## Lesson
# 6-2    The Slope of a Line

▶ **BIG IDEA**    A line has a constant slope equal to the rate of change between any two of its points.

## Constant-Decrease Situation

During a fire drill in a skyscraper with 50 floors, people were asked to move swiftly down the stairwell. To see how much time it would take to empty the building in a real emergency, the evacuation of the people on the top floor was monitored closely. They walked down the stairs at a rate of 5 floors every 2 minutes, or down $2\frac{1}{2}$ floors each minute.

This is a constant-decrease situation. The floor number decreases at a rate of $2\frac{1}{2}$ floors per minute. You can see the constant decrease by graphing the floor of the people after 0, 1, 2, 3, 4, … minutes of walking. Below is a table of ordered pairs (time, floor) charting their progress.

**Mental Math**

**Simplify.**

**a.** $\frac{1}{2}x + \frac{1}{2}x$

**b.** $\frac{1}{2}x \cdot \frac{1}{2}x$

**c.** $\dfrac{\frac{1}{2}x}{\frac{1}{2}x}$

| Time (min) | Floor |
|------------|-------|
| 0 | 50 |
| 1 | $47\frac{1}{2}$ |
| 2 | 45 |
| 3 | $42\frac{1}{2}$ |
| 4 | 40 |
| 5 | $37\frac{1}{2}$ |
| 6 | 35 |
| 7 | $32\frac{1}{2}$ |
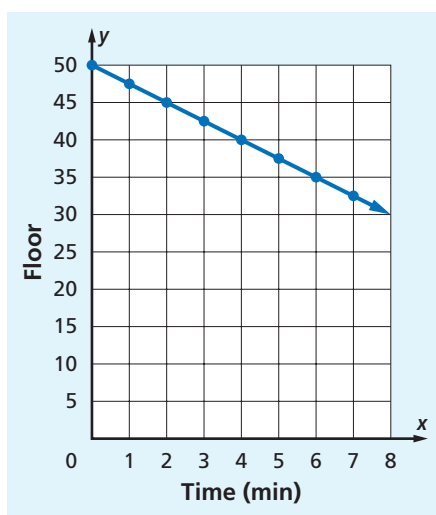


Pick any two points on this line, say (4, 40) and (6, 35). The rate of change between them is shown below.

$$\frac{\text{change in floor}}{\text{change in time}} = \frac{35\text{th floor} - 40\text{th floor}}{6\text{ min} - 4\text{ min}} = \frac{-5\text{ floors}}{2\text{ min}} = -2\frac{1}{2}\frac{\text{floors}}{\text{min}}$$

**STOP** QY

▶ **QY**

Pick two other nonconsecutive points on the line and calculate the rate of change of the floor.

# The Constant Slope of a Line

Notice that all the points on the graph on page 333 lie on the same line. In *any* situation in which there is a constant rate of change between points, the points lie on the same line. This constant rate of change is called the **slope** of the line.

> ## Slope
>
> The slope of the line through $(x_1, y_1)$ and $(x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$.
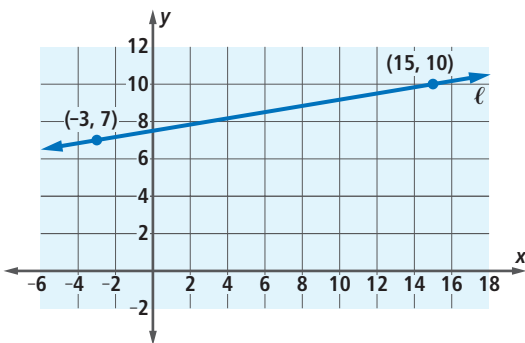
In contrast, consider the graph at the right. On this graph, no three points lie on the same line, and the rate of change between each pair of the points is different. This was also the case for the graph of the population of Chicago on page 325.

slope of $a = \frac{-3}{1} = -3$ \qquad slope of $b = \frac{-1}{1} = -1$

slope of $c = \frac{2}{1} = 2$ \qquad slope of $d = \frac{0}{1} = 0$
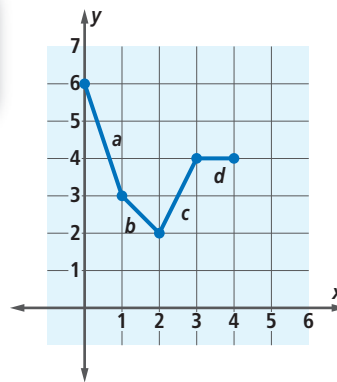
## Example 1

**Find the slope of line $\ell$ below.**

**Solution 1** You can choose either point to be $(x_1, y_1)$. The other will be $(x_2, y_2)$. We pick $(x_1, y_1) = (-3, 7)$, and so $(x_2, y_2) = (15, 10)$. Apply the formula for slope.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 7}{15 - -3} = \frac{3}{18} = \frac{1}{6}$$

**Solution 2** Instead, let $(x_1, y_1) = (15, 10)$ and $(x_2, y_2) = (-3, 7)$. Apply the formula for slope.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 10}{-3 - 15} = \frac{-3}{-18} = \frac{1}{6}$$

**Check** Because the line is going up to the right, the graph is expected to have a positive slope.

Example 1 shows that when you calculate a slope, it does not matter which of the two points you consider as $(x_1, y_1)$ and which as $(x_2, y_2)$. The slope is the same.
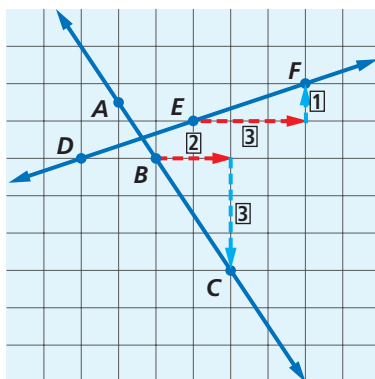
## Calculating Slope from a Graph

The subtractions in the slope formula allow you to find the vertical and horizontal change between any two points. This can also be done from a graph simply by counting the units of change in each direction.

### Example 2

In the coordinate grid below, each square of the grid is one unit. Find the slope of the line.

a. $\overleftrightarrow{AC}$

b. $\overleftrightarrow{DF}$



**Solutions**

a. Pick two points on $\overleftrightarrow{AC}$. We pick points *B* and *C*. The change in *x*-coordinates reading from left to right (*B* to *C*) is 2 units. The change in the *y*-coordinates is −3 units.

$$\text{slope of } \overleftrightarrow{AC} = \frac{-3}{2} = -\frac{3}{2}$$

b. We pick points *E* and *F*. The change in the *x*-coordinates reading from left to right (*E* to *F*) is 3 units. The change in the corresponding *y*-coordinates is 1 unit.

$$\text{slope of } \overleftrightarrow{DF} = \frac{1}{3}$$

Lines with negative slopes, such as $\overleftrightarrow{AC}$ in Example 2, go downward as they move to the right. Lines with positive slopes, such as $\overleftrightarrow{DF}$ in Example 2, go upward as they move to the right. If a line is horizontal, it is neither increasing or decreasing, and likewise the slope is zero.

If the rate of change, or slope, is the same for a set of points, then all the points lie on the same line. If the rate of change is different for different parts of a graph, then the graph is *not* a line.

**GUIDED**

## Example 3

a. Show that (−2, 5), (2, 3), and (−10, 9) lie on the same line.

b. Give the slope of that line.

**Solutions**

a. Pick pairs of points and calculate the slope between them.

The slope between (-2, 5) and (2, 3) is $\frac{?-5}{2-?} = \underline{\ ?\ }$.

The slope between (2, 3) and (-10, 9) is $\frac{9-?}{-10-?} = \underline{\ ?\ }$.

The slope between (-10, 9) and (-2, 5) is $\frac{?-?}{?-?} = \underline{\ ?\ }$.

Because the slope between any pair of the given points is $\underline{\ ?\ }$, the points lie on the same line.

b. The slope of the line is the constant rate of change, which is $\underline{\ ?\ }$.

# Finding the Slope of a Line from an Equation for the Line

You can find the slope of any line given its equation. Use the equation to find two points on the line and calculate the rate of change between them.

## Example 4

Find the slope of the line with equation $2x - 5y = 4$.

**Solution** First, find two points that satisfy the equation. Pick a value for $x$ or $y$ and then substitute it into $2x - 5y = 4$ to find the value of the other variable. To make the calculations easier, we let $x = 0$ for the first point and $y = 0$ for the second point.

| Let $x = 0$. | Let $y = 0$. |
|---|---|
| $2 \cdot 0 - 5y = 4$ | $2x - 5 \cdot 0 = 4$ |
| $0 - 5y = 4$ | $2x - 0 = 4$ |
| $\frac{-5y}{-5} = \frac{4}{-5}$ | $\frac{2x}{2} = \frac{4}{2}$ |
| $y = -\frac{4}{5}$ | $x = 2$ |
| The point $(0, -\frac{4}{5})$ is on the line. | The point $(2, 0)$ is on the line. |

Use the points $(0, -\frac{4}{5})$ and $(2, 0)$ in the slope formula.

$$\text{slope} = \frac{0 - \left(-\frac{4}{5}\right)}{2 - 0} = \frac{\frac{4}{5}}{2} = \frac{4}{5} \cdot \frac{1}{2} = \frac{2}{5}$$

**Check** Find a third point on the line $2x - 5y = 4$. For example, we let $y = 2$ and find that $x = 7$. Thus the point $(7, 2)$ is on the line, as shown on the graph at the right. Now calculate the slope determined by $(7, 2)$ and one of the points you used to find the original slope, say $(2, 0)$. This gives $\frac{2 - 0}{7 - 2}$, which equals $\frac{2}{5}$. So all three points give the same slope. It checks.
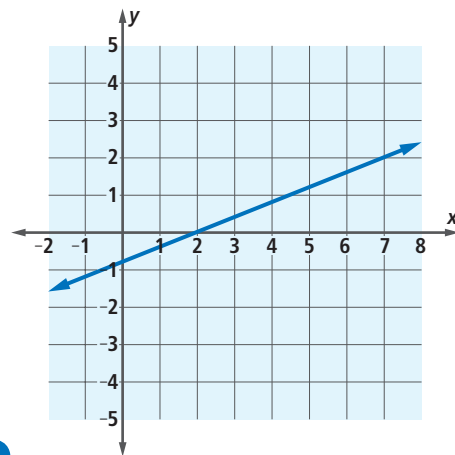
# Questions

## COVERING THE IDEAS

1. **Fill in the Blank** In a constant-increase or constant-decrease situation, all the points lie on the same ___?___.

2. What is the constant rate of change between any two points on a line called?

3. The table at the right lists the height of a burning birthday candle in centimeters based on time in seconds. The table shows the ordered pairs (time, height), and the relationship is linear.

   a. Find two more ordered pairs.
   b. Find the rate of change between any two points in the table.
   c. Explain the real-world meaning of your answer to Part b.

| Time (sec) | Height (cm) |
|------------|-------------|
| 0 | 5.7 |
| 1 | 5.4 |
| 2 | 5.1 |
| 3 | 4.8 |
| 4 | 4.5 |
| 5 | 4.2 |

4. The table at the right gives coordinates of points on a line.
   a. If the pattern in the table is continued, what ordered pair should be entered in the next row?
   b. What is the slope of the line?

| x | y |
|----|----|
| 6 | 23 |
| 8 | 29 |
| 10 | 35 |
| 12 | 41 |
| 14 | 47 |
| ? | ? |

In 5 and 6, calculate the slope of the line through the given pair of points.

5. $(0, 4)$ and $(3, 19)$    6. $(4, 3)$ and $(-6, 8)$

7. a. Calculate the slope of the line through $(2, 1)$ and $(5, 11)$.
   b. Calculate the slope of the line through $(5, 11)$ and $(-4, -18)$.
   c. Do the points $(2, 1)$, $(5, 11)$, and $(-4, -18)$ lie on the same line? How can you tell?

In 8 and 9, refer to the graph below. Find the slope of the line.
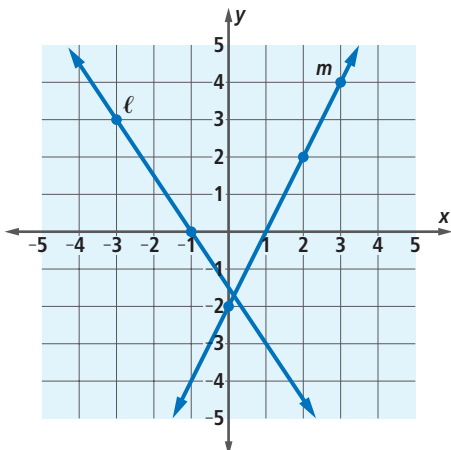


8. line $\ell$                     9. line $m$

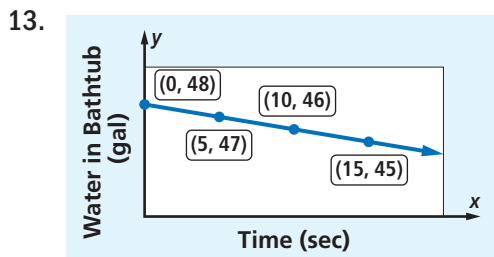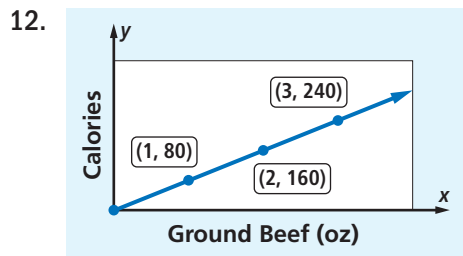In 10 and 11, an equation for a line is given.

a. Find two points on the line.
b. Find the slope of the line.
c. Check your work by graphing the line.

10. $y = -\frac{4}{3}x + 4$                     11. $6x - 5y = 30$

In 12 and 13, a graph is given.

a. Find the slope of the line.
b. Describe its real-world meaning.

12.



13.

## APPLYING THE MATHEMATICS

14. When the fire drill described on page 333 was over, the people returned to the top floor of the building, but this time via the elevator. From the 26th floor, the elevator went up 1 floor every 3 seconds. (Notice the elevator's time is given in seconds, not minutes as with the fire drill.)

   a. Make a table and graph the people's progress for the first 6 seconds of the elevator ride from the 26th floor up using ordered pairs (seconds ridden, floor number).

   b. Find the rate of change between any two points on the graph.

   c. How many seconds will it take for them to reach the 50th floor after leaving the 26th floor?

15. Suppose you graphed the data below.

| Time (x) | Temperature (y) |
|----------|-----------------|
| 8 A.M. | 62°F |
| 9 A.M. | 63°F |
| 10 A.M. | 67°F |
| 11 A.M. | 71°F |
| 12 P.M. | 73°F |

   a. What is the unit of the rate of change?

   b. Describe the real-world meaning of the rate of change.

16. Use the figure at the right. Which line could have the indicated slope?

   a. slope $\frac{3}{5}$

   b. slope $-\frac{3}{5}$

   c. slope 0
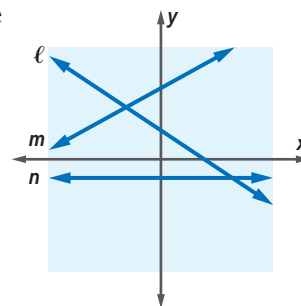
17. The points $(5, 7)$ and $(4, y)$ are on a line with slope $-4$. Find the value of $y$.

18. Coordinates of points are given in columns A and B in the spreadsheet at the right with a blank column for the rate of change between consecutive points.

   a. Complete the rate of change column, leaving C2 blank.

   b. Do the points lie on a line? Explain how you know.

| ◇ | A | B | C |
|---|---|---|---|
| 1 | x | y | rate of change |
| 2 | -4 | 22 | |
| 3 | 0 | 10 | $\frac{10-22}{0--4} = \frac{-12}{4} = -3$ |
| 4 | 5 | -5 | |
| 5 | 11 | -23 | |

19. In Lesson 1-4, the following sequence of toothpick designs was examined. A graph of ordered pairs (term number, number of toothpicks) is shown at the right.

**1st          2nd          3rd          4th**
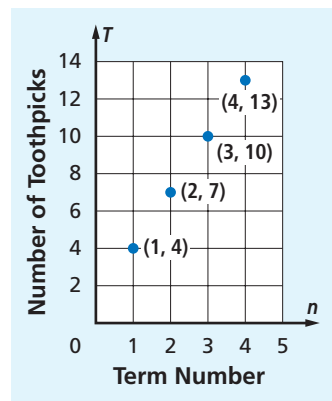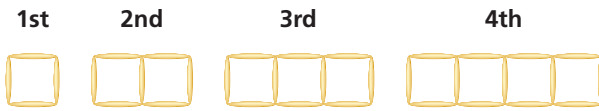


a. What is the rate of change of the line through these points? (Remember to include the rate unit.)

b. Let $n$ be the term number and $T$ be the number of toothpicks needed. Then $T = 1 + 3n$. Verify that the slope of the line with equation $T = 1 + 3n$ is the same as your answer to Part a.

c. What does the slope mean in this situation?

## REVIEW

20. A high-rise building was built at a constant rate. The builders completed the 18th floor on the 24th day of construction, and they finished the 81st floor on the 108th day of construction. Calculate the rate of construction in terms of floors per day. (**Lesson 5-3**)

21. Solve $-5 + 8p < 3(2p - 2) + 2p$. (**Lesson 4-5**)

22. Compute in your head. (**Lesson 4-1**)
    a. 16 is what percent of 64?      b. 75% of 80 is what number?

23. **Fill in the Blank** If $5a = \frac{1}{2}b$, then $20a = \underline{\ ?\ }$. (**Lesson 2-8**)

24. Determine whether $(x + y)(x + y)(x + y) = x^3 + y^3$ is *always, sometimes but not always,* or *never* true. Explain. (**Lesson 1-3**)

## EXPLORATION

25. Find a record of your height at some time over a year ago. Compare it with your height now. How fast (in inches or centimeters per year) has your height been changing from then until now?



A woman measures her daughter's height.