Lesson 6-3

Properties of Slope

BIG IDEA The slope of a line can be determined by examining the graph; only one line through a point has a particular slope.

Most houses have slanted, or pitched roofs. The *pitch* of a roof is its slope. In the United States, the pitch is usually measured in 12ths. Why are 12ths used? Because there are 12 inches in a foot. The number of 12ths tells you the number of inches the roof rises for each foot that the roof goes across. In areas where there is a lot of snow, roofs often have higher pitches. The steepest roof a carpenter can safely walk on without the aid of ropes has a pitch of $\frac{8}{12}$.

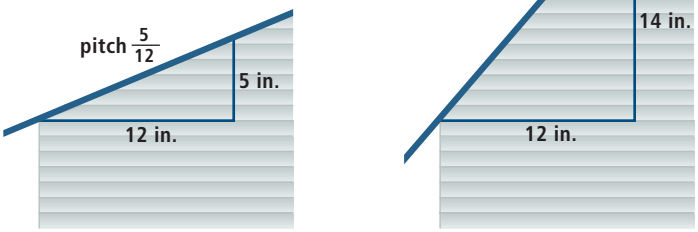
Mental Math

Consider the data set {3, 5, 9, 1, 7, 10, 16, 25, 24, 3, 18}. Find the

a. range.

b. mode.

c. median.



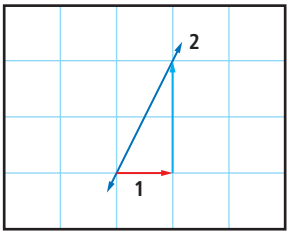
A roof with a pitch of $\frac{5}{12}$ goes up 5 inches for each 12 inches the roof goes across. But this ratio can also be expressed a different way. If the two inch measurements are converted to feet, the result is shown below.

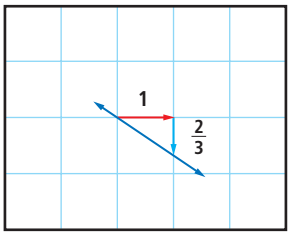
$$\frac{5 \text{ in.}}{12 \text{ in.}} = \frac{5 \text{ in.} \cdot \frac{1 \text{ ft}}{12 \text{ in.}}}{12 \text{ in.} \cdot \frac{1 \text{ ft}}{12 \text{ in.}}} = \frac{\frac{5}{12} \text{ ft}}{1 \text{ ft}} = \frac{5 \text{ in.}}{1 \text{ ft}}$$

So a roof with a pitch of $\frac{5}{12}$ goes up 5 inches ($\frac{5}{12}$ of a foot) for every foot the roof goes across.

The situation with roofs illustrates an important property of the slope of a line. The slope of a line is the amount of change in the height of the line for every change of one unit to the right.

For example, if a line has slope 2, as you move one unit to the right, the line goes up 2 units. So, if the line contains the point (a, b), it will also contain the point $(a + 1, b + 2)$.





If the slope is $-\frac{2}{3}$, for every change of 1 unit to the right, the line goes down $\frac{2}{3}$ of a unit. So, if the line contains the point (a, b), it will contain the point $(a + 1, b - \frac{2}{3})$.

STOP QY1

▶ **QY1**

Find another point on the line through (20, 80) that has a slope of $\frac{3}{4}$.

Graphing a Line by Using Its Slope

If you know one point on a line, you can find a second point using its slope. With the two points, you can graph the line.

Example 1

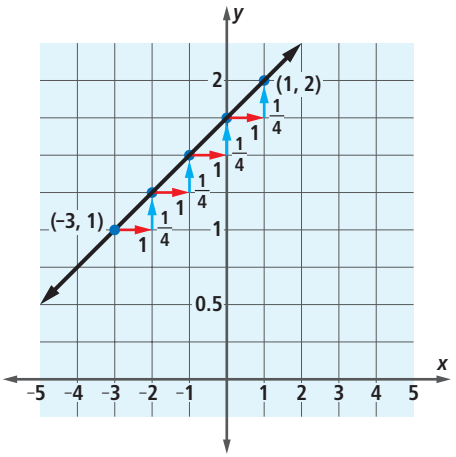
a. Graph the line through (−3, 1) with slope $\frac{1}{4}$.

b. Name another point on the line with integer coefficients.

Solutions

a. Plot (−3, 1), then move right 1 unit and up $\frac{1}{4}$ unit. Plot the resulting point $(-2, 1\frac{1}{4})$, and draw the line through the two points.

b. One point with integer coordinates (−3, 1) is given. Continue plotting points by moving right 1 unit and up $\frac{1}{4}$ unit until you reach another point with integer coordinates. As shown on the graph, the point (1, 2) is also on the line.



STOP QY2

▶ **QY2**

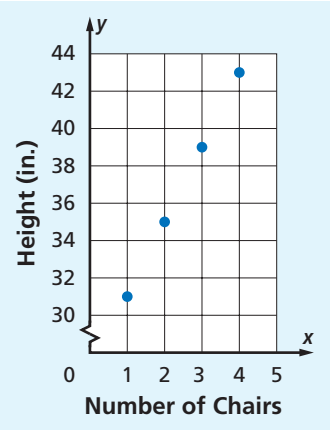
Find another point with integer coordinates on the line that contains (2, 6) and has a slope of $\frac{1}{2}$.

The idea of Examples 1 and 2 helps in drawing graphs of constant-increase or constant-decrease situations.

Example 2

A certain stacking chair is 31 inches tall. In a stack, each chair adds 4 inches to the height of the stack. Graph the relationship of the number of chairs to the height of the stack.

Solution The first point is (1, 31). Because the height increases 4 inches for each chair stacked, the slope is 4 inches per chair. The points are on a line because the situation is a constant-increase situation.



An equation for the height h of n stacking chairs is $h = 31 + 4(n - 1)$. This is an equation for the line that contains the points graphed in Example 2. Notice that the given numbers 31 and 4 both appear in this equation. In the next lesson, you will learn how the slope can be easily determined from an equation of a line.

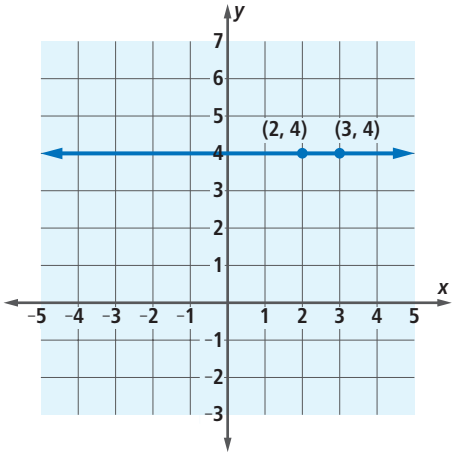
Zero Slope and Undefined Slopes

Lines with slope 0 can be drawn using the method of the previous two examples.

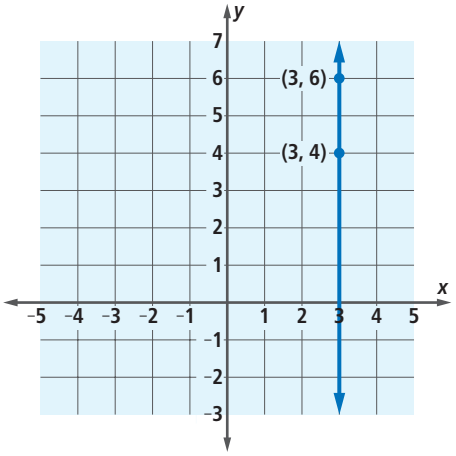
Example 3

Draw a line through (2, 4) with slope 0.

Solution Plot the point (2, 4). From this point move one unit right and 0 units up. Place a point there and draw a line through the points. You should get a horizontal line.



There is one type of line for which the methods used in Examples 1–3 do not work. If you have points on a vertical line, you cannot move one unit to the right and stay on the line. Also, if you try to calculate the slope using the formula $\frac{y_2 - y_1}{x_2 - x_1}$, the denominator will be 0. For example, the slope of a line through (3, 4) and (3, 6) would be $\frac{6-4}{3-3} = \frac{2}{0}$, which is not defined. Thus the slope of a vertical line is *undefined*.



Notice the important difference between slopes of horizontal and vertical lines.

Slope of Horizontal and Vertical Lines

1. The slope of every horizontal line is 0.
2. The slope of every vertical line is undefined.

Questions

COVERING THE IDEAS

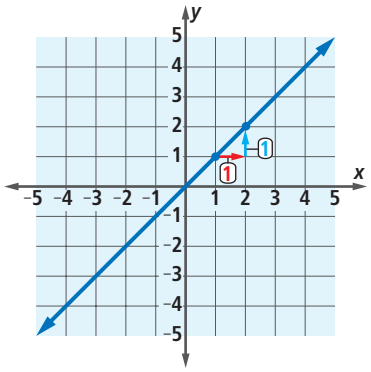
1. Suppose the pitch of a roof is $\frac{9}{12}$.
   a. Draw such a roof.
   b. Is this roof safe for a roofer to walk on without ropes? Why or why not?
   c. What is the slope of this roof?

In 2 and 3, a roof pitch is given. If the pitch is possible, draw the roof. If it is not possible, explain why.

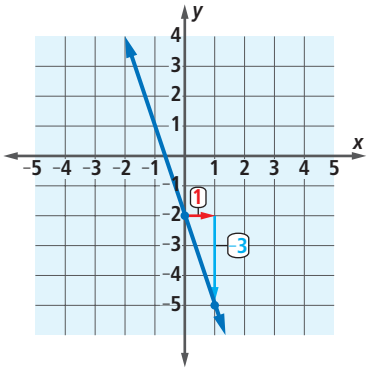
2. pitch $\frac{0}{12}$
3. pitch $\frac{12}{0}$
4. **Fill in the Blanks** The slope of a line is the amount of change in the ___?___ of the line for every change of ___?___ to the right.

In 5 and 6, find the slope of the line.

5.



6.



In 7 and 8, a line's slope and a point on it are given.

a. Graph the line.

b. Find one other point on this line.

c. Check that your second point is on the line using the slope formula.

7. point $(2, 2)$, slope 4

8. point $(4, -2)$, slope $-\frac{2}{3}$

In 9 and 10, find another point on the line with the given slope and containing the given point.

9. point $(-3, 4)$, slope -3

10. point $(4, 1)$, slope 0

11. Explain why the slope of a vertical line is not defined, by referring to a specific example that is not in this lesson.

12. Graph the line through $(-1, 4)$ with an undefined slope.

13. A small plastic bathroom drinking cup is about 5.70 cm tall. These cups stack tightly and each cup adds only about 0.28 cm to the stack. Graph the relationship between the number of cups and the height of the stack.

APPLYING THE MATHEMATICS

In 14–16, the slope of a line is given as are some coordinates of points on the line. Complete the remaining entries in the table.

14. slope $\frac{1}{5}$

x	*y*
4	8
5	?
6	?
7	?

15. slope -4

x	*y*
2	0
3	?
4	?
5	?

16. Slope is undefined.

x	*y*
−3	4
−2	?
−1	?
0	?

In 17 and 18, the question relates to the slope of a road. In the United States, the slope of a road is often called its *grade.* The grade is often given in percent. In the British Commonwealth, the word for slope is *gradient.*

17. The grade of Canton Avenue in Pittsburgh, Pennsylvania between Coast and Hampshire Streets is 37%. How much does that street rise for every 100 feet horizontally?

18. Baldwin Street in Dunedin, New Zealand has a maximum gradient of 1 in 2.86. This means that for every horizontal change of 2.86 units, the road goes 1 unit up. What is the slope of Baldwin Street? (*Caution:* The slope is not 2.86.)

A house sits on the steepest residential street in the world in Dunedin, New Zealand.

19. On a piece of graph paper, plot the point (0, 0). Draw the lines through (0, 0) with the following slopes on the same coordinate grid.

    line a: slope 3 line b: slope -3

    line c: slope $\frac{1}{3}$ line d: slope $-\frac{1}{3}$

    a. Which line(s) are slanted upward as you read from left to right?

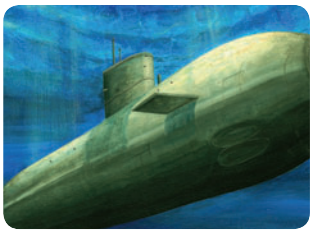
    b. Which line(s) are slanted downward as you read from left to right?

    c. Can you relate the slant of the line to the sign of the slope?

    d. Which line(s) are steepest?

    e. Give the value of a slope that is steeper than the slope of any of the four lines on the graph.

20. In Tom Clancy's best selling fiction novel, *The Hunt for Red October,* the American attack submarine *Dallas* is chasing the renegade Russian submarine *Red October*. The captain of the *Dallas* gave orders to descend to 1,200 feet below sea level. Engineer Butler "watched the depth gauge go below 600 feet. The diving officer would wait until they got to 900 feet before starting to level off, the object being to *zero the dive* out at exactly 1,200 feet". Explain, in terms of slope, what is meant by the italicized phrase.



Operators of a submarine control its buoyancy, allowing it to sink and surface at will.

Source: science.howstuffworks.com

REVIEW

21. Find the slope of the line with equation $12x - 8y = 4$. (**Lesson 6-2**)

22. The cost to rent a movie from Marcus's Movies is \$2.99 for two nights with a late fee of \$0.99 per day late. After x days late, the total cost will be y dollars. (**Lessons 6-2, 3-1, 1-2**)

    a. Write an equation relating x and y.

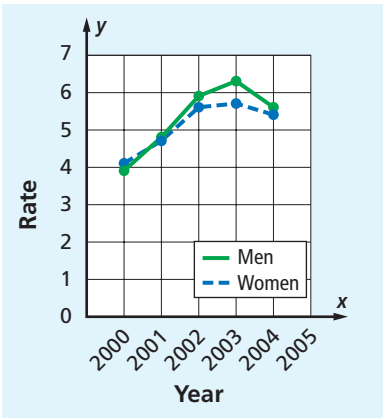
    b. Graph the line.

    c. Find the rate of change between any two points on the graph.

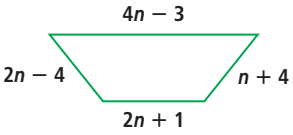
In 23 and 24, use the table and graph below. They show the unemployment rate, the percent of the United States labor force that was unemployed, for men and women. (Lesson 6-1)

	Unemployment Rate	
Year	Men	Women
2000	3.9	4.1
2001	4.8	4.7
2002	5.9	5.6
2003	6.3	5.7
2004	5.6	5.4

Source: *Statistical Abstract of the United States*

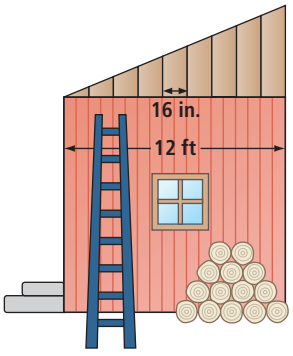


23. Between which two years was there the greatest increase in unemployment rate for
    a. men? b. women?
24. What was the rate of change from 2001 to 2003 for
    a. men? b. women?
25. Find b if $y = mx + b$, $y = -10$, $m = \frac{3}{4}$, and $x = 5$. **(Lessons 4-7, 1-1)**
26. If the quadrilateral at the right has perimeter 70, what is n? **(Lessons 3-4, 2-2)**



EXPLORATION

27. Canton Avenue and Baldwin Street, the roads in Questions 17 and 18, are among the steepest streets in the world. Search the Internet to find the grades of two other steep streets. Then make an accurate drawing indicating steepness of these roads.
28. Sam Saw wants to build a shed with a $\frac{5}{12}$ roof and the base of the roof being 12 feet long. To support the roof he needs to place a vertical support every 16 inches along the base. One way to do this is to climb the ladder, measure the length needed, go down the ladder and cut the board, then go back up the ladder to nail the board in place. Use your knowledge of slope to calculate the length of all the vertical supports so Sam does not need to go up and down the ladder constantly.



QY ANSWERS

1. Answers vary.
Sample answer: $(21, 80\frac{3}{4})$

2. Answers vary.
Sample answer: (4, 7)