Lesson 3-2

Linear Combinations and $Ax + By = C$

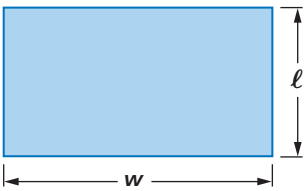
Vocabulary

linear combination

standard form

BIG IDEA When the sum of multiples of x and y is a constant, then $Ax + By = C$ for all (x, y).

The form $y = mx + b$ is convenient for graphing lines because the y-intercept and slope are obvious, but you have also seen many equations for lines in different forms. For example, in the rectangle with length ℓ, width w, and perimeter 20 inches, $20 = 2\ell + 2w$. This is a linear equation, and the expression $2\ell + 2w$ is called a *linear combination* of ℓ and w.



A **linear combination** is an expression in which all variables are raised to the first power and are not multiplied or divided by each other. Linear combinations may have 2, 3, 4, or more variables.

Mental Math

Solve for x.

a. $|x| < 12$

b. $|2x| < 12$

c. $|2x - 4| < 12$

d. $|2x - 4| \geq 12$

Activity

At Harry's Hamburger Hovel, hamburgers cost \$2.50 and hot dogs cost \$2. Suppose you have \$30 to spend for hamburgers and hot dogs for a family gathering.

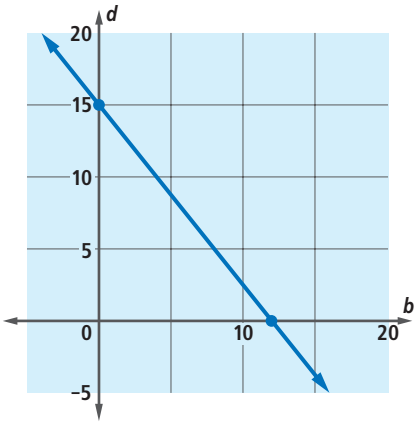
Step 1 Write an equation relating the number b of hamburgers and the number d of hot dogs you can purchase to spend all the money.

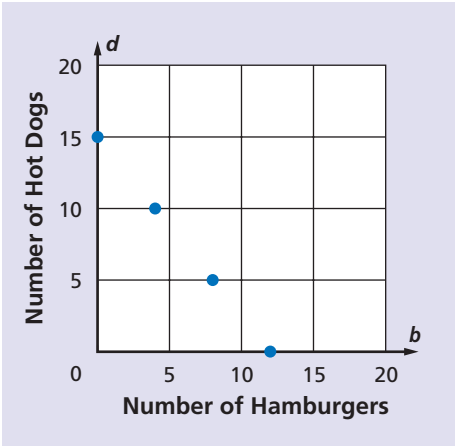
Step 2 Solve the equation in Step 1 for d.

Step 3 Graph the equation in Step 2 without taking the real-world context into account.

Step 4 Identify all points on the graph that correspond to a possible combination of hamburgers and hot dogs you can buy.

In Step 3 of the Activity you should have found that the graph of the equation is a line. This function is a *continuous* function whose domain and range are the set of real numbers. However, the entire line, as shown at the left below, is not an appropriate graph of this situation because counts of hamburgers and hot dogs are whole numbers. The linear function in this situation is *discrete; x* and *y* must be nonnegative integers. At the right below is a graph of the four ordered pairs of nonnegative integers that satisfy the equation.





STOP QY

▶ **QY**

Give the domain and range of the discrete function graphed above at the right.

GUIDED

Example 1

Give two different situations that could be modeled by the linear combination equation $6x + 3y = 30$. For each situation, state whether the linear function is discrete or continuous, and explain what the point (3, 4) represents.

Solution Each situation should specify what the numbers and variables in the equations represent. Here are two possible solutions.

a. You jog for x hours at 6 miles per hour and walk for y hours at 3 miles per hour, going a total of __?__ miles. The point (3, 4) represents __?__ hours of jogging and __?__ hours of walking.

You can jog and walk for any part of an hour; it does not make sense to restrict the domain to whole numbers, so the function is __?__.

b. You buy x adult tickets at __?__ each and y child tickets at __?__ each. The total cost is __?__. The point (3, 4) represents __?__.

You cannot buy part of a ticket. The function is __?__.

The equation $6x + 3y = 30$ is in the form $Ax + By = C$, with $A = 6$, $B = 3$, and $C = 30$. When A and B are not both zero, the equation $Ax + By = C$ is called the **standard form** of an equation for a line.

Linear equations commonly occur in chemistry. For example, chemists describe the concentration of a solution in terms of the number of moles of the substance per liter (mol/L). One mole of a substance is approximately 6.02×10^{23} molecules. So, for example, 2 liters of a solution of 5 mol/L hydrochloric acid contains $2 \text{ L} \cdot 5 \text{ mol/L} = 10$ moles, or about 6.02×10^{24} molecules, of hydrochloric acid. Although a well-equipped lab usually stocks solutions in various concentrations, a chemist often has to mix his or her own solution at a particular concentration by combining other stock solutions. Pharmacists, painters, and others who deal with mixtures use the same mathematics.

Example 2

Suppose a chemist wants to mix x liters of a 2.5 mol/L solution of acid with y liters of a 7.25 mol/L solution to obtain a mixture with 10 moles of acid.

a. **Write an equation that relates x, y, and the total number of moles of acid.**

b. **How many liters of the 7.25 mol/L solution must be added to 1.3 liters of the 2.5 mol/L solution to get 10 moles of acid in the final mixture? Answer to the nearest 0.1 liter.**

Solution

a. To find the number of moles of acid in each solution, multiply the concentration by the quantity.

(L)(mol//L) = moles, so

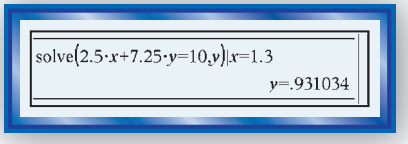
x liters of 2.5 mol/L solution contain 2.5x moles of acid.

y liters of 7.25 mol/L solution contain 7.25y moles of acid.

So, an equation representing the situation is $2.5x + 7.25y = 10$.

b. Think: I want to know y when $x = 1.3$. On one CAS this translates to

`solve(2.5x + 7.25y = 10,y)|x = 1.3.`



This CAS gives $y \approx 0.9$. So, you need to add approximately 0.9 liter of the 7.25 mol/liter solution.

Check 1 Substitute 1.3 for x and 0.9 for y in the original equation. $2.5(1.3) + 7.25(0.9) = 9.775$, which is close but not exactly equal because of rounding. It checks.

Check 2 Solve by hand. Substitute 1.3 for x. Then $2.5 \cdot 1.3 + 7.25y = 10$, so $7.25y = 6.75$. Then $y = \frac{6.75}{7.25} = \frac{27}{29} \approx 0.931$. It checks.

Questions

COVERING THE IDEAS

1. a. **Fill in the Blank** The expression $15P + 12L$ is called a ___?___ of P and L.
   b. Give a situation that could be modeled by the equation $15P + 12L = 600$.
2. **Multiple Choice** Which of the following is not a linear combination of x and y?
   A $x + y$ **B** $y - x$ **C** $4 \cdot x + x \cdot y$ **D** $x \cdot 2 + y \cdot -8$
3. At a vegetable market, Farmer Bob sells T tomatoes at \$0.50 each, S squash at \$0.75 each, and K bunches of kale at \$1.25 each. Write a linear combination to express the total amount of money he takes in.

In 4 and 5, a situation is given.

a. Write a linear combination equation describing the situation.
b. Determine whether the situation is discrete or continuous.

4. Ayani is planning a trip across Europe. He will travel b km by bus and t km by train over a 5,000 km route.
5. Jewel is planning a trip across the United States. She wants to visit 12 cities along the way, taking a bus to b cities and taking a train to t cities.
6. Refer to Example 2.
   a. **Fill in the Blanks** The point (0.8, 1.1) is an approximate solution to the equation $2.5x + 7.25y = 10$. This solution means that you used about ___?___ liters of 2.5 mol/L solution, about ___?___ liters of 7.25 mol/L solution, with a total of about ___?___ moles of acid.
   b. What is the domain of the relation $2.5x + 7.25y = 10$ in this situation?
   c. Find the y-intercept of the graph. What combination does it represent?
7. Sodium hydroxide, NaOH, is a common compound used in chemistry. Suppose that x liters of a solution that is 5.2 mol/L sodium hydroxide are combined with y liters of a solution that is 7.8 mol/L sodium hydroxide. In Parts a–c, write an expression for the number of moles of NaOH in
   a. the 5.2 mol/L solution.
   b. the 7.8 mol/L solution.
   c. the two solutions combined.

d. If Tyra wants 3.6 moles of NaOH in the final mixture, what equation relates x, y, and the 3.6 total moles of NaOH?

e. How many liters of the 7.8 mol/L solution must be added to 0.4 liter of the 5.2 mol/L solution to get 3.6 moles of NaOH in the final mixture?

APPLYING THE MATHEMATICS

8. Refer to the Activity. Suppose that you have \$40 to spend on hamburgers and hot dogs.
   a. In standard form, write a new equation relating the costs of hamburgers and hot dogs.
   b. Graph your equation on your graphing utility. Set the window to $\{x|\ 0 \le x \le 16\}$ and $\{y|\ 0 \le y \le 20\}$. Sketch the graph.
   c. Make a table showing all the possible combinations of hamburgers and hot dogs that you could purchase for exactly \$40.

9. A charity makes and sells piñatas as a fundraiser. Large piñatas cost \$25 and small piñatas cost \$15. Let L be the number of large piñatas and S be the number of small piñatas that are sold.
   a. What kinds of numbers make sense for S and L in this context?
   b. How much money will the charity take in if 4 large piñatas and 6 small piñatas are sold?
   c. If the charity takes in a total of \$225, write an equation relating S, L, and the amount of money taken in.
   d. Graph the equation from Part c. Use L as the independent variable.
   e. Make a discrete graph of the solution set appropriate to this situation.
   f. Give all possible pairs (L, S) of large and small piñatas the charity could have sold to earn \$225.

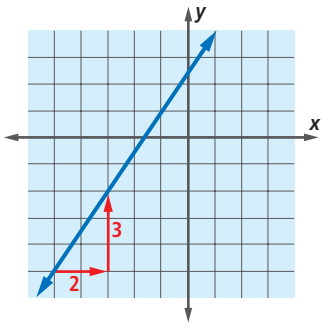
A traditional piñata is usually made of paper and is filled with toys or sweets.

10. Driving from the city to visit a friend in the country, suppose a person drives at an average speed of 15 miles per hour on city streets and 40 miles per hour on the highway. Suppose the person spends C hours on city streets and H hours driving on the highway, and the friend lives 60 miles away.
    a. Write an equation relating C, H, and the total distance.
    b. If the person spent 2 hours driving on city streets, how many minutes did the person spend on the highway?

11. How many pounds of cashews at \$3.29/lb should be mixed with peanuts at \$1.79/lb to create a mixture of 5 pounds of nuts worth \$2.29/lb?

REVIEW

12. **Multiple Choice** Which of the following could be an equation of the graph at the right? **(Lesson 3-1)**



A $y = \frac{2}{3}x + 1$ B $y = \frac{3}{2}x - 1$ C $y = 3x - 2$

D $y = x + \frac{2}{3}$ E $y = \frac{3}{2}x + \frac{5}{2}$

13. If t varies jointly with s and r, and r varies directly with m, show that t varies jointly with s and m. **(Lesson 2-9)**

14. Consider the points (3, 1), (–4, 4.5), and (5, 0). Determine if they lie on a line. If so, state the slope of the line; if not, explain why not. **(Lesson 2-4)**

15. Michelangelo's *David*, a statue in Florence, Italy, is 17 feet high and made out of marble. Over the years, many replicas have been made of this statue. The weight w of a marble replica varies directly as the cube of the replica's height h. **(Lesson 2-1)**
    a. Write an equation modeling this situation.
    b. **Fill in the Blank** The weight of a 12" marble replica would be __?__ the weight of the original statue.

16. Graph $y = \sqrt{x^2 - 4}$ on a calculator. **(Lesson 1-4)**
    a. Does this graph appear to be a graph of a function?
    b. What is the domain?
    c. What is the range?

EXPLORATION

17. In many schools, a student's grade point average is calculated using linear combinations. Some schools give 4 points for each A, 3 points for each B, 2 points for each C, and 1 point for each D. Suppose a person gets 7 As, 3 Bs and 2 Cs.
    a. Calculate this person's total number of points.
    b. Divide your answer in Part a by the total number of classes to get the grade point average.
    c. Calculate your own grade point average for last year using this method.

QY ANSWER

The domain is {0, 4, 8, 12} and the range is {0, 5, 10, 15}.