shown at the right.

In 4 and 5, find an equation for the line through the two given points. Check your answer.

4. $(3, 0)$, $(7, 16)$
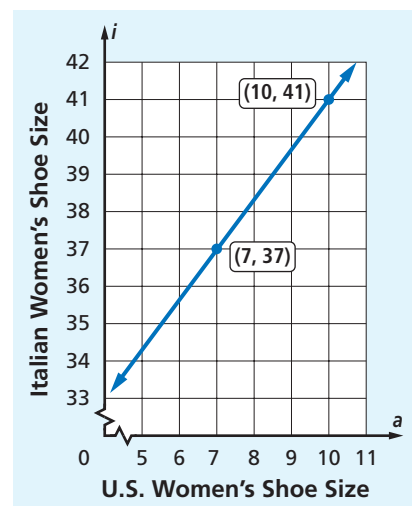5. $(4, 11)$, $(10, 5)$

APPLYING THE MATHEMATICS

6. Raini ran a marathon, which is 26.2 miles long. It took him 2 hours and 6 minutes to reach mile 13 on the race course and he crossed the finish line in 4 hours and 18 minutes. Raini tends to run at a constant speed.
   a. Express the given information as two ordered pairs (time, distance).
   b. Write an equation to find the distance Raini ran in terms of the time.
   c. What does the slope represent in this situation?
   d. How long did it take Raini to reach mile 7?

7. Write an equation for the line that produced the table of values shown below.

X	Y₁
30	27
40	32
50	37
60	42
70	47
80	52
90	57
X=30	

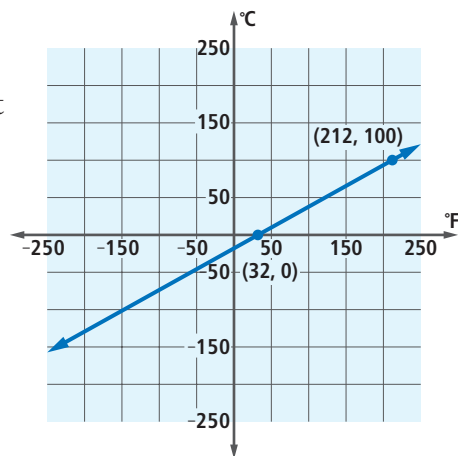8. Find an equation for the line with x-intercept 7 and y-intercept 3.

9. Penicillin was discovered in 1928. However, the medicine was not mass produced until the 1940s. In 1943, the price of a dose of penicillin was $20. By 1946, the price per dose was $0.55.

   a. Assuming the price of a dose of penicillin decreased at a constant rate, find an equation relating the year and price.

   b. According to the equation, what was the 1957 price of a dose of penicillin?

   c. Do you believe this equation gives good estimates for the price of penicillin? Why or why not?

10. a. Use the graph at the right to estimate the Italian shoe size that corresponds to a women's size 9 in the United States.

    b. Write an equation to relate women's shoe size in Italy to women's shoe size in the United States.

    c. Use your equation from Part b to find the Italian shoe size of a women's size 9 in the United States.

11. Old Faithful is a geyser in Yellowstone National Park that erupts often. The National Park Service has studied the length of the eruption and the amount of time between eruptions for many years. They have found that the relationship between how long an eruption lasts x and the amount of time until the next eruption y is approximately linear. One eruption lasted 2.1 minutes and the time before the next eruption was 59.24 minutes. Another eruption lasted 3.7 minutes and the time before the following eruption was 70.08 minutes.

    a. Write the data as two ordered pairs.

    b. Find the slope of this linear relationship.

    c. Explain what the slope represents in this situation.

    d. Find an equation for y in terms of x.

    e. If the park rangers posted a sign at the end of an eruption that said 94 minutes until the next eruption of Old Faithful, approximately how long had the previous eruption lasted?

An eruption at Old Faithful expels between 3,700 and 8,400 gallons of boiling water.

Source: National Park Service

12. The graph at the right shows the linear relationship between Fahrenheit and Celsius temperatures. The freezing point of water is 32°F and 0°C. The boiling point of water is 212°F and 100°C.

   a. Find an equation that relates Celsius temperatures C and Fahrenheit temperatures F.

   b. When it is 155°F, what is the temperature in degrees Celsius?

   c. When it is –10°C, what is the temperature in degrees Fahrenheit?

REVIEW

13. A cab company charges a base rate of $2.50 plus $1.60 per mile. A 15-mile cab ride costs $26.50.

   a. Write an equation relating the number of miles driven to the cost of the cab ride.

   b. Make up a question about this cab company that you can answer using your equation from Part a. (**Lesson 6-5**)

14. What is an equation for the line through the point (5, 5) with slope $-\frac{3}{5}$? (**Lesson 6-5**)

15. Graph $y = 3x + 9$ by using its slope and y-intercept. (**Lesson 6-4**)

16. The points (3, b) and (–2, 4) are on a line with slope $\frac{1}{2}$. Find b. (**Lesson 6-2**)

17. Interstate 70 leads from Denver, Colorado into the mountains. There are signs that post the elevation. (**Lesson 6-1**)

Miles from Denver	Elevation (ft)
0	5,260
33	7,524
45	8,512
47	9,100
72	9,042

Source: Colorado Tourism Office

A hiker at Alpine Ridge Trail's summit in Rocky Mountain National Park in Colorado.

   a. Between which two signs is the rate of change of elevation negative?

   b. Calculate the rate of change of elevation for the entire distance listed in the table.

18. Simplify $\frac{3a^2b}{c^2} \div \frac{12a}{c^4}$. (**Lesson 5-2**)

19. A rectangle is 3 cm longer than twice its width w. Its perimeter is 42 cm. (**Lesson 3-4**, **Previous Course**)

   a. Write an expression for the length ℓ in terms of w.

   b. Find its width and length.

   c. What is its area?

In 20–22, simplify the expression. (Lessons 2-2, 2-1)

20. $4(-7x) - 9(x - 1)$

21. $\frac{3}{4}(16g - h) + 4(2h)$

22. $-4p - 3(p - 2.5)$

EXPLORATION

23. In many places, a taxi ride costs a fixed number of dollars plus a constant charge per mile.

   a. Find a rate for taxi rides in your community or in a nearby place.

   b. Find an equation relating distance traveled and the cost of a ride.

QY ANSWER

Does $68 = \frac{1}{4} \cdot 124 + 37$?

$68 = 31 + 37$?

Yes, it checks.