Lesson 6-5

Equations for Lines with a Given Point and Slope

Vocabulary

point-slope form

x-intercept

BIG IDEA An equation for the line through any given point with any given slope can be determined with the slope-intercept form for the equation.

In Lesson 6-4, you saw that it is easy to find an equation of a line if you know its slope m and y-intercept b. This means that if you know the line's slope and the point $(0, b)$ where the line intersects the y-axis, then you can write an equation for the line. But what if you know the slope and some other point on the line, not necessarily the y-intercept? This is often the situation, as shown in Example 1.

Mental Math

Evaluate.

a. $3 \cdot 2\frac{2}{7}$

b. $\frac{15}{6} \cdot \frac{4}{5}$

c. $-1\frac{2}{3} \cdot 5$

Example 1

In 2006, the population of Alaska was 664,000. It was increasing by 7,800 people each year. Assuming this rate of increase remains steady, find an equation relating the population of Alaska y to the year x.

Solution 1 This is a constant-increase situation, so it can be described by a line with equation in the form $y = mx + b$. The rate 7,800 people/year is the slope, so $m = 7{,}800$. The population of 664,000 in 2006 is described by the point (2006, 664,000). You now have the slope and one point $(x, y) = (2006, 664{,}000)$. Follow these three steps.

Step 1 Substitute for m, x, and y. $664{,}000 = 7{,}800 \cdot 2006 + b$

Step 2 Solve for b.

$$664{,}000 = 15{,}646{,}800 + b$$

$$-14{,}982{,}800 = b$$

Step 3 Substitute the values for m and b in $y = mx + b$.

$$y = 7{,}800x - 14{,}982{,}800$$

Check After 3 years, the population should have increased by $7{,}800 + 7{,}800 + 7{,}800 = 23{,}400$ people to be 687,400.

When $x = 2009$, does $y = 687{,}400$?

Does $7{,}800 \cdot 2009 - 14{,}982{,}800 = 687{,}400$?

Yes, so it checks.

The solution to Example 1 begins with the slope-intercept form $y = mx + b$. Another method begins with the definition of slope.

Alaska's commercial fishing industry is the number one private sector employer in Alaska, providing more jobs than oil, gas, timber, and tourism.

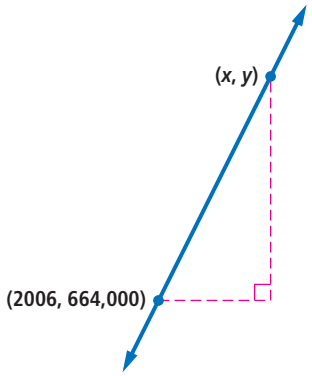
Source: Alaska Department of Fish and Game

Solution 2 This is a constant-increase situation. The amount of increase in a unit time, 7,800 $\frac{\text{people}}{\text{year}}$, is the slope, so $m = 7{,}800$. The given information means that the point (2006, 664,000) is on the line. Now picture the graph of the line, as shown at the right. If (x, y) is any point on the line, then

$$\frac{y - 664{,}000}{x - 2006} = 7{,}800.$$

Multiplying both sides by $x - 2006$, we obtain

$$y - 664{,}000 = 7{,}800(x - 2006).$$



The equation found for the line using the method of the second solution has the advantage of showing all the given information. It displays the slope and the given point on the line. The general form is called the **point-slope form** of an equation of a line.

Point-Slope Equation of a Line

The line through the point (h, k) with slope m has equation $y - k = m(x - h)$.

Either the slope-intercept method or the point-slope method of solving a problem like Example 1 enables you to find an equation for any nonvertical line if you know its slope and the coordinates of one point on it.

In Example 2, we use the point-slope equation method to find an equation for a line whose slope and *x-intercept* are known. An ***x*-intercept** of a graph is the x-coordinate of a point where the graph intersects the x-axis.

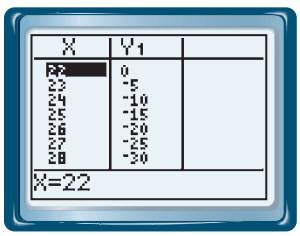
GUIDED

Example 2

A line has slope −5, and its *x*-intercept is 22. Find an equation for the line.

Solution Because the x-intercept is 22, the point (22, 0) is on the line. So $(h, k) = (22, 0)$. Since the slope is −5, $m = -5$. Use the point-slope form $y - \underline{?} = m(x - \underline{?})$.

Substitute for m, h and k. $y - \underline{?} = \underline{?}(x - \underline{?})$.



When you find the equation of a line from a point and a slope, you can use a graphing calculator to check your answer. In Example 2, the slope of −5 and the point (22, 0) are indicated by the problem. Enter the equation you found into your [Y=] menu. To see if the point (22, 0) lies on the line, set up a table whose first x-coordinate is 22. The table shows that indeed the line passes through the point.

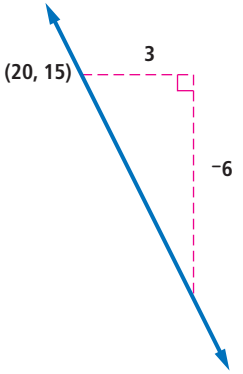
Questions

COVERING THE IDEAS

1. In 2000, Vancouver, Washington, had a population of 143,560 and was growing at a rate of about 9,700 people per year. Suppose that this rate of increase stays steady.
   a. Find an equation relating y, the population of Vancouver, to the year x, where x is the number of years since 2000.
   b. Predict the population of Vancouver in the year 2015.
   c. Predict which year the population of Vancouver will exceed 400,000.

In 2–5, find an equation of the line given the slope and one point on the line.

2. point (h, k); slope m
3. point $(-8, 6)$; slope -2
4. point $(-2, 0)$; slope $\frac{1}{3}$
5. point $(5, -\frac{1}{2})$; slope 0
6. The slope of a line is -5 and the x-intercept is 4. Find an equation for the line.
7. Write the equation of the line graphed at the right.
8. What is an equation of the horizontal line through the point $(3, 9)$?
9. What is an equation for the line with slope -4.3 that contains the point $(6.8, -3.0)$?



APPLYING THE MATHEMATICS

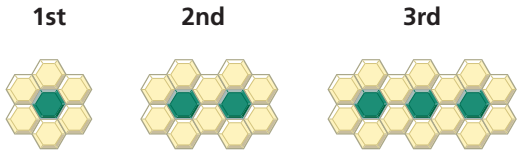
10. A mountain climber leaves his camp and hikes up the side of a mountain. His altitude increases at an average rate of 200 feet per hour. After 3 hours his altitude is 4,100 feet. Write an equation to find his altitude y after he has been hiking for x hours.
11. A newborn koala (age 0 months) is 2 cm long. Until maturity, it grows at an average rate of 1.5 cm per month. Koalas mature at about 4 years of age.
    a. From the given information, find an equation estimating the length y of a koala at age x months.
    b. About how long are mature koalas?
12. The slopes of two lines are reciprocals.
    a. An equation of one of the lines is $y = -8x - 9$. What is the slope of the second line?
    b. Find the equation of the second line if it passes through the point $(4, 6)$.

Koala bears are not bears. They are marsupials, like kangaroos. Baby koalas, called joeys, are the size of a jellybean at birth. A joey remains in the mother's pouch for about 22 weeks before emerging.

Source: *National Geographic*

13. Suppose a 5-week-old baby weighs 12 pounds and is gaining weight at the rate of 0.3 pound per week.
    a. The given information represents the slope m, age x, and weight y. Find the values of m, x, and y.
    b. Determine an equation for the baby's weight y at age x weeks.
    c. Determine the baby's weight at 12 weeks.

14. The following sequence, seen in Lesson 1-3, is made up of yellow and green hexagonal tiles. Find a formula for the number of tiles y in terms of n, and the term of the sequence.



15. Determine an equation for the line that contains (0, 3) and is parallel to the line with equation $y = 4x - 9.89$.

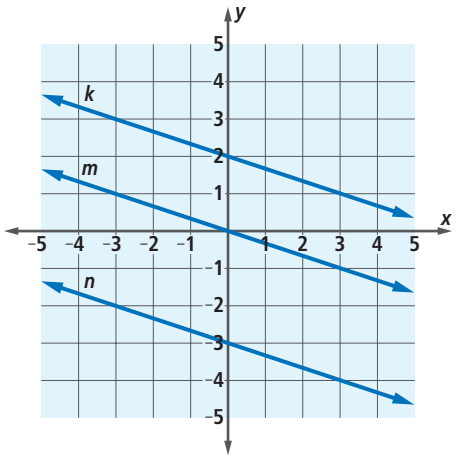
In 16 and 17, some information is given.
a. Write the slope and an ordered pair described by the information.
b. Write an equation relating x and y.

16. Kathy is on vacation and is spending \$75 per day. After 4 days she has \$320 left.

17. To buy a pass for the city bus system, you must pay an initial fee and then pay \$1.75 per ride. Dante paid \$29.25 for 15 rides.

REVIEW

18. Match each of the lines k, m, and n at the right with its equation. **(Lesson 6-4)**
    a. $y = -\frac{1}{3}x$
    b. $y = -\frac{1}{3}x - 3$
    c. $y = 2 - \frac{1}{3}x$

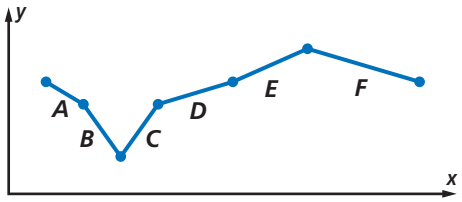


19. Graph the line with equation $y = -x$. **(Lesson 6-4)**

20. Do the points (–4, 3), (0, 2), and (4, –7) lie on the same line? Justify your answer. **(Lesson 6-2)**

21. The following two points give information about prices of corn: (4 bushels, \$9.24), (12 bushels, \$27.72). Calculate the rate of change and describe what it stands for. **(Lesson 6-1)**

22. Which section(s) of the graph below shows the
    a. fastest increase?
    b. slowest decrease? **(Lesson 6-1)**



23. ***Skill Sequence*** Simplify each expression. **(Lesson 5-2)**
    a. $\frac{12}{5} \div \frac{12}{6}$ b. $\frac{x}{5} \div \frac{x}{6}$ c. $\frac{x}{y} \div \frac{x}{z}$

24. Phone Company A charges \$4.99 per month for a special 7¢ per minute rate. Company B charges 10¢ per minute with no monthly fee. Suppose you talk for T minutes in a particular month.
    a. What will it cost you if you have signed up with Company A?
    b. What will it cost you if you have signed up with Company B?
    c. How many minutes must you talk before you pay less with Company A's rate than with Company B's rate? **(Lessons 4–5, 3–4, 1–2)**

EXPLORATION

25. Use the Internet or another resource to find the population at the last census of the state where you live.
    a. Find how much your state population has changed since the previous census. Assume the rate of change is constant and that it continues.
    b. Using your estimate, find an equation relating the population y to the year x.
    c. Use this equation to estimate what the population of your state will be when you are 50 years old.