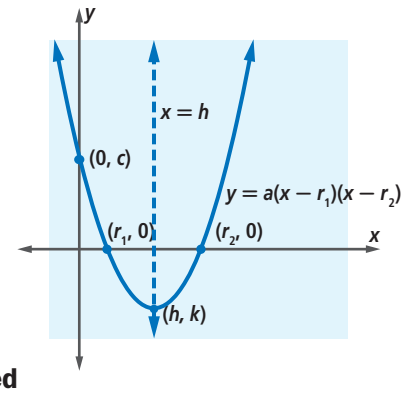
Chapter 12 Summary and Vocabulary

- From the **standard form of a quadratic equation,** $y = ax^2 + bx + c$, the vertex of the parabola is not visible, but if the form is converted to $y - k = a(x - h)^2$, then the vertex is (h, k). The process of converting is called **completing the square.**

- The x-intercepts of this parabola are the solutions to the equation $ax^2 + bx + c = 0$. These can be found using the Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Letting $r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$, then $y = a(x - r_1)(x - r_2)$. This **factored form** is a special case of a more general theorem about polynomials: If $P(x)$ is a polynomial and $P(r) = 0$, then $x - r$ is a factor of $P(x)$.



- Because the factored form of a polynomial enables you to see the x-intercepts, it is useful to be able to factor quadratic expressions $ax^2 + bx + c$ into two linear factors $dx + e$ and $fx + g$. In this chapter, we considered only factoring situations where a, b, c, d, e, f, and g are integers. Most quadratic expressions with integer coefficients do *not* factor into linear factors with integer coefficients. The key is the value of the **discriminant $b^2 - 4ac$.** If a, b, and c are real numbers and $b^2 - 4ac > 0$, then there are two real solutions to the equation $ax^2 + bx + c = 0$. If $b^2 - 4ac = 0$, then $r_1 = r_2$; there is exactly one solution to the equation, and the vertex of the parabola is on the x-axis. If $b^2 - 4ac < 0$, then there are no real solutions; the parabola does not intersect the x-axis.

- The factoring of polynomials also helps in work with **rational expressions.** By dividing out common factors from the numerator and denominator, you can write rational expressions in **lowest terms** and can be added or subtracted.

Vocabulary

12-1
vertex form of an equation for a parabola

12-2
complete the square

12-3
factored form (of a quadratic function)

12-4
square term
linear term
constant term
prime polynomial over the integers

12-7
cubic polynomial

12-8
rational expression
lowest terms

Theorems and Properties

Parabola Vertex Theorem (p. 716)
Factor Theorem for Quadratic Functions (p. 731)
Discriminant Theorem (p.749)
Factor Theorem (p. 755)

Chapter 12

Self-Test

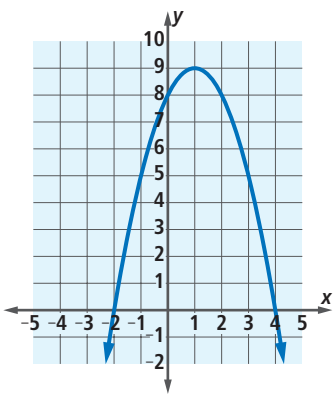
Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

In 1–3, factor completely.

1. $x^2 + 3x - 40$
2. $m^2 - 17m + 72$
3. $-9h^2 + 9h - 2$
4. **Multiple Choice** Which of the following can be factored over the integers?

   A $x^2 + 22x - 9$ **B** $x^2 + 9x - 22$

   C $x^2 - 9x + 22$ **D** $x^2 - 22x + 9$
5. Determine what number must be added to $z^2 - 12z$ to complete the square.
6. Simplify the expression $\frac{3x^2 - 75}{2x^2 - 7x - 15}$ and indicate all restrictions on x.
7. **Multiple Choice** Which equation is graphed below?



   A $y = (x + 2)(x - 4)$

   B $y = (x - 2)(x + 4)$

   C $y = (-x - 2)(x + 4)$

   D $y = -(x + 2)(x - 4)$
8. **a.** Give an equation in factored form for a polynomial function with 3 x-intercepts: 0, –3, and 9.

   b. Rewrite your equation from Part a in standard form.
9. A square frame is x feet on each side. The painting it holds is 1 foot shorter and 2 feet narrower than the frame. If the area of the painting is 12 square feet, what are the dimensions of the frame?

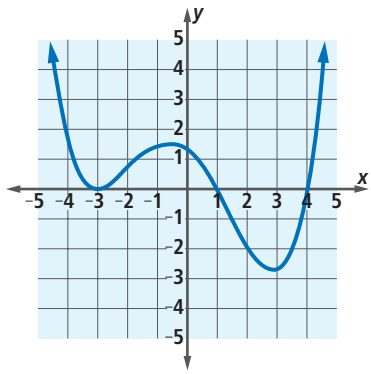
In 10 and 11, determine the vertex of the parabola with the given equation.

10. $y = x^2 - 2x - 3$
11. $y = 7 - 4x - 2x^2$

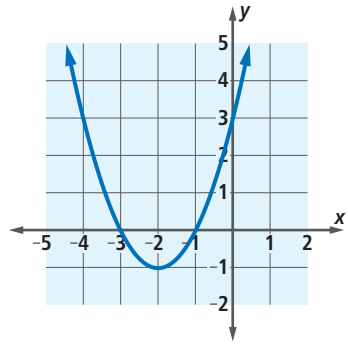
In 12 and 13, sketch the graph of the equation without a calculator.

12. $y - 4 = 2(x - 3)^2$
13. $y = (x - 5)(x + 4)$
14. Factor $3n^4 - 15n^3 + 18n^2$ completely over the integers.
15. Suppose a rectangle has an area of 486 cm^2, and one side is 9 cm longer than the other side. Find the dimensions of the rectangle.
16. **True or False** A quadratic expression with a positive discriminant is always factorable over the integers. Justify your answer.

17. Write a possible equation for the polynomial graphed below.



18. For the graph of the parabola, Marie wrote $y = (x + 3)(x + 1)$. Anthony wrote $y = (x + 2)^2 - 1$.



a. Are Marie and Anthony's equations equivalent? Explain.

b. What is Anthony's equation useful for finding on the graph?

Chapter 12

Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A **Complete the square on a quadratic expression.** **(Lesson 12-2)**

In 1–4, a quadratic expression is given.

a. Determine what number must be added to the expression to complete the square.

b. Complete the square.

1. $x^2 + 4x$
2. $t^2 - 5t$
3. $z^2 + bz$
4. $w^2 - \frac{3}{4}w$

OBJECTIVE B **Factor quadratic expressions of the form $x^2 + bx + c$ and $ax^2 + bx + c$.** **(Lessons 12-4, 12-5)**

In 5–8, factor the quadratic expression.

5. $x^2 - x - 6$
6. $y^2 + 60y + 800$
7. $m^2 - 2m - 24$
8. $n^2 + 4n - 12$

In 9–12, factor the trinomial.

9. $3x^2 - 2x - 8$
10. $5x^2 + 16x + 3$
11. $6d^2 - 8d - 8$
12. $8n^2 + 21 + 34n$

OBJECTIVE C **Find the product of three or more binomials.** **(Lesson 12-7)**

In 13–16, an equation of a function is given.

a. Identify the x-intercepts of its graph.

b. Put the equation into standard form.

13. $y = 6(x + 1)(x - 3)(2x - 11)$
14. $y = x(x - 5)(x + 2)$
15. $f(x) = (x - 4)(2x - 7)(x + 2)$
16. $g(x) = (3 - x)(4x - 1)(x + 3)(x + 1)$

OBJECTIVE D **Use factoring to write rational expressions in lowest terms.** **(Lesson 12-8)**

In 17 and 18, simplify the rational expression and indicate all restrictions on values of the variables.

17. $\frac{4n^2m - 8nm^2}{nm}$
18. $\frac{-7rst^2q^4}{(s + 1)tq}$

In 19–21, write as a single rational expression. You may need a calculator or a CAS.

19. $\frac{-4n^2 + 23n - 15}{n^2 - 8n + 15}$
20. $\frac{3}{x + 4} - \frac{3x - 7}{x^2 + 3x - 4}$
21. $\frac{-m^3 + 5m^2 - 2m - 8}{2m^3 - 9m^2 + 3m + 14}$

PROPERTIES Principles behind the mathematics

OBJECTIVE E **Determine whether a quadratic polynomial can be factored over the integers.** **(Lessons 12-4, 12-5, 12-6)**

22. **True or False** Every quadratic polynomial whose discriminant is an integer is factorable over the integers.
23. For what values of the discriminant is a quadratic polynomial factorable?

In 24–27, determine whether the quadratic polynomial is factorable over the integers. If it is not, label it *prime*. If it is, factor the polynomial.

24. $5x^2 + 33x + 12$
25. $15x^2 + 69x + 72$
26. $4c^2 - 14c + 8$
27. $8x^2 + 34x + 182$

OBJECTIVE F Apply the Factor Theorem. (Lesson 12-7)

28. $f(x) = 3x^3 - 15x^2 - 42x$.
    a. Write $f(x)$ in factored form.
    b. What does the Factor Theorem tell you about the graph of $f(x)$?

In 29 and 30, x-intercepts are given.

a. Give an equation in factored form for a polynomial function with the given x-intercepts.

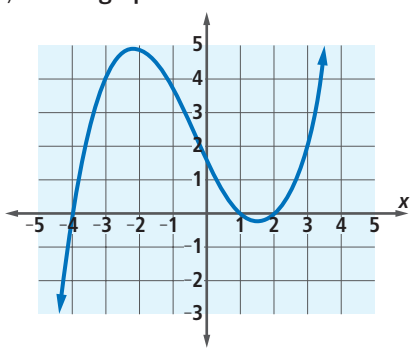
b. Rewrite your equation in standard form.

29. 2 and –2

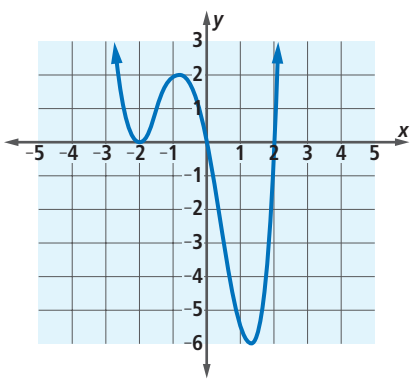
30. 0, 7, and –10

In 31 and 32, give a possible equation, in factored form, for the graph.

31.



32.



USES Applications of mathematics in real-world situations

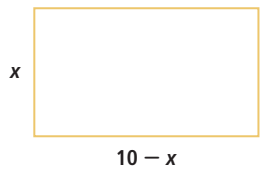
OBJECTIVE G Solve problems involving areas and perimeters of rectangles that lead to quadratic functions or equations. (Lesson 12-1)

In 33 and 34, a rectangle is given.

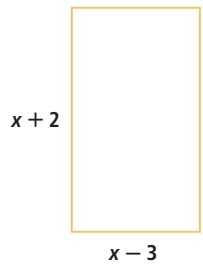
a. Find its area A and perimeter P.

b. Determine the dimensions of the rectangle so that the perimeter and area are numerically equal.

33.



34.



35. A rectangular soccer field has a perimeter of 390 yards.
    a. Write a formula for the area of the field in terms of its length x.
    b. If the width of the field must be at least 50 yards and at most 75 yards, what is the maximum area of the field?

36. A dairy cow is walking around the outside of her rectangular pasture at an average speed of 2 feet per second.
    a. If it takes the cow 20 minutes to walk around the pasture, what is the perimeter of the pasture?
    b. Write a formula for the area A of the pasture in terms of the length of one side L, and determine the dimensions of the rectangle that will maximize the cow's pasture.

37. The Irish sport of hurling can be played on a rectangular field with an area of 12,600 m^2. If the longer side of a hurling field is 50 m longer than the shorter side, what are the dimensions of the field?

38. The perimeter of a rectangle is 24 cm.
   a. Write a formula for the area A of the rectangle in vertex form in terms of the length x of one of its sides.
   b. What is the meaning of the vertex in this scenario?

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE H **Graph quadratic functions whose equations are given in vertex form.** (Lesson 12-1)

In 39–42, sketch the graph of the equation without a calculator.

39. $y - 3 = (x - 5)^2$

40. $y - 5 = (x + 2)^2$

41. $y + 4 = 2(x + 8)^2$

42. $y + 2 = -3(x - 7)^2$

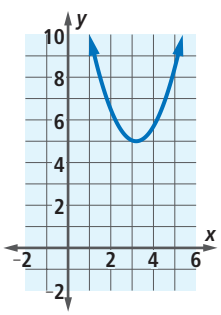
OBJECTIVE I **Find the vertex of a parabola whose equation is given in standard form.** (Lesson 12-2)

In 43–46, match the graph with one of the equations i. through iv. below.

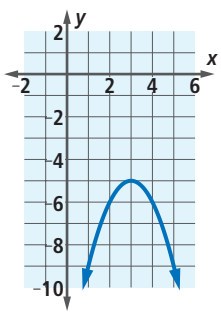
i. $y = -x^2 + 6x - 14$ ii. $y = x^2 - 6x + 14$

iii. $y = -x^2 - 6x - 4$ iv. $y = x^2 + 6x + 9$

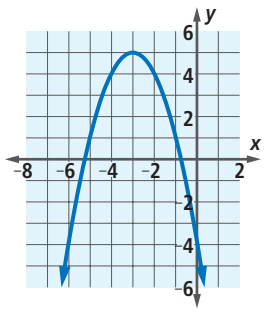
43.



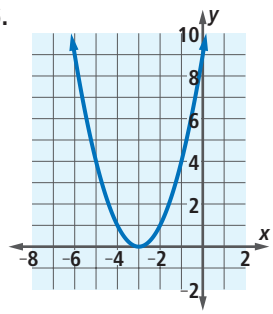
44.



45.



46.



In 47–50, find the vertex of the parabola.

47. $y = 2x^2$

48. $y = x^2 + 3x - 2$

49. $y = -3x^2 + 2x - 5$

50. $y = -x^2 + 6x + 4$

OBJECTIVE J **Graph quadratic functions whose equations are given in factored form.** (Lesson 12-3)

In 51–54, sketch the graph of the equation without using a calculator.

51. $y = (x - 2)(x + 3)$

52. $y = (x + 4)(x - 6)$

53. $y = (2x - 1)(x + 4)$

54. $y = (3x + 7)(x + 8)$