Chapter 3 Summary and Vocabulary

- A **linear equation** is one that is equivalent to an equation of the form $Ax + By = C$, where A and B are not both zero. The graph of every linear equation is a line. If the line is not vertical, then its equation represents a linear function and can be put into the **slope-intercept form** $y = mx + b$, where m is its slope and b is its **y-intercept**. Horizontal lines have slope 0 and equations of the form $y = b$. Slope is not defined for vertical lines, which have equations of the form $x = a$. In these last three forms, m, b, and a can be any real numbers.
- If the slope m and one point (x_1, y_1) on a line are known, then an equation for the line is $y - y_1 = m(x - x_1)$, or $y = m(x - x_1) + y_1$.
- In many real-world situations, a set of data points is roughly linear. In such cases, a regression or least squares line, **the line of best fit**, can be used to describe the data and make predictions.
- Linear equations can model two basic kinds of real-world situations: constant increase or decrease, and **linear combinations**. The graph of a constant-increase or constant-decrease situation is a line, with the slope of the line representing the constant change and the y-intercept representing the initial condition.
- Sequences with a constant difference between terms are called **linear** or **arithmetic sequences**. If a_n is the nth term of an arithmetic sequence with constant difference d, then the sequence can be described explicitly as $a_n = a_1 + (n - 1)d$, for integers $n \geq 1$, or recursively as $\begin{cases} a_1 \\ a_n = a_{n-1} + d \end{cases}$, for integers $n \geq 2$.
- A function whose graph is the union of segments or rays is called **piecewise linear**. **Step functions** are instances of piecewise linear functions. Step functions represent situations in which rates are constant for a while but change to a different constant rate at known points.

Theorems

Parallel Lines and Slope Theorem (p. 158)
Standard Form of an Equation of a Line Theorem (p. 167)
Point-Slope Theorem (p. 174)
*n*th Term of an Arithmetic Sequence Theorem (p. 201)
Constant-Difference Sequence Theorem (p. 202)

Vocabulary

Lesson 3-1
y-intercept
*linear function
*slope-intercept form
x-intercept

Lesson 3-2
*linear combination
*standard form

Lesson 3-4
*point-slope form
piecewise linear

Lesson 3-5
linear regression
least-squares line, regression line, line of best fit
deviation

Lesson 3-6
*recursive formula
recursive definition

Lesson 3-7
Fibonacci sequence

Lesson 3-8
linear sequence, *arithmetic sequence

Lesson 3-9
*step function
floor symbol $\lfloor \; \rfloor$
ceiling symbol $\lceil \; \rceil$
*floor function, greatest-integer function, rounding-down function, int function
ceiling function, rounding-up function

Chapter 3 Self-Test

Take this test as you would take a test in class. Then use the Selected Answers section in the back of the book to check your work.

1. Graph the line with equation $y = \frac{1}{2}x - 3$.

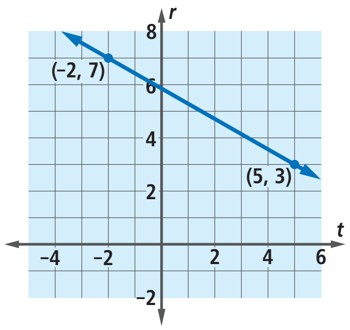
2. Consider the line with equation $5x - 3y = 10$.

a. What is its slope?

b. What are its x- and y-intercepts?

3. If $m < 0$, does the equation $y = mx + b$ model a constant-increase or a constant-decrease situation?

4. Write an equation for the line graphed below.



5. Write an equation for the line parallel to $y = -\frac{2}{3}x - 7$ that contains (–6, 3).

6. a. Slope is undefined for which type of line?

b. Which type of line has a slope of zero?

7. A restaurant sells hot dogs for \$2.25 and tacos for \$3.75. Write an expression that gives the amount of money the restaurant takes in by selling H hot dogs and T tacos.

8. A crane lowers a 500-pound load from the top of a 310-foot tall building at the rate of 20 feet per minute. Write a formula for the height h of the load above the ground after t seconds.

9. Consider the sequence defined by $a_n = 50 - 3(n - 1)$.

a. Is this an explicit or recursive formula for the sequence? Explain your answer.

b. Write the sequence's first five terms.

c. Is the sequence arithmetic? Explain your answer.

10. A cell-phone plan costs \$23.50 a month for the first 200 minutes of calls, plus 8 cents per minute for all calls after that.

a. In July, Michelle's calls total 453 minutes. How much will she be charged?

b. Write an equation for the piecewise linear function that gives the monthly cost C in terms of the minutes m spent on the phone.

11. The Lucas sequence L begins with the terms 1 and 3. After that, each term is the sum of the two preceding terms. The first six Lucas numbers are 1, 3, 4, 7, 11, and 18.

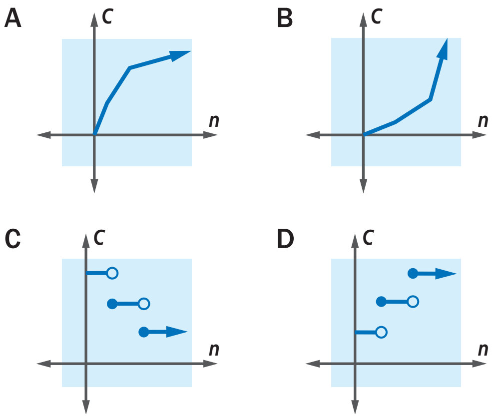
a. Write the next four Lucas numbers.

b. Let L_n be the nth Lucas number. Write a recursive formula for the Lucas sequence L.

12. Temperature F in degrees Fahrenheit and temperature K in kelvins are related by a linear equation. Two pairs of corresponding temperatures are $32°F = 273.2$ kelvins and $90°F = 305.4$ kelvins. Write a linear equation relating F and K, and solve for K.

13. Write an explicit formula for the arithmetic sequence –25, –45, –65, –85, ...

14. **Multiple Choice** Which graph most closely describes a piecewise function where each piece has a slope greater than the previous piece? Explain your response. (The scales on all four sets of axes are the same.)



15. Suppose the U.S. Postal Service sought to set the price of mailing a letter weighing 1 ounce or less at 50¢, with each additional ounce or part of an ounce costing an extra 30¢. Graph the function that maps weight w (in ounces) of a letter onto the cost c (in cents) to mail the letter.

16. a. **Multiple Choice** Which equation describes the function in Question 15?

   A $c = 50 - 30\lceil 1 - w \rceil$

   B $c = 50 + 30\lceil 1 - w \rceil$

   C $c = 50 - 30\lceil w - 1 \rceil$

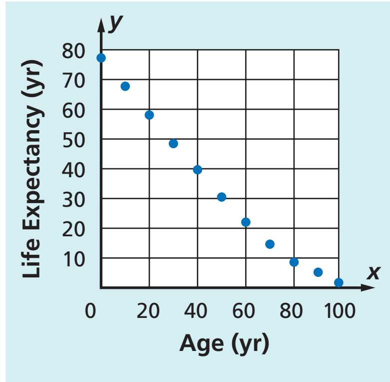
   D $c = 50 + 30\lceil w - 1 \rceil$

   b. Find the cost of mailing a 3.2-ounce letter.

17. Sketch a graph of $y = \lfloor x \rfloor + 1$ for $-2 \leq x \leq 2$.

18. The table and the scatterplot below give the life expectancy in the U.S. at selected ages in 2003.

Age	Expected Years of Life
0	77.4
10	68.1
20	58.4
30	48.9
40	39.5
50	30.5
60	22.2
70	14.8
80	8.9
90	4.8
100	2.5



Source: Centers for Disease Control and Prevention

a. Using the data points for ages 0 to 80, find an equation of the regression line.

b. Using your answer to Part a, estimate the life expectancy of someone who is currently 42.

c. Which data point is farthest vertically from the regression line? What does this mean in context?

d. Why is it unreasonable to use a linear model for ages over 80?

Chapter 3 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Determine the slope and intercepts of a line given its equation. (Lessons 3-1, 3-3)

In 1–6, an equation for a line is given.

a. Give its slope.

b. Give its x-intercept.

c. Give its y-intercept.

1. $y = 3x - 12$ **2.** $4y = 6 + 5x$

3. $y = -17$ **4.** $x = 8$

5. $300x - 250y = -100$

6. $x + y = 1.46$

OBJECTIVE B Find an equation of a line given two points on it or given a point on it and its slope. (Lesson 3-4)

In 7–10, find an equation of the line satisfying the given conditions.

7. The line has a slope of $\frac{2}{5}$ and contains $(-5, 10)$.

8. The line contains the point $(8, 2)$ and goes through the origin.

9. The line contains the points $(-3, 4)$ and $(5, -4)$.

10. The line is parallel to $y = 6x - 1$ and contains the point $(7, 1)$.

OBJECTIVE C Evaluate expressions based on step functions. (Lesson 3-9)

In 11–13, evaluate the expression.

11. a. $\lfloor 13.5 \rfloor$ **b.** $\lceil -13.5 \rceil$

12. a. $\lfloor x - 0.4 \rfloor$ when $x = 3.6$

b. $\lceil x + 0.4 \rceil$ when $x = 3.6$

13. $4\lfloor 2n + 1.4 \rfloor$ when $n = 0.4$

OBJECTIVE D Evaluate or find explicit and recursive formulas for sequences. (Lessons 3-6, 3-8)

In 14 and 15, an arithmetic (linear) sequence is given. For the sequence:

a. find an explicit formula.

b. write a recursive formula.

c. find the fourth through the eighth terms.

14. $-2, 1, 4, \ldots$ **15.** $37, 16, -5, \ldots$

16. Write a recursive definition of the sequence whose nth term is $a_n = -4n + 15$.

17. Use this recursively-defined sequence.

$$\begin{cases} a_1 = -\frac{2}{3} \\ a_n = a_{n-1} + \frac{1}{4} \text{ for integers } n \geq 2 \end{cases}$$

a. Write an explicit formula for the nth term of this sequence.

b. Generate the first five terms.

PROPERTIES Principles behind the mathematics

OBJECTIVE E Recognize properties of linear functions. (Lessons 3-1, 3-2, 3-3)

18. Multiple Choice Which of the following does *not* mean a line has a slope of $-\frac{2}{3}$?

A It has a vertical change of $\frac{2}{3}$ unit for a horizontal change of -1 unit.

B It has the equation $y = -\frac{2}{3}x + 5$.

C It has the equation $2x - 3y = 7$.

D It is parallel to the line with equation $2x + 3y = 7$.

19. **True or False** Two lines in a plane are parallel if they have equal slopes.

20. Consider $Ax + By = C$. For what values of A and B does this equation not represent a function? Explain your response.

21. What is the x-intercept of $y = mx + b$?

22. **Multiple Choice** Three points A, B, and C are on a line with B between A and C and $AB = 3(BC)$. The slope determined by B and C

    A is 3 times the slope determined by A and B.

    B is $\frac{1}{3}$ the slope determined by A and B.

    C equals the slope determined by A and B.

    D None of the above is true.

23. Suppose a line has slope $-\frac{3}{4}$. Then if (x, y) is a point on the line, name another point on the line.

OBJECTIVE F **Recognize properties of linear or arithmetic sequences.** (Lesson 3-8)

24. What is a linear sequence?

25. Describe the graph of an arithmetic sequence.

In 26 and 27, tell whether the numbers could be the first four terms of an arithmetic sequence.

26. –4, –6, –8, –10

27. 3, 4, 6, 9

In 28 and 29, does the formula generate an arithmetic sequence?

28. $\begin{cases} a_1 = 3 \\ a_n = \frac{1}{2}a_{n-1} - 1, \text{ for integers } n \geq 2 \end{cases}$

29. $\begin{cases} a_1 = 3 \\ a_{n+1} = a_n - 5, \text{ for integers } n \geq 1 \end{cases}$

30. Find the nth term of a linear sequence whose 1st term is 11 and whose constant difference is –4.

31. If the 10th term of an arithmetic sequence is 8 and the 20th term is 16, what is the first term?

USES Applications of mathematics in real-world situations

OBJECTIVE G **Model constant-increase or constant-decrease situations or situations involving arithmetic sequences.** (Lessons 3-1, 3-8)

32. A truck weighs 2000 kg when empty. It is loaded with crates of oranges weighing 17 kg each.

    a. Write an equation relating the total weight w and the number c of crates.

    b. Find the weight when there are 112 crates in the truck.

33. On his way to work, Rusty drives over a nail that punctures his tire. The tire begins to lose pressure at about 2 pounds per square inch (psi) per hour.

    a. If Rusty's tire had 44 psi of pressure before the puncture, write an equation to show how much pressure p the tire has after t hours.

    b. In order to drive safely, the tire must have at least 26 psi of pressure. How long can Rusty wait to replace his tire?

OBJECTIVE H **Model linear combination situations.** (Lesson 3-2)

34. A crate contains grapefruits and oranges. On average, an orange weighs 0.3 pound and a grapefruit weighs 1.1 pounds. The contents of the crate weigh a total of 30 pounds. Let x be the number of oranges and let y be the number of grapefruits.

    a. Write an equation to model this situation.

    b. If there are 15 grapefruits in the crate, how many oranges are there?

35. A chemist combines A liters of a solution that is 2.5 moles/liter bromic acid with B liters of a solution that is 6.25 moles/liter bromic acid.

a. Write an expression for how many liters of solution there are altogether.

b. How many moles of bromic acid are there altogether?

c. The chemist preparing the solution needs a total of 0.75 mole of bromic acid in a solution. Write an equation that describes this situation.

d. List three ordered pairs that are realistic solutions to the equation in Part c.

OBJECTIVE I **In a real-world context, find an equation for a line containing two points.** (Lesson 3-4)

36. On a trip abroad, Salena buys 7000 Indian rupees for 150 U.S. dollars, and then buys 11,200 Indian rupees for 250 U.S. dollars.

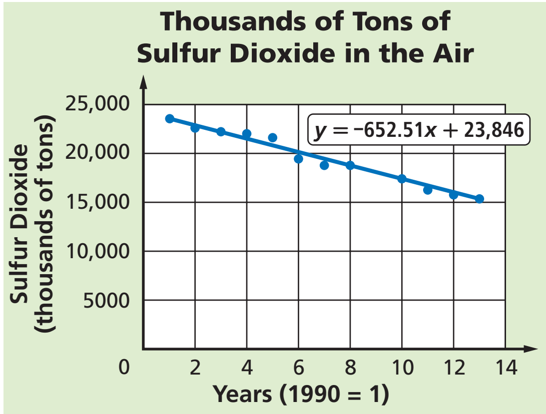
a. Assume a linear relationship exists between the number of Indian rupees and the cost in U.S. dollars. Write an equation representing the relationship.

b. How much will it cost to buy 20,000 Indian rupees?

37. Gerald finds that it takes 30 ml of a standard solution to neutralize 12 ml of a solution of unknown concentration. If he starts with 20 ml of the unknown solution, it takes 50 ml of standard solution to neutralize it. Assuming a linear relationship exists between the amount of unknown solution and the amount of standard solution required to neutralize it, how much unknown solution can be neutralized with 175 ml of standard solution?

OBJECTIVE J **Fit lines to data.** (Lesson 3-5)

38. The display below shows the number of tons of sulfur dioxide, a major form of air pollution, in the United States from 1990 to 2002. An equation of the regression line is $y = -652.51x + 23{,}846$, with $x = 1$ in 1990.



a. What value does the regression equation predict for 1998 (year 9)?

b. The actual value for 1998 is 18,944. What is the difference between the value predicted by the regression equation and the actual value?

c. What is the percent decrease from 1990 (23,760,000 tons) to 2002 (15,353,000 tons)?

d. If this linear trend continued, what would have been the approximate number of tons of sulfur dioxide in the air in 2006?

39. The table below gives a measure of the purchasing power of the U.S. dollar from 1991 to 2004. Here year 1 = 1991 and year 14 = 2004.

Year	Purchasing Power	Year	Purchasing Power
1	0.734	8	0.613
2	0.713	9	0.600
3	0.692	10	0.581
4	0.675	11	0.565
5	0.656	12	0.556
6	0.638	13	0.543
7	0.623	14	0.529

a. Make a scatterplot of these data.

b. Find an equation for the regression line of these data.

c. Does it appear that a linear equation would be a good model for these data? Explain why or why not.

OBJECTIVE K **Model situations leading to piecewise linear functions or to step functions.** (Lessons 3-4, 3-9)

In 40 and 41, an online music service charges 99¢ for each of the first 20 music downloads and then 89¢ for each additional download.

40. Find the total cost for a user who completes

a. 15 downloads. **b.** 27 downloads.

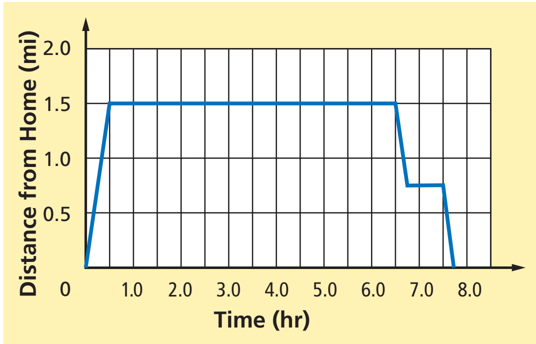
41. Describe the situation with a function mapping the number d of downloads onto the total cost C.

42. **Multiple Choice** Suppose a salesperson earns a \$50 bonus for each \$1000 in sales that he or she makes. What is a rule for the function that relates sales s to the amount b of bonus?

A $b = 50 \cdot \left\lfloor \frac{s}{1000} \right\rfloor$ **B** $b = \left\lfloor \frac{50s}{1000} \right\rfloor$

C $b = 50 + \left\lfloor \frac{s}{1000} \right\rfloor$ **D** $b = 50 + \left\lfloor \frac{1000}{s} \right\rfloor$

43. The graph below shows Imani's trip directly from home to school. After attending school all day she went to her friend's house where they did their math homework. Then she returned to her house.



a. At what rate did she walk to school?

b. At what rate did she walk to her friend's house?

c. Write a piecewise linear function for this graph.

d. Was her speed walking home the same, faster, or slower than her speed walking to school? Explain how you know.

44. A cell-phone plan charges \$39.99 for the first 450 minutes plus \$0.45 for each additional minute or part of a minute. Write an equation to model this situation.

OBJECTIVE L **Model situations with recursive formulas.** (Lessons 3-6, 3-7)

45. On the first day of winter, Charo Chipmunk has a pile of 2300 nuts. Each day, she eats 26 nuts. Let a_n be the number of nuts in her pile on day n.

a. Write a recursive formula for a_n.

b. Write an explicit formula for a_n.

c. The winter typically lasts 80 days. Will Charo's supply of nuts last her through the winter?

46. Martin raises emus. The number a_n of emus on Martin's farm in year n is given by the table below.

n	1	2	3	4
a_n	10	20	40	80

a. Assume the doubling pattern in the table continues. Write a recursive formula for a_n.

b. How many emus should he expect to have in year 10?

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE M **Graph or interpret graphs of linear equations.** (Lessons 3-1, 3-3)

47. Graph the line with slope 4 and y-intercept 8.

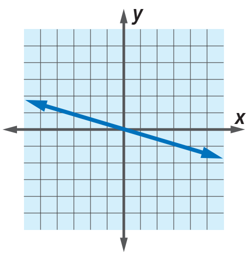
48. Graph the line $-2x + 8y = 12$ using its intercepts.

49. Graph $x = -5$ in the coordinate plane.

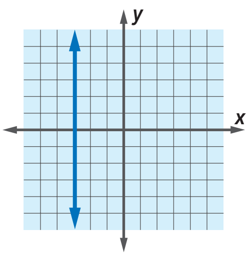
50. Graph $y = 2$ in the coordinate plane.

In 51 and 52, tell whether the slope of the line is positive, negative, zero, or undefined.

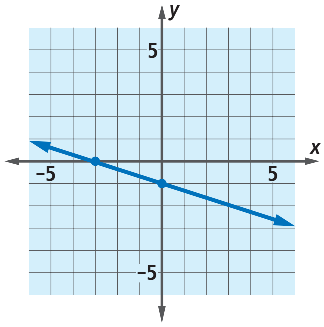
51.



52.



53. What is an equation of the line graphed below?



OBJECTIVE N **Graph or interpret graphs of piecewise linear functions, step functions, or sequences.** (Lessons 3-4, 3-7, 3-9)

54. Consider the function f, where

$$f(x) = \begin{cases} x - 2, \text{ for } x < 0 \\ 2, \text{ for } x \geq 0 \end{cases}.$$

a. Draw a graph of $y = f(x)$.

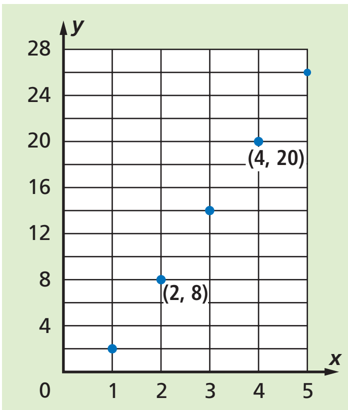
b. Find the domain and range of f.

55. a. Graph the function h, where $h(x) = 3 + \lfloor x \rfloor$.

b. Give the domain and range of h.

56. A personal trainer earns a bonus of \$150 for every 2 pounds of weight a client loses. Draw a graph of the bonuses as a function of the number of pounds lost.

57. A graph of the first five terms of an arithmetic sequence is shown at the right. Write an explicit formula for the sequence.



58. Graph this sequence.

$$\begin{cases} a_1 = -5 \\ a_n = a_{n-1} + 3.5, \text{ for integers } n \geq 2 \end{cases}$$