To all students enrolled in AP Pre-Calculus Students,

Congratulations on your decision to join the thousands of other students across the country that will be enrolled in AP Pre-Calculus in the upcoming school year. AP Precalculus prepares students for other college-level mathematics and science courses. Through regular practice, students build deep mastery of modeling and functions, and they examine scenarios through multiple representations. The course framework delineates content and skills common to college precalculus courses that are foundational for careers in mathematics, physics, biology, health science, social science, and data science.

AP Pre-Calculus is not an easy class. No Advanced Placement class is easy. You can expect to spend time studying outside of class, as well as in class. How can you prepare for this class? Complete the summer packet by August 14, 2025. Get the required supplies which include graph paper, pencil, and a graphing calculator such as the TI-84+. You may also want to go to a bookstore this summer and pick up an AP preparation guide for the AP Exam.

If you have any questions, do not hesitate to e-mail Mr. Thaemert at jthaemert@usd232.org.

Welcome to AP PreCalculus!

AP Pre-Calculus Summer Assignment 2025
Due the August 14, 2025!!!!

Welcome to AP Pre-Calculus!

All of your work is attached with instructions. Be sure to finish all work by the beginning of school in August as it will be collected and graded. There will also be an assessment on the materials in this packet within the first two weeks of returning to school.

If you are struggling and need more help with any topics, Khan Academy is a great free resource.

Upon your return to school in August, this packet is expected to be turned in the first day of class. You are expected to complete each part notes and each assignment. These assignments will be counted as two grades as follows:

1. **Homework Grade: Complete ALL sections of the notes packet – 25 pts**
2. **Quiz Grade: Complete ALL of the Delta Math assignments at 100% and the one worksheet, will be graded in class. – 75 pts**

Assignments will be completed using Delta Math plus one worksheet.

Summer Outline:

1. **You will need to enroll AP Pre-Calc Summer Course 2025 course on Delta Math. To do this, click the following link and follow the prompts.**
   https://www.deltamath.com/students?code=QT6K-E4LW

   Class Code 55QD-4PK9

2. **Lesson 1:**
   a) Watch the following video and complete the 1.1: Interval Notation Notes in the notes packet:
   https://youtu.be/JsuxrBKd4Jg

   b) Complete the Inequality & Interval notation assignment on Delta Math.

3. **Lesson 2:**
   a) Watch the following video and complete the 1.2: Exponents and Radicals Notes in the notes packet:
   https://www.youtube.com/watch?v=3iYHVzWT-pM
   https://www.youtube.com/watch?v=bOzE_g9FOl8

b) Complete the following two assignments on Delta Math.

- Exponent Rules
- Radicals

4. **Lesson 3:**
   a) Watch the following video and complete the 1.3: Algebraic Expression Notes in the notes packet: https://www.youtube.com/watch?v=WvAtlZyOKas

   c) Complete the following two assignments on Delta Math.

   - Polynomial Operations
   - Factoring

5. **Lesson 4:**
   a) Watch the following videos and complete the 1.4: Rational Expressions Notes in the notes packet: https://youtu.be/ydQldroUL1E

   b) Complete the Rational Expressions assignment on Delta Math.

6. **Lesson 5:**
   c) Watch the following videos and complete the 1.9: Circles Notes in the notes packet:

   https://www.youtube.com/watch?v=NU5rZp8n_5c

   d) Complete the 1.9 Worksheet assignment at the end of the packet. You may print it off or do the work on a separate sheet of paper.

AP
PRE-CALCULUS
Chapter 1
Lecture Notes

Section 1.1: Interval Notation

Instructional Objectives: Students will be able to:
(1) Write between Interval and Inequality Notation.

Notation for Inequalities

Interval Notation Inequality Notation

Using Interval Notation

Interval notation is another way to write an inequality. Interval notation is often used as a way to explain the domain or range of a function. In order to use interval notation you must first be familiar with the symbols that will be used.

- $<$ and $>$ ________________
- $\geq and \leq$ ____________________
- U ______________________
- $\pm\infty$ ____________________

Example 1: Write each of the following inequalities in interval notation.

a) $x < -4$

b) $x \geq 12$

c) $0 < x < 59$

d) $-10 \leq x \leq -1$

e) $x < -21 \ or \ x \geq 17$

f) $-100 \leq x \leq 15 \ or \ x > 35$

Example 2: Write each interval in inequality notation.

a) $[-3,5)$

b) $[1,9] \ U \ [16,\infty)$

c) $(-\infty,3] \ U \ (5,22)$

d) $(-15,0) \ U \ [3,\infty)$

e) $(5,10]$

Section 1.2: Exponents and Radicals

Instructional Objectives: Students will be able to:

(1) Use properties of exponents.
(2) Use properties of radicals.
(3) Simplify and combine radicals.
(4) Rationalize denominators.
(5) Use properties of rational exponents.

Integer Exponents

Repeated Multiplication → **Exponential Form**

$a \cdot a \cdot a \cdot a \cdot a$ →

$(-4)(-4)(-4)$ →

$(2x)(2x)(2x)(2x)$ →

Exponential Notation

If a is a real number and n is a positive integer, then

$$a^n = \underbrace{a \cdot a \cdot a \cdot a \cdots\cdots a}_{\text{n factors}}$$

where n is the **exponent** and a is the **base.**

Properties of Exponents

Let a and b be real numbers, variables, or algebraic expressions, and let m and n be integers. (All denominators and bases are nonzero.)

Property		Example
1. $a^m a^n = a^{m+n}$	→	1.
2. $\frac{a^m}{a^n} = a^{m-n}$	→	2.
3. $a^{-n} = \frac{1}{a^n}$	→	3.
4. $a^0 = 1$	→	4.
5. $(ab)^m = a^m b^m$	→	5.
6. $(a^m)^n = a^{mn}$	→	6.
7. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	→	7.
8. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$	→	8.
9. $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$	→	9.

Integer Exponents

It is important to recognize the difference between expressions such as

$$(-2)^4 \ and \ -2^4$$

Example 1: Use the properties of exponents to simplify each expression.

a. $(2a^3b^2)(3ab^4)^3$

b. $3a(-4a^2)^0$

c. $\left(\frac{x}{y}\right)^3 \left(\frac{y^2x}{z}\right)^4$

d. $\frac{6st^2}{2s^{-2}t^2}$

e. $\left(\frac{y}{3z^2}\right)^{-2}$

Radicals and Their Properties

Properties of Radicals

Let *a* and *b* be real numbers, variables, or algebraic expressions such that the indicated roots are real numbers, and let *m* and *n* be positive integers.

Property	Example
1. $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ →	**1.**
2. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ →	**2.**
3. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ →	**3.**
4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ →	**4.**
5. $(\sqrt[n]{a})^n = a\text{, if } n \text{ is odd}$ →	**5.**
6. $(\sqrt[n]{a})^n = \lvert a\rvert\text{, if } n \text{ is even}$ →	**6.**

Example 2: Use the properties of radicals to simplify each expression.

a. $\sqrt{8} \cdot \sqrt{2}$

b. $\left(\sqrt[3]{5}\right)^3$

c. $\sqrt[3]{x^3}$

d. $\sqrt[6]{x^6}$

Simplifying Radicals

An expression involving radicals is in simplest form when the following conditions are satisfied.

1. All possible factors have been removed from the radical.
2. All fractions have radical-free denominators (a process called *rationalizing the denominator* accomplishes this).
3. The index of the radical is reduced.

Example 3: Simplifying Radicals

a. $\sqrt[3]{24}$

b. $\sqrt[4]{48}$

c. $\sqrt{75x^3}$

d. $\sqrt[3]{24a^4}$

Simplifying Radicals

- ✓ Radical expressions can be combined (added or subtracted) when they are like radicals—that is, when they have the same index and radicand.
- ✓ For instance, $\sqrt{2}, 3\sqrt{2},\ and\ \frac{1}{2}\sqrt{2}$ are like radicals, but $\sqrt{3}\ and\ \sqrt{2}$ are unlike radicals.
- ✓ To determine whether two radicals can be combined, you should first simplify each radical.

Example 4: Combining Radicals

a. $2\sqrt{48} - 3\sqrt{27}$

b. $\sqrt[3]{16x} - \sqrt[3]{54x^4}$

Rationalizing Denominators

- ✓ To rationalize a denominator of the form $a - b\sqrt{m}$ *or* $a + b\sqrt{m}$ multiply both numerator and denominator by a conjugate: $a + b\sqrt{m}$ *or* $a - b\sqrt{m}$
- ✓ If a = 0, then the rationalizing factor for $\sqrt{m}$ *is itself*, $\sqrt{m}$

Example 5: Rationalize the denominator of each expression.

a. $\frac{5}{2\sqrt{3}}$

b. $\frac{2}{1-\sqrt{5}}$

Rational Exponents

Definition of Rational Exponents

If a is a real number and n is a positive integer such that the principal nth root of a exists, then $a^{1/n}$ is defined as $a^{1/n} = \sqrt[n]{a}$, *Where* $1/n$ is the **rational exponent** of a.

Moreover, if m is a positive integer that has no common factor with n, then
$a^{m/n} = \left(a^{1/n}\right)^m = \left(\sqrt[n]{a}\right)^m$ and $a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$

Example 6: Simplifying with Radical Exponents

a. $(-32)^{-4/5}$

b. $\left(-5x^{5/3}\right)\left(3x^{-3/4}\right)$

Section 1.3: Algebraic Expressions

Instructional Objectives: Students will be able to:
(1) Write polynomials in standard form.
(2) Add, subtract, and multiply polynomials.
(3) Use polynomials to solve real life problems.
(4) Factoring out the Greatest Common Factor.
(5) Factor Difference of Squares.
(6) Factor by Grouping.
(7) Factor Trinomials

Polynomials

One of the most common types of algebraic expressions is the polynomial. Some examples are

$$2x + 5, \quad 3x^4 - 7x^2 + 2x + 4, \quad and \quad 5x^2y^2 - xy + 3$$

The first two are *polynomials in x* and the third is a *polynomial in x and y*.

The terms of a polynomial in *x* have the form ax^k where *a* is the coefficient and *k* is the degree of the term.

Polynomials with one, two, and three terms are called monomials, binomials, and trinomials, respectively. A polynomial written with descending powers of *x* is in **standard form.**

Writing polynomials in Standard Form

Polynomial	Standard Form	Degree	Leading Coefficient
a. $4x^2 - 5x^7 - 2 + 3x$	$-5x^7 + 4x^2 + 3x - 2$	7	−5
b. $4 - 9x^2$	$-9x^2 + 4$	2	−9
c. 8	$8\left(8 = 8x^0\right)$	0	8

Operations with Polynomials

You can add and subtract polynomials in much the same way you add and subtract real numbers. Add or subtract the *like terms* (terms having the same variables to the same powers) by adding or subtracting their coefficients.

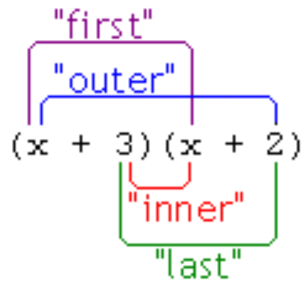
Example 1: Sum and Differences of Polynomials

a. $(5x^3 - 7x^2 - 3) + (x^3 + 2x^2 - x + 8)$

b. $(7x^4 - x^2 - 4x + 2) - (3x^4 - 4x^2 + 3x)$

FOILING:

- ✓ Useful ONLY for a two-term polynomial times another two-term polynomial.
- ✓ The method is called "FOIL".
- ✓ The letters F-O-I-L come from the words **first, outer, inner, last**.
- ✓ Always combine like terms in expression if possible



Example 2: Simplify each expression using the FOIL method.

a.) $(2x + 4)(3x + 5)$

b.) $(x - 4)(4x + 6)$

c.) $(8x - 1)(7x - 4)$

d.) $(2x + 1)(2x - 1)$

e.) $(2x + 3)(x^2 - 5x + 4)$

Factoring

The process of writing a polynomial as a product is called **factoring.** It is an important tool for solving equations and for simplifying rational expressions.

Factoring out the Greatest Common Factor (GCF):

Example 1: Factor out the GCF.

a. $4x^2 - 18x$ b. $3x^5 - 9x^3$ c. $12a^3bd - 54a^2b^4c^3$ d. $2x^2 + 3y + 9$

Factoring the Difference of Squares:

Formula: $u^2 - v^2 =$

Example 2: Factor

a. $x^2 - 36$ b. $16x^2 - 81$

Factor by Grouping:

Example 3: Factor $5x^2 + 20x - 8x - 32$

<u>Factoring with a leading coefficient of 1:</u>

Example 4: Factor

$a.\ x^2 - 7x - 18$ b. $x^2 - 5x - 14$ c. $x^2 - 9x + 8$

Factoring when the Leading Coefficient is *not* one:

Steps:

1) Factor out the GCF if there is one.
2) If the expression has **three terms (trinomial)** and is in the form **$ax^2 + bx + c$**, then determine factors of **a** and **c** whose sum is ***b***.
3) Rewrite your middle term with the two factors from step 2.
4) Factor by grouping.

Example 5: Factoring with a leading coefficient greater than 1

a. $3x^2 - 16x + 20$ b. $2x^2 + 7x - 30$

c. $12x^2 + 26x - 10$

Section 1.4: Rational Expressions

Instructional Objectives: Students will be able to:
(1) Simplify Rational Expressions.
(2) Add, Subtract, Multiply and Divide Rational Expressions.
(3) Simplify Complex Fractions.

Rational Expressions

A rational expression is a fractional expression in which both the numerator and the denominator are polynomials.

Simplify a Rational Expression:

- ✓ Recall that a fraction is in simplest form when its numerator and denominator have no factors in common aside from ±1.
- ✓ To write a fraction in simplest form, divide out common factors.
- ✓ The key to success in simplifying rational expressions lies in your ability to ***factor*** polynomials.
- ✓ When simplifying rational expressions, factor each polynomial completely to determine whether the numerator and denominator have factors in common.

Example 1: Simplify.

b. $\frac{x^2+4x-12}{3x-6}$

b. $\frac{x^2-1}{x^2+x-2}$

Multiplying and Dividing Rational Expressions:

Example 2: Multiply

$$\frac{2x^2+x-6}{x^2+4x-5} \cdot \frac{x^3-3x^2+2x}{4x^2-6x}$$

Example 3: Divide

a. $\frac{x^2-14x+49}{x^2-49} \div \frac{3x-21}{x+7}$

b. $\frac{x-4}{x^2-4} \div \frac{x^2-3x-4}{x^2+5x+6}$

Complex Fractions

Example 4: Simplify

a. $\frac{\frac{x^2}{(x+1)^2}}{\frac{x}{(x+1)^3}}$

b. $\frac{\frac{2}{x}-3}{1-\frac{1}{x-1}}$

<u>Adding and Subtracting Rational Expressions:</u>

To add or subtract rational expressions, use the LCD (least common denominator) method.

Example 5: Perform the operation and simplify.

a. $\frac{x}{x-3} - \frac{2}{3x+4}$

b. $\frac{3}{x-1} - \frac{2}{x} + \frac{x+3}{x^2-1}$

c. $\frac{1}{x^2-1} - \frac{2}{(x+1)^2}$

Section 1.9: Circles

Instructional Objectives: Students will be able to:
(1) Write equations of circle.

Circles

A circle is defined by the location of its center and the length of its radius. A circle with a center of __________ and a radius of _____ has an equation of ____.

Example 1 - Write the standard form of the equation of the specified circle.

a. C (-7, 4); $r = 7$

b. C (0, -9); $r = 4.2$

Example 2 - Find the center and radius of the circle.

a. $x^2 + (y - 5)^2 = 4$

b. $(x - 2)^2 + (y + 1)^2 = 12$

Review: Completing the Square

- Steps:

1. Move __ to the ______ side of the equation.
2. Divide all terms by ____.
3. Add ____ to both sides of the equation.
4. Factor the ____ side.

Example 3 – Complete the square.

a) $x^2 - 8x + 13 = 0$

b) $3x^2 - 12x + 6 = 0$

Completing the Square: Circle Equations

Example 4: Write the equation in standard from. Then find the center and radius of the circle.

a) $x^2 + y^2 - 8x + 4y - 16 = 0$

b) $4x^2 + 4y^2 - 16x - 24y + 51 = 0$

Integrated Math II

Name________________________________

1.9 Assignment

Use the information provided to write the standard form equation of each circle.

1) Center: $(3, -15)$
Radius: 1

2) Center: $(11, 14)$
Radius: 3

3) Center: $(-2, 13)$
Radius: 6

4) Center: $(14, 12)$
Radius: 3

Use the information provided to write Center and Radius.

5) $(x + 14)^2 + (y + 4)^2 = 4$

6) $(x + 15)^2 + (y + 1)^2 = 16$

7) $(x - 3)^2 + (y + 12)^2 = 36$

8) $(x + 14)^2 + (y + 8)^2 = 16$

Use the information provided to write the standard form equation of each circle.

9) $x^2 + y^2 + 20y + 63 = 0$

10) $x^2 + y^2 - 14x + 2y - 53 = 0$

11) $x^2 + y^2 - 32x - 14y + 304 = 0$

12) $x^2 + y^2 + 18x + 8y + 61 = 0$

13) $x^2 + y^2 + 2x + 20y + 85 = 0$

14) $x^2 + y^2 - 30x + 22y + 330 = 0$