Greater Atlanta Christian School

Welcome to AP Calculus BC!

I am very glad that again this year the Honors Analysis class was able to get through derivatives. This is good on many levels. 1) The students from AB won't be bored the first six weeks as we make our way through limits and derivatives. 2) We will have more time on the BC topics. This is a huge advantage and you will be very appreciative of this during the 2nd semester.

However, it all depends upon you actually having the knowledge of the limits and derivatives, which is the foundation of everything else we do. So do this packet as a review of most of the material. I have listed below the concepts I am assuming you know from limits and derivatives. Mark any that you don't know or need to discuss:

1. Limits of functions (including one-sided limits)

Calculating limits using algebra (examples of the various types of these problems including straight substitution, factoring, conjugates, special limits).
Estimating limits from graphs or tables of data
Describing asymptotic behavior in terms of limits involving infinity
Dominance…which functions dominate others

2. Continuity

When is a function continuous at a point x = a? (3 parts to this definition)
(Understanding continuity in terms of limits)
Geometric understanding of graphs of continuous functions
Intermediate Value Theorem
Extreme Value Theorem

3. Differential Calculus

What is a derivative?
Definition of the derivative of a function
Definition of the derivative of a function at a point x = a
Relationship between differentiability and continuity
What is the symmetric difference quotient?
How does the calculator compute the derivative of a function and why is that important?
Tangent lines vs. Normal Lines
Local linearization of a function at a point
Instantaneous rate of change vs. average rate of change
Approximate rate of change from graphs and tables of values
Theorems on differentiation (all of the derivative rules)
Implicit differentiation

Come to class on Day 1 ready to discuss and turn in this packet.
I am looking forward to an excellent year!
~Mr. Washington

Limits: Someone gave this simplistic definition of a limit: It is the height a function is headed for as you get close to a specific x-value. (Nice way to visualize a limit.)

1) Consider the function $f(x)=\dfrac{x}{\sqrt{x+1}-1}$.

a) Create a table of values evaluating the function near x = 0 and use the results to estimate the limit. $\lim\limits_{x\to 0}\dfrac{x}{\sqrt{x+1}-1}\approx$ ____________

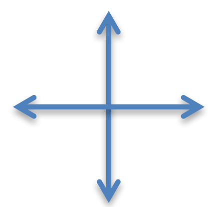
(Hint: Fastest way to do this is to enter the function in $y_1 =$. Then do TABLE SETUP on Indpt: ASK. Then go to TABLE and enter desired x values.)

x	-.01	-.001	-.0001	.0001	.001	.01
$\dfrac{x}{\sqrt{x+1}-1}$						

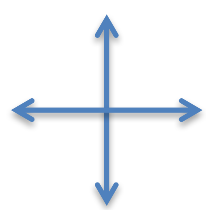
b) Find the limit of $f(x)$ algebraically.

2) We need to know these limits. They can be easily remembered by the graphs of the functions. Answer the questions and sketch a simple graph of the function to illustrate.

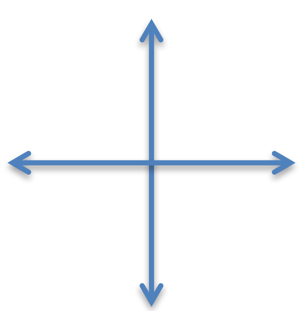
a) $\lim\limits_{x\to\pm\infty}\dfrac{1}{x}=$ b) $\lim\limits_{x\to 0+}\dfrac{1}{x}=$ c) $\lim\limits_{x\to 0^-}\dfrac{1}{x}=$



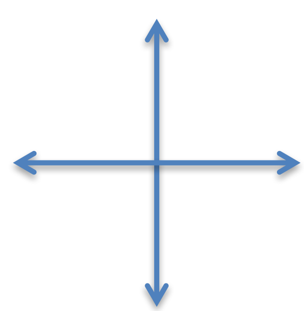
d) $\lim\limits_{x\to 0}\dfrac{\sin x}{x}$ e) $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}$



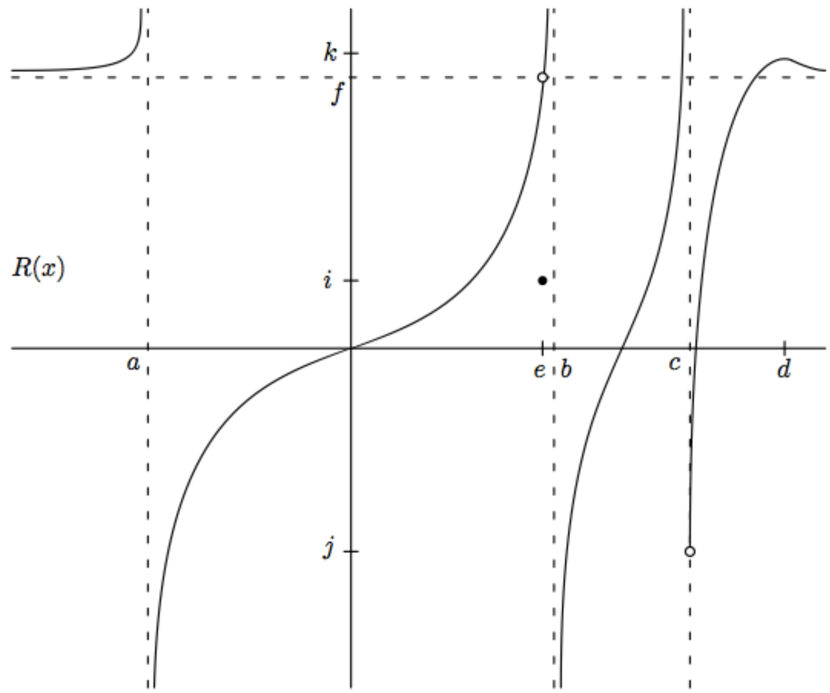
f) $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}$



g) $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{x}$



3)Use the graph to answer the following:



a) $\lim_{x\to\infty} R(x)$

b) $\lim_{x\to-\infty} R(x)$

c) $\lim_{x\to a^+} R(x)$

d) $\lim_{x\to a^-} R(x)$

e) $\lim_{x\to a} R(x)$

f) $\lim_{x\to b^+} R(x)$

g) $\lim_{x\to b^-} R(x)$

h) $\lim_{x\to b} R(x)$

i) $\lim_{x\to c} R(x)$

j) $\lim_{x\to d} R(x)$

k) $\lim_{x\to e} R(x)$

l) $R(e)$

m) $R(0)$

n) $R(b)$

o) Justify your answer in part e).

p) Is $R(x)$ continuous at x = e? Justify your answer using the definition of continuity.

Find the following limits:

4) $\lim_{x\to 3} x^2 - 2x + 3$

5) $\lim_{x\to -1} 5^x$

6) $\lim_{x\to e} \arctan(\ln x)$

7) $\lim_{x\to -\frac{1}{2}^-} \frac{2x^2 - 3x - 2}{2x+1}$

8) $\lim_{x\to -2} \frac{\sqrt{2x+5} - 1}{x+2}$

9) For the function $f(x) = \frac{2x-1}{|x|}$, find the following:

a) $\lim_{x\to\infty} f(x)$

b) $\lim_{x\to-\infty} f(x)$

c) $\lim_{x\to 0^+} f(x)$

d) $\lim_{x\to 0^-} f(x)$

e) All horizontal asymptotes

f) All vertical asymptotes

10) The function $G(x) = \begin{cases} x^2 & x > 2 \\ 4 - 2x & x < 2 \end{cases}$ is not continuous at $x = 2$ because

A) $G(2)$ does not exist
B) $\lim_{x\to 2} G(x)$ does not exist
C) $\lim_{x\to 2} G(x) \neq G(2)$
D) All three statements A,B, and C
E) none of the above

11) The function $G(x) = \begin{cases} x - 3 & x > 2 \\ -5 & x = 2 \\ 3x - 7 & x < 2 \end{cases}$ is not continuous at $x = 2$ because

A) $G(2)$ is not defined
B) $\lim_{x\to 2} G(x)$ does not exist
C) $\lim_{x\to 2} G(x) \neq G(2)$
D) $G(2) \neq -5$
E) All of the above

12) Find k so that $f(x)=\begin{cases}\dfrac{x^2-16}{x-4} & x\neq 4\\ k & x=4\end{cases}$ is continuous for all x.

Find the following limits:

13) $\lim\limits_{x\to 16}\dfrac{x-16}{\sqrt{x}-4}$

14) $\lim\limits_{x\to -4}\dfrac{-2+\sqrt{-x}}{8+2x}$

15) $\lim\limits_{\theta\to 0}\dfrac{\sin 7\theta}{\theta}$

16) $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{x(\ln 8)}$

17) Find the average rate of change of the $R(x)=\sqrt{4x+1}$ over the interval $\left[0,\dfrac{3}{4}\right]$.

18) Find the average rate of change of $h(t)=\dfrac{1}{\tan t}$ over the interval $\left[\dfrac{\pi}{4},\dfrac{3\pi}{4}\right]$.

19) The position $p(t)$, in meters, of an object at time t, in seconds, along a line is given by $p(t)=3t^2+1$.

a) Find the change in position between times $t=1$ and $t=3$.

b) Find the average velocity of the object between times $t=1$ and $t=4$.

c) Find the average velocity of the object between any time t and another time $t+\Delta t$.

20) State the Intermediate Value Theorem.

21) Use the Intermediate Value Theorem to show that the polynomial function $f(x)=x^3+2x-1$ has a zero in the interval $[0,1]$.

22) Use the Intermediate Value Theorem to show that for all spheres with radii in the interval $[1,5]$, there is one with a volume of 275 cubic centimeters.

23) Complete these properties of infinite limits:

Let c and L be real numbers and let f and g be functions such that $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = L$.

a) $\lim_{x \to c} \left[f(x) \pm g(x) \right] =$ ______

b) If $L > 0$ $\lim_{x \to c} \left[f(x)g(x) \right] =$ ________

c) If $L < 0$ $\lim_{x \to c} \left[f(x)g(x) \right] =$ ________

d) $\lim_{x \to c} \frac{g(x)}{f(x)} =$ _________

Determine the following limits using these properties:

24) $\lim_{x \to 0} \left(1 + \frac{1}{x^2} \right) =$

25) $\lim_{x \to 1^-} \frac{x^2 + 1}{\cot \pi x}$

26) $\lim_{x \to 0^+} 3 \cot x =$

27) $\lim_{x \to 1/2} x^2 \tan \pi x$

28) $\lim_{x \to 2^+} \frac{x - 3}{x - 2}$

29) $\lim_{x \to 2^+} \frac{x - 3}{x - 2}$

30) The concept of "dominance" is sometimes important in finding limits. This is a comparison of the relative magnitude of functions and their rates of change. As x grows larger, exponential functions will eventually be larger than polynomial functions, which will be larger than logarithmic functions. With this in mind, find the following limits:

a) $\lim_{x \to \infty} \frac{x^2}{2^x}$

c) $\lim_{x \to -\infty} \frac{x^2}{2^x}$

b) $\lim_{x \to \infty} \frac{2^x}{x^2}$

d) $\lim_{x \to \infty} \frac{\ln x}{2^x}$

Derivatives:
Find the derivative of each function:

1) $f(x)=\left(3x^5-2x^3-3x-\frac{1}{3}\right)^3$

2) $y=\left(\frac{x+5}{x^2+2}\right)^2$

3) $f(x)=x(3x-9)^3$

4) $y=\frac{x}{\sqrt{x^2+1}}$

5) $g(x)=3\tan 4x$

6) $y=\sin(\pi x)^2$

7) $g(t)=\frac{1}{2}x^2\sqrt{16-x^2}$

8) $f(x)=-3\sqrt[4]{2-9x}$

9) $2\cot^2(\pi t+2)$

10) $f(x)=\ln\sqrt{x+1}$

11) $f(x)=\ln\frac{x(x^2+1)^2}{\sqrt{2x^3-1}}$ (Hint: Use properties of logarithms to rewrite an easier expression to differentiate.)

12) $f(t)=\sin^{-1}t^2$

13) $h(x)=x^2\arctan x$

14) $k(x) = \log_3\left(x^2 + e^x\right)$

15) $Q(x) = \ln\left(e^x + 1\right)$

16) $y = e^{\sin 2x}$

17) $y = 3\sin 8x \cos 8x$

18) $y = \dfrac{\ln x}{\sin x}$

19) $y = 3^{\cos x}$

20) $g(x) = 3^{2x} 2^{3x^2}$

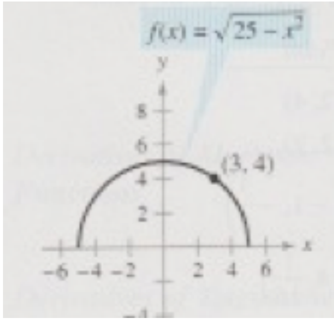
21) $A(x) = \dfrac{\ln x}{x - 2}$

Find $\dfrac{dy}{dx}$ for each of the following:

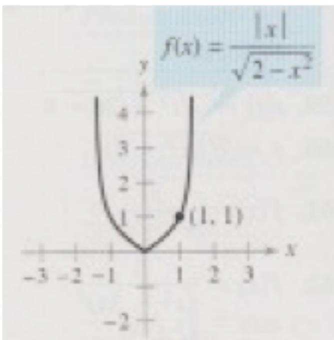
22) $x^2 - y^2 = 7$

23) $x^3 - xy + y^3 = 1$

24) $x = \tan y$

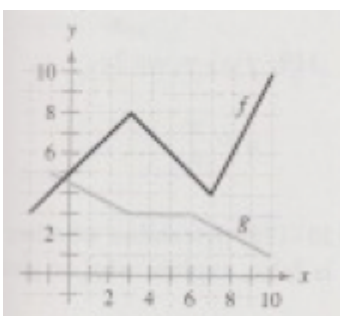


25) Be able to recognize this equation as the top half of a circle. Find the equation of the tangent line at the point (3,4).

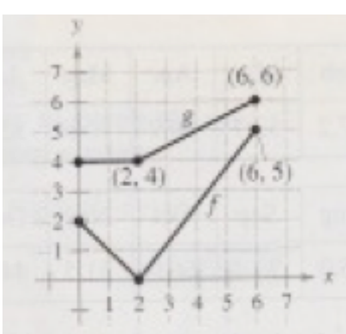


26) This curve is called the “bullet-nose curve.” Find the equation of the tangent line at (1,1).

For 27 and 28, let $h(x) = f(g(x))$ and $s(x) = g(f(x))$



27) Find $h'(1)$ and $s'(5)$.



28) Find $h'(3)$ and $s'(9)$.

29) Find the equation of the both the tangent and the normal lines to the curve $x^2 \cos^2 y - \sin y = 0$ at the point $(0, \pi)$.

Find the following derivatives:

30) $r = \ln(\cos^{-1} x)$

31) $s = \log_5(t-7)$

32) $y = e^{\tan^{-1} x}$

33) $y = t\sec^{-1} t - \frac{1}{2}\ln t$

35) $y = z\cos^{-1} z - \sqrt{1-z^2}$

36) $r = \left(\frac{1+\sin\theta}{1-\cos\theta}\right)^2$

37) On earth, if you shoot a paper clip 64 feet straight up into the air with a rubber band, the paper clip will be $s(t) = 64t - 16t^2$ feet above your hand at *t* seconds after firing.

a) Find $\frac{ds}{dt}$ and $\frac{d^2s}{dt^2}$.

b) How long does it take the paper clip to reach its maximum height?

c) With what velocity does it leave your hand?

d) On the moon the same force will send the paper clip to a height of $s(t) = 64t - 2.6t^2$ feet in t seconds. About how long will it take the paper clip to reach its maximum height, and how high will it go?

38) If a hemispherical bowl of radius 10 inches is filled with water to a depth of x in., the volume of water is given by $V = \pi[10 - (x/3)]x^2$. Find the rate of increase of the volume per inch increase in depth.

Have fun and I will see you in August.