PRACTICE CALCULUS MIDTERM

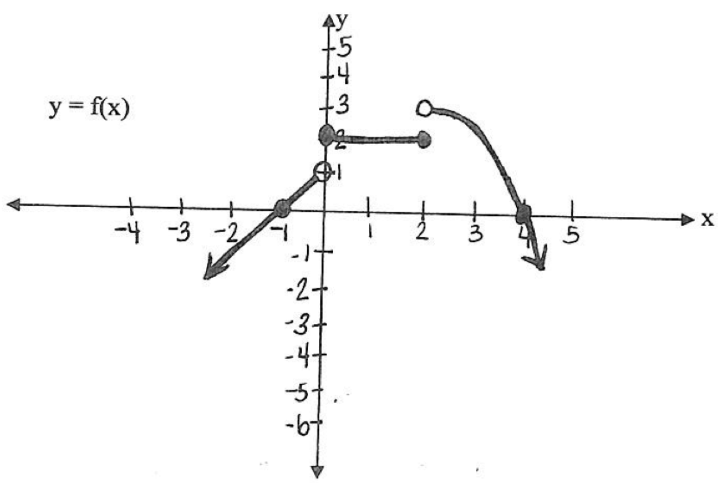
Part I – *place answers on scantron.*

1. Determine $\lim_{x \to 2} (5x^2 - 3x - 12)$ by substitution.

   $5(2)^2 - 3(2) - 12 = 20 - 6 - 12 = \boxed{2}$

2. Given the following graph for f(x), Find:

y = f(x)



A) $\lim_{x \to 0^-} f(x) =$ 1

B) $\lim_{x \to 0^+} f(x) =$ 2

C) $\lim_{x \to 0} f(x) =$ DNE

D) $f(0) =$ 2

E) $\lim_{x \to 2^-} f(x) =$ 2

F) $\lim_{x \to 2} f(x) =$ DNE

G) $\lim_{x \to 1^+} f(x) =$ 2

H) $\lim_{x \to 1} f(x) =$ 2

3. Let $f(x) = \begin{cases} 3x^2 & , x \le 2 \\ 7x & , x > 2 \end{cases}$ Determine:

   A.) $\lim_{x \to 2^-} f(x) = 3(2)^2 = 12$

   B.) $\lim_{x \to 2^+} f(x) = 7(2) = 14$

   C.) $\lim_{x \to 2} f(x) =$ DNE

   D.) $f(2) = 12$

4. Let f(x) = $\begin{cases} x^2 - 2x & , x < 4 \\ 2x & , x > 4 \end{cases}$ Determine:

E.) $\lim_{x \to 4^-} f(x) = (4)^2 - 2(4) = 8$

F.) $\lim_{x \to 4^+} f(x) = 2(4) = 8$

G.) $\lim_{x \to 4} f(x) = 8$

H.) $f(4)$ = DNE

5. Determine the vertical and horizontal asymptotes for the following functions:

A) $f(x) = \frac{4x^2 - 3x + 2}{9x^2 + 5x}$

VA: $9x^2 + 5x = 0$

$x(9x+5) = 0$

$\boxed{x=0}$ | $9x+5=0$

$9x=-5$

$\boxed{x=-\frac{5}{9}}$

HA: $y = \frac{4}{9}$

B) $g(x) = \frac{x^2}{x+4}$

VA: $x+4=0$

$\boxed{x=-4}$

HA: none

C) $h(x) = \frac{7x}{x^2 - 25}$

VA: $x^2 - 25 = 0$

$(x+5)(x-5) = 0$

$x+5=0$ | $x-5=0$

$\boxed{x=-5}$ | $\boxed{x=5}$

HA: $y=0$

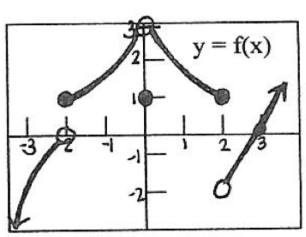
6. Find the average rate of change of the function $f(x) = 2x^2 + 1$ over the interval [1, 3].

$$\frac{F(B)-F(A)}{B-A} = \frac{f(3)-f(1)}{3-1}$$

$$= \frac{(2(3)^2+1) - (2(1)^2+1)}{3-1}$$

$$= \frac{19-3}{2} = \boxed{8}$$

7. (A) State the points of discontinuity, given the following graph for f(x).



[-4,4] by [-3,3]

$\boxed{\text{discontinuity at } x = -2, 0, 2}$

at $x=-2$ it is a jump discontinuity

at $x=0$ it is a removable discontinuity

at $x=2$ it is a jump discontinuity

(B) The function f whose graph is show in above has a removable discontinuity at which points?

$x=-2$

8. Let f(x) = $\begin{cases} x & ,x \le 1 \\ 2-x & ,x > 1 \end{cases}$

A) If f(x) continuous at x = 1? Explain.
B) Is f(x) differentiable at x = 1? Explain.

A) $f(1) = \begin{cases} 1 \\ 2-1=1 \end{cases}$

f(x) is continuous at x=1

B) $f'(x) = \begin{cases} 1 & , x \le 1 \\ -1 & , x > 1 \end{cases}$

$f'(-1) = \begin{cases} 1 \\ -1 \end{cases}$

f(x) is not differential at x = 1. derivatives are different

9. The equation for free fall on a certain planet is s = $-6.1t^2$ ft, where t is in seconds. Assume a rock is dropped from a 50 foot cliff. Find the *speed* of the rock at t = 0.75 sec.

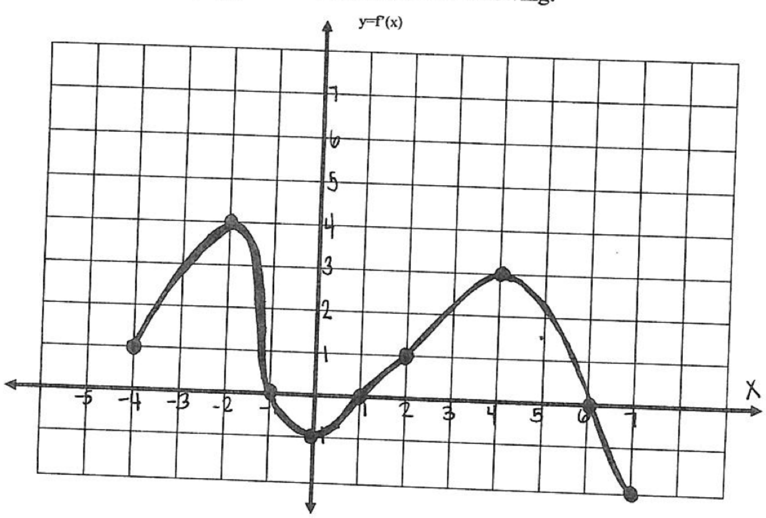
$S'(t) = -12.2t$

$S'(0.75) = -12.2(0.75) = -9.15$

the speed of the rock at t = 0.75 sec is 9.15 ft/sec

☆ Speed is always positive!

10. Given the graph of *f'(x)*, the derivative, find the following:



A) For what values of x does f(x) have a relative maximum? Relative minimum?
relative max at x=-2, 4 , relative min at x=0
B) For what values of x does f(x) have a horizontal tangent? slope = 0
x= -2, 0, 4
C) On what intervals is f(x) increasing? Decreasing?
Inc: (-4,-2) U (0,4) Dec: (-2,0) U (4,7)
D) On what intervals is f(x) concave up? Concave down?
concave up: (-1,1) U (6,7) concave down: (-4,-1) U (1,6)

11. Given $f(x) = x^3 - 6x^2 + 15$, for what interval(s) is the function increasing? Decreasing?

$f'(x) = 3x^2 - 12x$

$3x^2 - 12x = 0$

$3x(x-4) = 0$

$3x = 0 \quad | \quad x - 4 = 0$

$x = 0 \quad | \quad x = 4$

+++++ , - - - - - , +++++ (0, 4)

$f'(-1) = 15$

$f'(2) = -12$

$f'(5) = 15$

Increasing: $(-\infty, 0) \cup (4, \infty)$

Decreasing: $(0, 4)$

12. Given $\lim_{x\to a} f(x) = 6$ and $\lim_{x\to a} g(x) = -4$, find:

A) $\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = 6 - 4 = 2$

B) $\lim_{x\to a}(f(x)-g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = 6 + 4 = 10$

C) $\lim_{x\to a}(f(x)\cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = 6(-4) = -24$

D) $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to b} g(x)} = \frac{6}{-4} = -\frac{3}{2}$

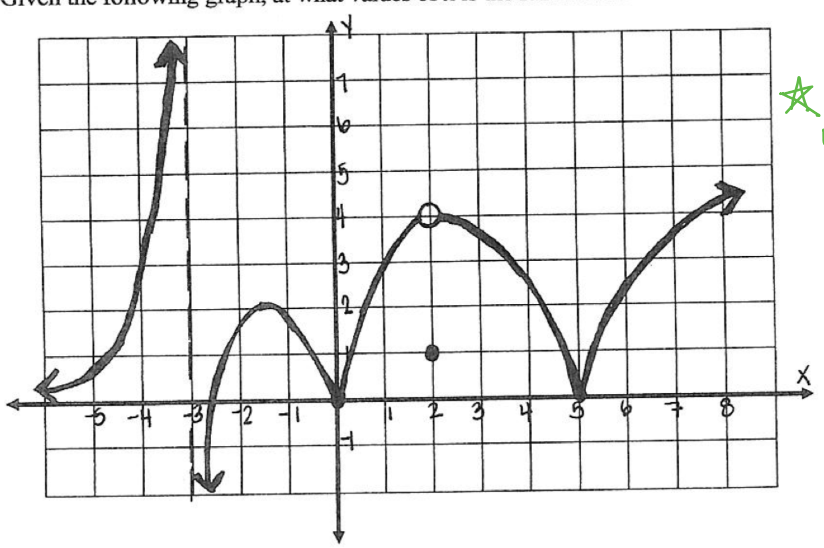
13. Find dy, the approximate change in y, if $y = x^3 + 2x - 7$, $x = 4$ and $dx = 0.01$

$\frac{dy}{dx} = 3x^2 + 2$

$dy = (3x^2 + 2)dx$

$dy = (3 \cdot 4^2 + 2)(0.01) = 0.5$

14. Given the following graph, at what values of x is the function not differentiable?



☆ f(x) is differentiable when its graph is smooth and continuous. Smooth means no sharp points, continuous means no holes or gaps.

$\boxed{x = -3, 0, 2, 5}$

15. Find $\frac{dy}{dx}$ if $y = 5x^4 - 3x^3 + 2x + 7$

$\frac{dy}{dx} = 20x^3 - 9x^2 + 2$

16. Find $\frac{dy}{dx}$ if $y = \frac{4x+7}{2x-3}$

$\frac{lo\,d\,hi - hi\,d\,lo}{lo^2}$

$\frac{dy}{dx} = \frac{(2x-3)(4) - (4x+7)(2)}{(2x-3)^2} = \frac{(\cancel{8x}-12) - (\cancel{8x}+14)}{(2x-3)^2} = \frac{-26}{(2x-3)^2}$

17. Find $\frac{dy}{dx}$ if $y = \cos x - \tan^2 x = \cos x - (\tan x)^2$

$\frac{dy}{dx} = -\sin x - 2\tan x \sec^2 x$

18. Find $\frac{dy}{dx}$ if $y = (4x - 8)^5$

$\frac{dy}{dx} = 5(4x-8)^4(4) = 20(4x-8)^4$

19. Let $f(x) = (3x-5)^4$. Write an equation for the linearization L(x) at *(1, -16)*

$y - y_1 = m(x - x_1)$

$f'(x) = 4(3x-5)^3(3) = 12(3x-5)^3$

$m = f'(1) = 12(3\cdot 1 - 5)^3 = -96$

$y + 16 = -96(x-1)$

$L(x) + 16 = -96(x-1)$

$L(x) = -96x + 96 - 16$

$\boxed{L(x) = -96x + 80}$

20. Find $\frac{dy}{dx}$ if you $= x^4 e^{5x}$

$\frac{dy}{dx} = x^4 e^{5x}(5) + e^{5x} 4x^3 = 5x^4 e^{5x} + 4x^3 e^{5x}$

21. Find $\frac{dy}{dx}$ if $y = \ln(7x^2 - 3x + 5)$.

$\frac{dy}{dx} = \frac{1}{7x^2 - 3x + 5} \cdot 14x - 3 = \frac{14x-3}{7x^2-3x+5}$

22. Find $f'(x)$ if $f(x) = 4\sin^5 7x$.

$f(x) = 4(\sin 7x)^5$

$f'(x) = 20(\sin 7x)^4 \cos 7x \cdot 7$

$\boxed{f'(x) = 140\cos 7x(\sin 7x)^4}$

23. Given $f(x) = 2x^3 + 3x^2 - 12x$, find the values of x where the absolute extrema occur on the interval [-2, 2]

$f'(x) = 6x^2 + 6x - 12$

$6x^2 + 6x - 12 = 0$

$6(x^2 + x - 2) = 0$

$6(x+2)(x-1) = 0$

$x+2=0 \quad x-1=0$

$x=-2 \quad x=1$

$\boxed{x=-2, 1}$

24. Find $\lim_{x \to -1} \frac{x^2 - 5x - 6}{x+1}$, if it exists.

$\lim_{x\to -1} \frac{x^2-5x-6}{x+1} = \lim_{x\to -1} \frac{(x-6)\cancel{(x+1)}}{\cancel{x+1}} = \lim_{x\to -1} x-6 = -1-6 = \boxed{-7}$

25. Find a value c so that the function $f(x) = \begin{cases} 3x + 2c & , x \le 4 \\ 5x + c & , x > 4 \end{cases}$ is continuous.

$3(4) + 2C = 5(4) + C$

$12 + 2C = 20 + C$

$12 + C = 20$

$\boxed{C = 8}$

26. For the function $f(x) = 3x^2 + 2$ at the point (1, 5), find

a) the slope of the curve $f'(x) = 6x$

b) an equation of the tangent line at the given point

c) an equation of the normal line at the given point

(b) $y - y_1 = m(x - x_1)$

$m = f'(1) = 6(1) = 6$

$\boxed{y - 5 = 6(x-1)}$

(c) $m = -\frac{1}{6}$

$\boxed{y - 5 = -\frac{1}{6}(x-1)}$

27. Given $f(x) = (x-4)^3$, find the points of inflection.

$f'(x) = 3(x-4)^2$

$f''(x) = 6(x-4)$

$6(x-4) = 0$

$x - 4 = 0$

$x = 4$ I.V.

$f(4) = (4-4)^3 = 0$

Inflection point $\boxed{(4,0)}$

28. Given $f(x) = x^3 - 12x$, on what interval(s) is the function concave up? Concave down?

$f'(x) = 3x^2 - 12$

$f''(x) = 6x$

$6x = 0$

$x = 0$

$- - - - - - \; + + + + + +$

0

$\boxed{\begin{array}{l} \text{concave up: } (0, \infty) \\ \text{concave down: } (-\infty, 0) \end{array}}$

29. Suppose that u and v are differentiable at x = 5 and that u(5) = 7, v (5) = 4, u´(5) = 1, and v´ (5) = -6.

Find **(a)** $\frac{d}{dx}\left(\frac{u}{v}\right)$ and **(b)** $\frac{d}{dx}(3uv)$ at x = 5

(a) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2} = \frac{4 \cdot 1 - 7 \cdot -6}{4^2} = \frac{23}{8}$

(b) $\frac{d}{dx}(3uv) = 3(uv' + vu') = 3(7 \cdot -6 + 4 \cdot 1) = -114$

30. Find $\frac{dy}{dx}$ if $y = 3^{x^2 - 2x}$

$\ln y = \ln 3^{x^2 - 2x}$

$\ln y = (x^2 - 2x) \ln 3$

$\frac{1}{y} \frac{dy}{dx} = \ln 3 \, (2x - 2)$

$\frac{dy}{dx} = y \cdot \ln 3 (2x - 2)$

$\boxed{\frac{dy}{dx} = 3^{x^2 - 2x} \ln 3 \, (2x - 2)}$

31. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^3 - 27t + 4$, where s is measured in feet and t is measured in seconds.

a) Find the displacement during the first 2 seconds.
b) Find the average velocity during the first 2 seconds.
c) Find the instantaneous velocity when t = 2.
d) Find the acceleration of the particle when t = 2
e) At what value or values of t does the particle change direction?

(a) $f(b) - f(a) = S(2) - S(0) = (2^3 - 27 \cdot 2 + 4) - (0^3 - 27 \cdot 0 + 4) = -42 - 4 = -46$

(b) $\frac{f(b) - f(a)}{b - a} = \frac{-46}{2} = -23$ ft/sec

(c) $v(t) = 3t^2 - 27$ $\quad v(2) = 3(2)^2 - 27 = -15$ ft/sec

(d) $a(t) = 6t$ $\quad a(2) = 6(2) = 12$ ft/sec^2

(e) $3t^2 - 27 = 0$

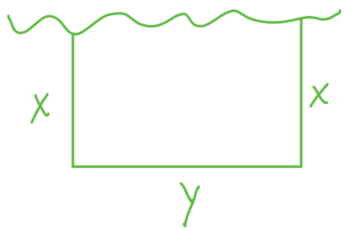
$3(t^2 - 9) = 0$

$3(t+3)(t-3) = 0$

$\cancel{t = -3}$ | $t = 3$

the particle change direction at $\boxed{t = 3 \text{ sec}}$

32. A farmer plans to fence in a rectangular field adjacent to a river. The field must contain 120,000 square meters in order to provide enough soil for farm. What dimensions would require the least amount of fencing if no fencing is needed along the river?



$\boxed{\begin{array}{l} x = 245\text{m} \\ y = 490\text{m} \end{array}}$

our goal is to minimize perimeter (i.e. fencing)

$P = 2x + y$

we are given area $= 120{,}000\ m^2$, so we have

$A = xy = 120{,}000$

$y = \dfrac{120{,}000}{x}$

$P = 2x + \dfrac{120000}{x}$

$P = 2x + 120000x^{-1}$

$P' = 2 - 120{,}000x^{-2} = 2 - \dfrac{120000}{x^2}$

$2 - \dfrac{120000}{x^2} = 0$

$2 = \dfrac{120000}{x^2}$

$\dfrac{120000}{2} = x^2$

$60000 = x^2$

$x = \sqrt{60000} = \boxed{245\text{m}}$

$y = \dfrac{120000}{245}$

$\boxed{y = 490\text{m}}$