San Mateo Union High School District
Course of Study

Algebra II

Approved by
BOARD OF TRUSTEES
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I. Course Description

A. UC/CSU "a-g" Subject Area: C) Mathematics
B. Rationale for Course: The California state standards in mathematics have changed. We need new courses of study that reflect the changes in content, practice, and rigor. The overall goal will be to increase A-G completion rates. Algebra II is the third course that meet college and career readiness standards in mathematics.
C. Grade Level: 9 - 12
D. Credits: 10 Math
E. Pre-Requisites: Successful completion of Geometry
F. Brief Course Description: The Algebra II course extends students' understanding of functions and real numbers providing the student with the tools needed to model the real world. Students in Algebra II extend their notion of *number* to include complex numbers and see how the introduction of this set of numbers yields the solutions of polynomial equations and the Fundamental Theorem of Algebra. Students deepen their understanding of the concept of *function* and apply equation-solving and function concepts to many different types of functions. The system of polynomial functions, analogous to integers, is extended to the field of rational functions, which is analogous to rational numbers. Students explore the relationship between exponential functions and their inverses, the logarithmic functions. Trigonometric functions are extended to all real numbers, and their graphs and properties are studied. Finally, students' knowledge of statistics is expanded to include understanding *the normal distribution*, and students are challenged to make inferences based on sampling, experiments, and observational studies.

II. Course Purpose: Goals and Student Outcomes

Students demonstrate the mathematical practice standards by:

- Students apply their understanding of various functions to real-world problems. They approach complex mathematics problems and break them down into smaller problems, synthesizing the results when presenting solutions.
- Students deepen their understanding of variables—for example, by understanding that changing the values of the parameters in the expression has consequences for the graph of the function. They interpret these parameters in a real-world context.

- Students continue to reason through the solution of an equation and justify their reasoning to their peers. Students defend their choice of a function when modeling a real-world situation.
- Students apply their new mathematical understanding to real-world problems, making use of their expanding repertoire of functions in modeling. Students also discover mathematics through experimentation and by examining patterns in data from real-world contexts.
- Students continue to use graphing technology to deepen their understanding of the behavior of polynomial, rational, square root, and trigonometric functions.
- Students make note of the precise definition of complex number, understanding that real numbers are a subset of complex numbers. They pay attention to units in real-world problems and use unit analysis as a method for verifying their answers.
- Students see the operations of complex numbers as extensions of the
- operations for real numbers. They understand the periodicity of sine and cosine and use these functions to model periodic phenomena.
- Students observe patterns in geometric sums—for example, that the first several sums of the form $\sum_{k=0}^{n} 2^k$ can be written as follows:

$$1 = 2^1 - 1$$
$$1 + 2 = 2^2 - 1$$
$$1 + 2 + 4 = 2^3 - 1$$
$$1 + 2 + 4 + 8 = 2^4 - 1$$

Students use this observation to make a conjecture about any such sum.

Students demonstrate the content standards:

- Students will interpret functions that arise in applications in terms of the context.
- Students will analyze functions using different representations.
- Students will build a function that models a relationship between two quantities.
- Students will build new functions from existing functions.
- Students will construct and compare linear, quadratic, and exponential models and solve problems.
- Students will extend the domain of trigonometric functions using the unit circle.
- Students will model periodic phenomena with trigonometric functions.
- Students will prove and apply trigonometric identities.

- Students will perform arithmetic operations with complex numbers.
- Students will use complex numbers in polynomial identities and equations.
- Students will interpret the structure of expressions.
- Students will write expressions in equivalent forms to solve problems.
- Students will perform arithmetic operations on polynomials.
- Students will understand the relationship between zeros and factors of polynomials.
- Students will use polynomial identities to solve problems.
- Students will rewrite rational expressions.
- Students will create equations that describe numbers or relationships.
- Students will understand solving equations as a process of reasoning and explain the reasoning.
- Students will solve equations and inequalities in one variable.
- Students will represent and solve equations and inequalities graphically.
- Students will translate between the geometric description and the equation for a conic section.
- Students will summarize, represent, and interpret data on a single count or measurement variable.
- Students will understand and evaluate random processes underlying statistical experiments.
- Students will make inferences and justify conclusions from sample surveys, experiments, and observational studies.
- Students will use probability to evaluate outcomes of decisions.

III. Course Outline

I. Linear and Quadractic Functions: Students will explore and transform linear, absolute value and quadratic parent functions. Characteristics of the functions are given both abstract and contextual meaning. Applying their knowledge of these functions in a real world situations, including writing equations based on given data, using lines of best fit to approximate data and solving 3-by-3 systems of equations, will give significance and relevance to these functions, their graphs, and their equations.

II. Quadratic Equations and Polynomial Functions: Students will build on their prior knowledge of how to solve quadratic equations. Solutions are no longer limited to real numbers, but now include the set of complex numbers. Students will now have five strategies to use to solve quadratic equations: graphing, factoring, square rooting, completing the square and the Quadratic Formula. Solving nonlinear systems and quadratic inequalities broaden the students' knowledge of linear functions and quadratic equations. Viewing polynomials through the lens of quadratics, students already have a familiarity with factoring, graph behavior and transformations. Extending from second degree functions to those of third and fourth degree, students work with the end behaviors of graphs, the number of zeros of an *n*th-

degree polynomial and the Fundamental Theorem of Algebra. Important for student understanding of polynomials is that polynomials form a system analogous to integers, namely, they are closed under the operations of addition, subtraction and multiplication.

III. Radical and Rational Functions: A theme in this unit is that the arithmetic of rational expressions is governed by the same rules of the arithmetic of rational numbers. Connecting to the properties of exponents learned in Algebra 1, students now see that exponents can be rational numbers and are no longer restricted to being nonzero integers. Graphs help to illustrate the solutions to radical equations and inequalities. Even and odd functions and domains are investigated and defined. Function operations lead to solving for the inverses of functions where possible. The graphs of functions compared to the graphs of their inverses add a visual component to understanding inverse relationships. From direct variation in middle school, the students in Algebra 2 move on to rational functions, the simplest of which is inverse variation. Graphs play an important role in understanding rational functions as students are introduced to asymptotes and note the effect of simple transformations. Operations with rational expressions are primarily symbolic manipulation, but graphs can be used to confirm results.

IV. Exponential and Logarithmic Functions: Students add to their list of function families with exponential and logarithmic functions. Drawing on their experience with exponential growth and decay functions in Algebra 1, students will be working with the natural base, *e*, along with other bases, and applying them to compound interest, continuous compounding, and other applications. Logarithmic functions follow naturally from an exploration of the properties of exponents. Graphs confirm the inverse relationship between exponential and logarithmic functions. Transformations of the graphs of these functions reinforce that transformations on a graph always have the same effect regardless of the type of underlying function. Exponential functions lead naturally to geometric sequences. New this year is the skill of adding the terms of a sequence. Along with arithmetic and geometric sequences and series, partial sums and sums of infinite geometric series will be explored numerically and graphically.

V. Trigonometry: Students worked with trigonometric ratios and circles in Geometry and were introduced briefly to radian measure. Developing their understanding of trigonometric functions, students will define radian measure and the six trigonometric functions in terms of the unit circle. The concept of a periodic function is developed as students graph sine and cosine by plotting functional values for benchmark angles. The graphs of the remaining four trigonometric functions are deduced from the students' knowledge of sine and cosine. Graphs are again transformed beyond the parent functions. Students will complete Unit 5 with an introduction to the trigonometric identities and the sum and difference formulas.

VI. Probability and Data: Drawing on their knowledge of probability from Algebra 1, students construct sample spaces from a given data set and calculate the probability of an independent, dependent or compound event. Two-way frequency tables are used to compute joint and marginal relative frequencies. Permutations and combinations are used to solve for probabilities of compound events and real-world applications. Binomial and Normal probability distributions are introduced, defined, described, and used to solve for probabilities. In a broad overview, students look at sampling populations and collecting data without introducing bias. Experiments and observational studies are discussed in the context of design, correlation and causation. Unit 6 finishes with the students drawing inferences from experiments and sample surveys where possible.

IV. Key Assignments

I. Representing Quadratic Functions Graphically: This lesson assesses how well students are able to understand what the different algebraic forms of a quadratic function reveal about the properties of its graphical representation. In particular, the lesson will help you identify and help students who have the following difficulties: 1) Understanding how the factored form of the function can identify a graph's roots, 2) Understanding how the completed square form of the function can identify a graph's maximum or minimum point, and 3) Understanding how the standard form of the function can identify a graph's intercept. Before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. The teacher then reviews their work and creates questions for students to answer in order to improve their solutions. After a whole-class interactive introduction, students work in pairs on a collaborative discussion task in which they match quadratic graphs to their algebraic representation. As they do this, they begin to link different algebraic forms of a quadratic function to particular properties of its graph. At the end of the lesson there is a whole-class discussion. In a follow-up lesson students attempt to improve their original response to the assessment task.
http://map.mathshell.org/lessons.php?unit=9245&collection=8

II. Representing Polynomials Graphically: This lesson assesses how well students are able to translate between graphs and algebraic representations of polynomials. In particular, this unit aims to help you identify and assist students who have difficulties: 1) Recognizing the connection between the zeros of polynomials when suitable factorizations are available and graphs of the functions are defined by polynomials, and 2) Recognizing the connection between transformations of the graphs and transformations of the functions obtained by replacing $f(x)$ by $f(x + k)$, $f(x) + k$, $-f(x)$, $f(-x)$. Before the lesson, students attempt the assessment task individually. The teacher

then reviews their work and formulates questions that will help them improve their solutions. During the lesson, students work collaboratively in pairs or threes, matching functions to their graphs and creating new examples. Throughout their work students justify and explain their decisions to peers. During a whole-class discussion, students explain their reasoning. Finally, students improve their solutions to the initial task and complete a second, similar task. http://map.mathshell.org/lessons.php?unit=9270&collection=8

III. Evaluating Statements about Radicals: This lesson assesses how well students are able to use the properties of exponents, including rational exponents and manipulate algebraic statements involving radicals, and discriminate between equations and identities. Before the lesson, students work alone on an assessment task designed to reveal their current understanding. The teacher reviews their work, creating questions to help students improve their solutions. During the lesson, students first work in small groups on a collaborative discussion task. After sharing their solutions with another group, students extend and generalize the math. An optional collaborative task focuses on imaginary numbers. During a whole-class discussion, students review the main mathematical concepts of the lesson. In a follow-up lesson, students review their initial solutions and then use what they have learned to revise the same introductory assessment task and complete a second, similar task.
hhttp://map.mathshell.org/lessons.php?unit=9115&collection=8

IV. Representing Linear and Exponential Growth: This lesson assesses how well students are able to interpret exponential and linear functions and in particular, to identify and help students who have the difficulty translating between descriptive, algebraic, tabular, and graphical representation of the functions, and recognizing how and why a quantity changes per unit interval. Before the lesson, students work individually on an assessment task designed to reveal their current understanding and difficulties working with linear and exponential functions. The teacher reviews their responses and creates questions for students to consider, to help them improve their work. After a whole-class interactive introduction, students work in small groups on a series of collaborative card matching tasks involving simple and compound interest. In a whole-class discussion, students review the main mathematical concepts of the lesson and the strategies used. Students then return to the original task, consider their own responses and the questions posed and use what they have learned to complete a similar task.
http://map.mathshell.org/lessons.php?unit=9240&collection=8

V. Representing Trigonometric Functions: This lesson assesses how well students are able to model a periodic situation, the height of a person on a Ferris wheel, using trigonometric functions, and interpret the constants a, b, c in the formula $h = a + b \cos ct$ in terms of the physical situation, where h is the height of the person above the ground and t is

the elapsed time. Before the lesson, students attempt the assessment task individually. The teacher then reviews their solutions and formulates questions for students to answer in order for them to improve their work. In the lesson, students engage in pairs or threes on a related card-matching task involving graphs, functions and verbal descriptions of Ferris Wheels. Throughout their work they justify and explain their decisions to peers. In a whole-class discussion, students explain and extend their solutions and methods. Finally, students work alone on a task similar to the assessment task.
http://map.mathshell.org/lessons.php?unit=9255&collection=8

VI. Representing Data with Frequency Graphs: This lesson assesses how well students are able to use frequency graphs to identify a range of measures and make sense of this data in a real-world context, and understand that a large number of data points allow a frequency graph to be approximated by a continuous distribution. Before the lesson, students work alone to complete an assessment task designed to reveal their current understanding. After a whole-class introduction, students work in pairs or threes on a collaborative discussion task, matching written interpretations and graphs as they begin to link these two representations. There is a whole-class discussion to end the lesson. In a follow-up lesson, students work alone on a similar task to the assessment task.
http://map.mathshell.org/lessons.php?unit=9415&collection=8

V. Instructional Methods and/or Strategies

I. Think - Pair - Share: Teacher asks a question or assigns a problem and allow students to think or work with a partner for one to five minutes before requesting an answer.
II. Show thinking: Either verbally or written students need to explain how they arrived at a solution to a problem.
III. Questioning and Wait Time: Teachers provide a thought provoking question to students, then allow the students time to think and work toward an answer.
IV. Group Work: Students are given challenging problems to work, and allowed to work on the problem in a group of two, three, or four. Challenging mathematics problems take time, effort, reasoning, and thinking to solve. Group work is an important teaching strategy for having students work on the common core performance tasks.
V. Allowing Students to Struggle: Students learn to persevere in solving challenging mathematics problems by being allowed to struggle with challenging problems. Students need to understand that mathematical problems do not usually have a quick, easy solution. Effective effort is a life-skill and should be learned interdependently and independently.

VI. Assessment Methods and/or Tools

Formative assessments will be used with constructed response questions. Each chapter will have a summative exam that includes no more than 50% new material, projects will be included in the grade and there will be two final exams, one at the end of the first and second semester.

VII. Textbook(s) and Supplemental Instructional Materials

CPM Core Connections; http://map.mathshell.org; MARS/MAC tasks; NCTM Illuminations, Math Vision Project, Ed Puzzle, IXL, That Quiz, Graphing calculators TI-84 Silver Edition, Math Type Software, Smart View Software, graph paper, rulers, individual student whiteboards, LCD projector, document reader, Algebra tiles, and any other CCSS aligned materials