Lesson 6-4 Slope-Intercept Equations for Lines

Vocabulary

y-intercept
slope-intercept form
direct variation

BIG IDEA The line that contains the point (0, *b*) and slope *m* has equation $y = mx + b$.

You have already worked with equations whose graphs are lines. In the following activity, you will use *dynamic graphing software* to experiment with equations. As you change the equation, watch how its graph changes in response. In many software applications, *sliders* allow you to change values in an equation. A slider consists of a portion of a number line that is used to control numeric values for a specific variable. As you drag a point along a slider, the value of a corresponding variable changes automatically.

Mental Math

Express as a mixed number.

a. 175 cm in meters

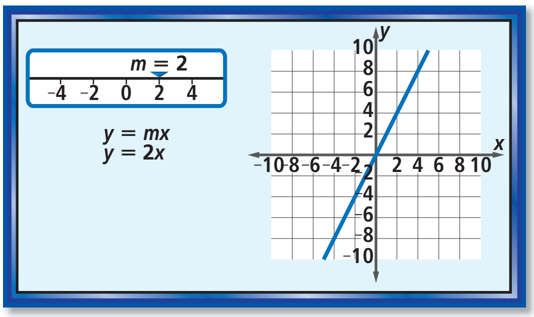
b. 35 ounces in cups

c. 5 feet, 7 inches in feet

Activity

Step 1 Create a slider for the variable *m*. Set the software so that *m* can vary in the interval $-5 \le m \le 5$. Move the slider so that $m = 2$.

Step 2 Enter the equation $y = mx$. Since $m = 2$, you have created the graph of $y = 2x$.



Step 3 Slowly move the slider to increase the value of *m*. What happens to the graph as *m* increases?

Step 4 Now move the slider so $m = 0$. What is the graph like when $m = 0$? Write an equation for this line.

Step 5 Move the *m* slider to the left of zero (into the negatives). What happens to the graph when *m* is negative?
What happens to the graph as *m* moves more and more to the left (“farther” into the negative values)?

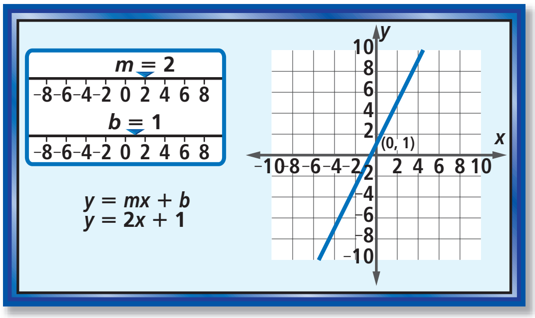
What appears to be true about the line when m is

a. positive?

b. negative?

c. zero (which is neither positive nor negative)?

Step 6 Move the slider so that $m = 2$. Enter the equation $y = mx + b$ and create a second slider for the variable b, using the interval $-5 \le m \le 5$. Move this slider so $b = 1$. Also, graph the point $(0, b)$.



Step 7 Slowly move the b slider to the right (toward greater values of b). What happens to the graph as b increases?

Step 8 Now move the b slider so $b = 0$. Describe the graph when $b = 0$ and write the equation for this line.

Step 9 Slide the b slider to the left of zero (into the negatives). What happens to the graph when b is negative? What happens to the graph as b moves farther to the left? What appears to be true about the line when b is

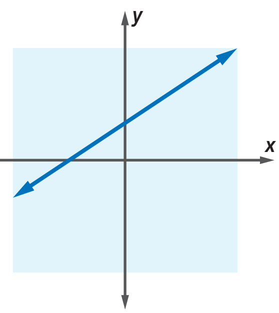
a. positive?

b. negative?

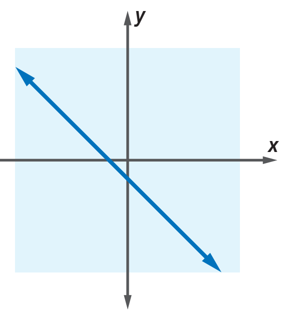
c. zero (which is neither positive nor negative)?

Step 10 Refer to the graphs below. Move both sliders until you have a graph that resembles the line pictured. Give the values of m and b. Write the equation for the line that is shown by using your slider values.

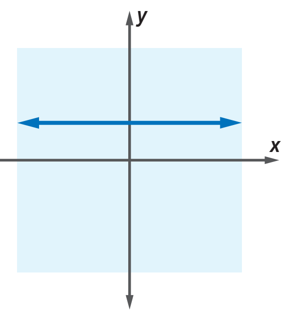
a.



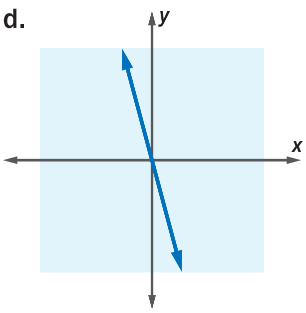
b.



c.



d.



In the Activity, you should have seen that the graph of a line whose equation is of the form $y = mx + b$ is determined by the values of m and b. In Steps 1–5, as you changed the value of m, the slope of the line changed.

When $m = 2$ and you varied the value of b in Steps 6–10, the line shifted and the point at which it crossed the y-axis changed. That number is the *y-intercept* of the line. In general, when a graph intersects the y-axis at the point $(0, b)$, the number b is a **y-intercept** for the graph. Each line at the right has slope $\frac{1}{2}$, but the y-intercepts are different.

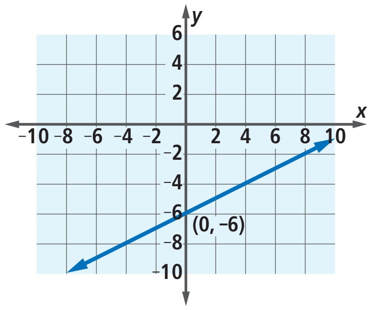
So the equation $y = mx + b$ shows the slope of the line and its y-intercept. For this reason, $y = mx + b$ is called the **slope-intercept form** for an equation for a line.

Slope-Intercept Equation of a LIne

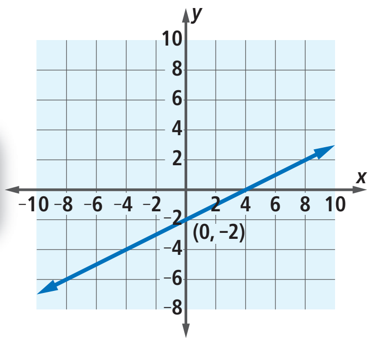
The line with equation $y = mx + b$ has slope m and y-intercept b.

STOP **QY1**

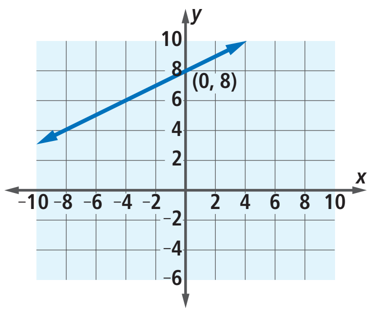
$b = -6$



$b = -2$



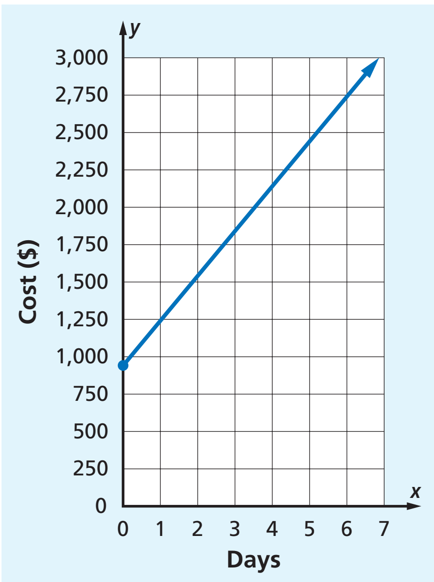
$b = 8$



Using the Slope-Intercept Form of the Equation of a Line

Slopes and y-intercepts can be seen in equations of real-world situations. Suppose Jody is going on a vacation to Istanbul. The airfare is \$940 and she expects to spend \$300 per day for hotel and expenses. Then after x days, the total cost y of her trip is given by $y = 940 + 300x$. With the Commutative Property of Addition, $y = 940 + 300x$ becomes $y = 300x + 940$, which fits the slope-intercept form $y = mx + b$.

Notice how the two key numbers in Jody's expenses appear in the equation. Her starting cost of \$940 is the y-intercept. Her \$300 cost per day is the slope.



QY1

a. Find the slope and y-intercept of $y = \frac{1}{4}x + 11$.

b. Write an equation in slope-intercept form for the line with slope 3 and y-intercept of -7.

The same line can have many equations. For example, $3x + y = 7$, $30x + 10y = 70$, and $y = -3x + 7$ are all equations for the same line.

QY2

Slope-intercept equations allow us to get information about a line quickly. For this reason, it is often helpful to convert other equations for lines into slope-intercept form.

QY2

Give the slope and y-intercept of $y = -2 - 6x$.

Example 1

Write the equation $3x + 7y = 9$ in slope-intercept form. Give the slope and y-intercept of the graph.

Solution Solve $3x + 7y = 9$ for y.

$$3x + 7y = 9$$
$$7y = -3x + 9$$
$$\frac{7y}{7} = \frac{-3x + 9}{7}$$
$$y = -\frac{3}{7}x + \frac{9}{7}$$

The slope is $-\frac{3}{7}$. The y-intercept is $\frac{9}{7}$ or $1\frac{2}{7}$.

Recall that every vertical line has an equation of the form $x = h$, where h is a fixed number. Equations of this form clearly cannot be solved for y. Thus, equations of vertical lines cannot be written in slope-intercept form (and they cannot be graphed on many graphing calculators). This confirms that the slope of vertical lines cannot be defined.

Writing an Equation for a Line from Its Graph

The graph of a line gives information that can be used to write its equation.

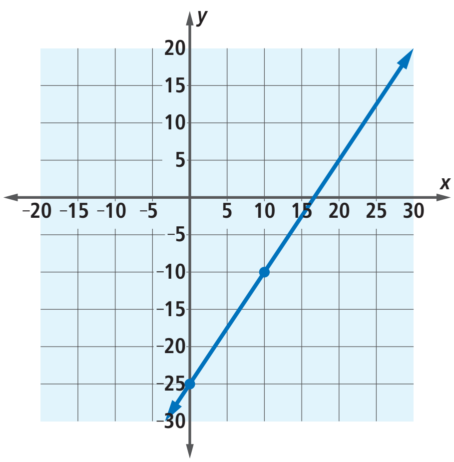
Example 2

Find the equation of the line graphed at the right.

Solution The graph crosses the y-axis at −25, so −25 is the y-intercept. The line contains (0, −25) and (10, −10). Its slope is found below.

$$m = \frac{-25 - -10}{0 - 10} = \frac{-15}{-10} = 1.5$$

The slope-intercept equation of the line is $y = 1.5x - 25$.



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Check Do the coordinates of the two known points (0, −25) and (10, −10) satisfy the equation?

Does $-25 = 1.5(0) - 25$?
$-25 = 0 - 25$

Does $-10 = 1.5(10) - 25$?
$-10 = 15 - 25$

Yes, both points check.

Every constant-increase or constant-decrease situation can be described by an equation whose graph is a line. The *y*-intercept of that line can be interpreted as the starting amount. The slope of that line is the amount of increase or decrease per unit.

Example 3

Assume that a skydiver opens a parachute and falls at a speed of 10 feet per second from 5,000 feet above the ground.

a. Find a slope-intercept equation for the height *h* of the skydiver after *x* seconds.

b. How high is the skydiver after 5 minutes (300 seconds)?

Solutions

a. The skydiver falls at 10 feet per second, so the slope is −10. The *y*-intercept is the starting height, 5,000 feet.

$y = 5{,}000 - 10x$ Write the equation.

$y = -10x + 5{,}000$ Rewrite in slope-intercept form.

b. Use the equation and substitute 300 for *x* because 5 minutes equals 300 seconds.

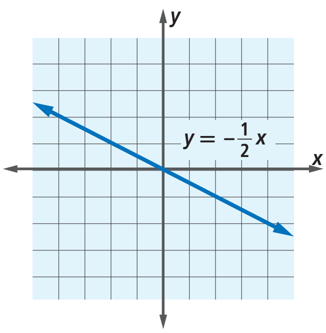
$y = 5{,}000 - 10(300)$

$y = 2{,}000$

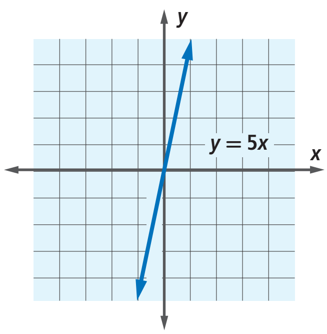
After 5 minutes, the height of the skydiver is 2,000 feet.

Direct Variation

A special case of a linear equation occurs when the *y*-intercept is at the origin. Then the *y*-intercept is 0 and $y = mx + 0$ becomes $y = mx$. This means that *y* is a constant multiple of *x*, as shown below.



x	*y*
−4	2
0	0
1	$-\frac{1}{2}$
10	−5



x	*y*
−2	−10
0	0
3	15
7	35

When y is a constant multiple of x, it is said that y varies directly as x. This situation is called **direct variation.** The amount you multiply each x by to get y is the slope, also called the *constant of variation*. Direct variation equations arise in large numbers of real-world situations. For example, the distance driven at a constant speed varies directly as the time driven. The circumference of a circle varies directly as the circle's radius.

STOP **QY3**

> **QY3**
>
> Determine which equation is an example of direct variation and give its constant of variation.
>
> **a.** $y = -4x + 1$
>
> **b.** $y = \frac{3}{4}x$
>
> **c.** $y = 8$

Questions

COVERING THE IDEAS

1. **Fill in the Blanks** The set of ordered pairs (x, y) that satisfy $y = mx + b$ is a line with slope __?__ and y-intercept __?__.

2. A family is taking a car trip. They expect to pay $150 for someone to look after their pet dog while they are gone. Then they think it will cost $200 per day for a room and meals and $80 per day for other expenses.
   a. What is the expected cost y for x days of the trip?
   b. If the equation in Part a is graphed, what are the slope and y-intercept of the graph?

In 3 and 4, an equation of a line in slope-intercept form is given.
a. Determine the slope.
b. Determine the *y*-intercept.
c. Graph the line.

3. $y = 2x + 3$

4. $y = \frac{1}{4}x - 2$

In 5 and 6, an equation of a line is given.
a. Rewrite the equation in slope-intercept form.
b. Determine the slope of the line.
c. Determine the *y*-intercept of the line.

5. $y = 5.6 - 1.3x$

6. $x + 5y = 7$

Hot air balloons were invented in France in 1783.

Source: www.hotairballoons.com

7. A hot air balloon begins 3 feet above the ground. It then climbs at a constant rate of 2 feet per second.
   a. Determine an equation for the height h of the balloon at time t.
   b. Draw a graph of the equation in Part a.
   c. What are the slope and y-intercept of the graph in Part b?
   d. How high is the balloon after 60 seconds?

8. **a.** Find the equation in slope-intercept form using the data in the table at the right.
   b. Is this an example of direct variation? Explain.

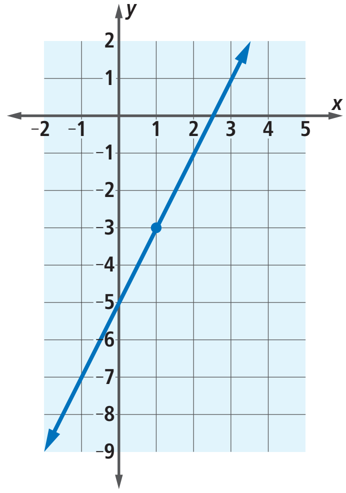
x	y
1	6.5
2	11
3	15.5
4	20
5	24.5

In 9 and 10, write an equation of a line in slope-intercept form with the following characteristics.

9. slope 4, *y*-intercept 3

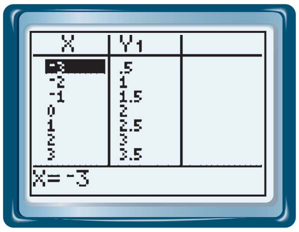
10. slope 0, *y*-intercept –2

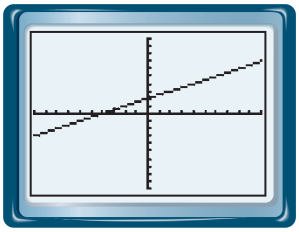
11. Write the equation of the line graphed at the right in slope-intercept form.



APPLYING THE MATHEMATICS

12. The equation $y = \frac{1}{2}x + 2$ was used to make the following table and graph. Find the slope and *y*-intercept. Explain how they are seen in the table and in the graph.





In 13–15, match the equation and graph with the situation.

Situation

13. Spencer has \$80 in the bank and is spending \$6 per week.

14. Owen borrowed \$80 from his uncle and is paying him back \$6 per week.

15. Savina has \$80 in the bank and is adding \$6 per week to the account.

Equation

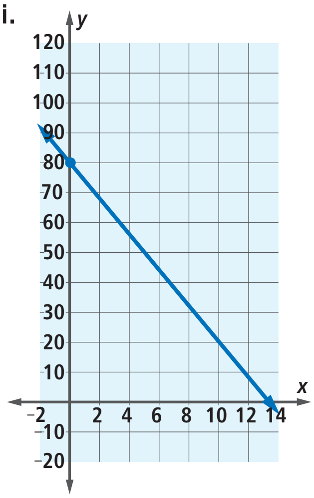
a. $y = 6x - 80$

b. $y = 6x + 80$

c. $y = -6x + 80$

Graph

i.



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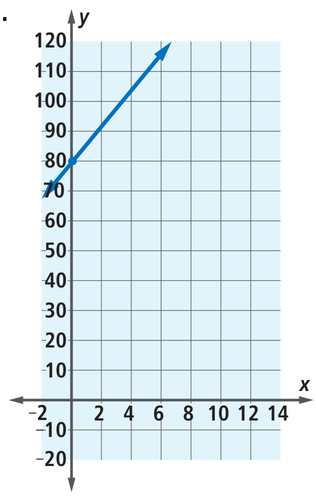

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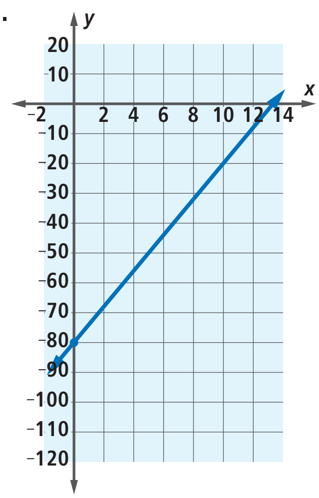


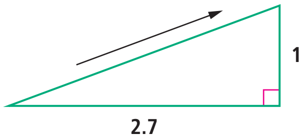
16. Consider the line through the points (3, 4) and (3, 7).
   a. Why does this line *not* have an equation in $y = mx + b$ form?
   b. What is an equation for this line?

17. a. Graph the equation $y = 3x + 4$ on a piece of graph paper.
   b. Draw a line through (2, 2) that is parallel to $y = 3x + 4$.
   c. Write an equation of the parallel line in slope-intercept form.
   d. How are the two equations the same?

18. A line has the equation $y = -3x + b$. For what value of b does the line pass through exactly 2 quadrants? (The axes are not considered to be in any quadrant.)

REVIEW

19. The Mount Washington Cog Railway in New Hampshire is one of the steepest mountain-climbing trains in the world with a gradient of 1 in 2.7. Find the slope of the path made as the train goes up the incline. **(Lesson 6-3)**



20. Determine if the following statement is *always, sometimes but not always,* or *never true* and explain your reasoning. *Vertical lines have a slope of zero.* **(Lesson 6-3)**

In 21 and 22, solve the equation. (Lesson 5-9)

21. $\frac{6x - 4}{3 + 2x} = \frac{5}{6}$

22. $\frac{w + 3}{3} = 3w - 1$

23. The sum of two consecutive integers is 8 less than three times the difference of the two numbers. Find the two numbers. **(Lesson 4-4)**

24. What is the area of the rectangle formed by the lines $y = 6$, $y = 2$, $x = -3$, and $x = 4$? **(Lesson 4-2)**

The Mount Washington Cog Railway is the world's first mountain-climbing cog railway.

Source: Mount Washington Cog Railway

EXPLORATION

25. A line t has slope 4 and y-intercept 3.
   a. What is an equation for line t?
   b. By experimenting with a graphing calculator, find an equation for the line that has y-intercept 3 and seems to be perpendicular to line t.

QY ANSWERS

1a. slope $= \frac{1}{4}$, y-intercept $= 11$

b. $y = 3x - 7$

2. slope $= -6$, y-intercept $= -2$

3. b.; $\frac{3}{4}$