A. Find All critical numbers. $\to f'(x)=0$ $\to f'(x)$ DNE

1. $f(x) = 3x^4 - 4x^3$

$f'(x) = 12x^3 - 12x^2$

$12x^3 - 12x^2 = 0$

$12x^2(x-1) = 0$

$12x^2 = 0$ | $x - 1 = 0$

$\boxed{x=0}$ | $\boxed{x=1}$

2. $g(x) = x^3 - 12x^2$

$g'(x) = 3x^2 - 24x$

$3x^2 - 24x = 0$

$3x(x-8) = 0$

$3x = 0$ | $x - 8 = 0$

$\boxed{x=0}$ | $\boxed{x=8}$

3. $f(x) = \frac{x-1}{x+3}$

* Quotient Rule!!

$f'(x) = \frac{(x+3)(1)-(1)(x-1)}{(x+3)^2}$

$f'(x) = \frac{x+3-x+1}{(x+3)^2}$

$f'(x) = \frac{4}{(x+3)^2}$

$\frac{4}{(x+3)^2} = 0$

$4 \neq 0$ Never

But $f'(x)$ DNE if

$(x+3)^2 = 0$

$x+3 = 0$

$\boxed{x=-3}$

4. $f(x) = (9-x^2)^{3/5}$

$f'(x) = \frac{3}{5}(9-x^2)^{-2/5}(-2x)$

$f'(x) = \frac{3(-2x)}{5(9-x^2)^{2/5}} = \frac{-6x}{5(9-x^2)^{2/5}}$

$\frac{-6x}{5(9-x^2)^{2/5}} = 0$

$-6x = 0$

$\boxed{x=0}$

$f'(x)$ DNE if

$5(9-x^2)^{2/5} = 0$

$(9-x^2)^{2/5} = 0$

$9 - x^2 = 0$

$(3-x)(3+x) = 0$

$\boxed{x=3}$ $\boxed{x=-3}$

B. 1) $f(x) = x^2 - 4x + 7$

$f'(x) = 2x - 4 = 0$

$x = 2$

C.V.

$f(0) = 7 \leftarrow$ max

$f(3) = 4$

$f(2) = 3 \leftarrow$ min

(2) $f(x) = \frac{10}{x^2+1}$ on $[-1,2]$

* Quotient Rule

$f'(x) = \frac{(x^2+1)(0) - (2x)(10)}{(x^2+1)^2}$

$= \frac{0 - 20x}{(x^2+1)^2}$

$f'(x) = \frac{-20x}{(x^2+1)^2}$

$\frac{-20x}{(x^2+1)^2} = 0$

$-20x = 0$

$x = 0$

$f'(x)$ DNE if

$(x^2+1)^2 = 0$

$x^2+1 = 0$

$x^2 = -1$

NEVER!!

$f(-1) = \frac{10}{(-1)^2+1} = \frac{10}{1+1} = \frac{10}{2} = 5$

$f(0) = \frac{10}{0^2+1} = \frac{10}{1} = 10$ ← MAXIMUM

$f(2) = \frac{10}{2^2+1} = \frac{10}{5} = 2$ ← MINIMUM

B

(4) a) Maximum at $x = c$

b) Minimum at $x = a$

c) minimum at $x = b$

(5) $f(x) = x^2 - 2x + 1$ on $[0,3]$

$f'(x) = 2x - 2$

$2x - 2 = 0$

$2x = 2$

$x = 1$

$f(0) = 0^2 - 2(0) + 1 = 1$

$f(1) = 1^2 - 2(1) + 1$

$= 1 - 2 + 1$

$= 0$ ← MINIMUM

$f(3) = 3^2 - 3(2) + 1$

$= 9 - 6 + 1$

$= 4$ ← Maximum

B

]

)-3^2

9

)

C. 1. $f(x) = x^3 + 2x$

$f'(x) = 3x^2 + 2$

$3x^2 + 2 = 0$

$x = \sqrt{-2/3}$

Never true!!

+ + + +

-1 0 1

$\boxed{\text{increasing } (-\infty, \infty)}$

C. 2. $f(x) = x^2 + x - 1$

$f'(x) = 2x + 1$

$2x + 1 = 0$

$x = -\frac{1}{2}$

- - - + + +

-1 $-\frac{1}{2}$ 0

$\boxed{\begin{array}{l}\text{inc } (-\frac{1}{2}, \infty) \\ \text{dec } (-\infty, -\frac{1}{2})\end{array}}$

C 3. $f(x) = -x^3 - 2x^2 - 11$

$f'(x) = -3x^2 - 4x$

$-3x^2 - 4x = 0$

$x = 0 \quad x = -4/3$

- - - + + + -

-2 $-\frac{4}{3}$ -1 0

$\boxed{\begin{array}{l}\text{inc } (-4/3, 0) \\ \text{dec } (-\infty, -4/3) \\ \quad (0, \infty)\end{array}}$

C 4. $f(x) = \frac{1}{x}$

$f'(x) = -\frac{1}{x^2}$

$-\frac{1}{x^2} = 0$

Never

$f'(x)$ DNE

$x = 0$

- - - + + +

0

$\boxed{\begin{array}{l}\text{inc } (0, \infty) \\ \text{dec } (-\infty, 0)\end{array}}$

D Give the relative extrema

1) $f(x) = 3x^5 - 5x^3$

$f'(x) = 15x^4 - 15x^2$

$= 15x^2(x^2 - 1)$

$= 15x^2(x-1)(x+1)$

$\boxed{x = 0}$ $\boxed{x = 1}$ $\boxed{x = -1}$

+++ - - - - - - - ++++

-1 0 1

Max @ $x = -1$

min @ $x = 1$

$f'(-2) = 180$

$f'(-.5) = -2.8$

$f'(.5) = -2.8$

$f'(2) = 180$

D 2) $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12$

$6(x^2 + x - 2) = 0$

$6(x+2)(x-1) = 0$

$\boxed{x = -2}$ $\boxed{x = 1}$

+++ - - - - - ++++

-2 1

Max @ $x = -2$

min @ $x = 1$

$f'(-3) = 24$

$f'(0) = -12$

$f'(2) = 24$

E. $f(x) = -x^2 + 18x - 78$

$f'(x) = -2x + 18$

$-2x + 18 = 0$

$x = 9$

+++ - - -

9

max

inc → dec

$\boxed{\begin{array}{l}\text{b) } f'(x) < 0 \\ \quad (9, \infty)\end{array}}$

F

$$f(x) = \frac{x^4}{4} + 3x^3$$

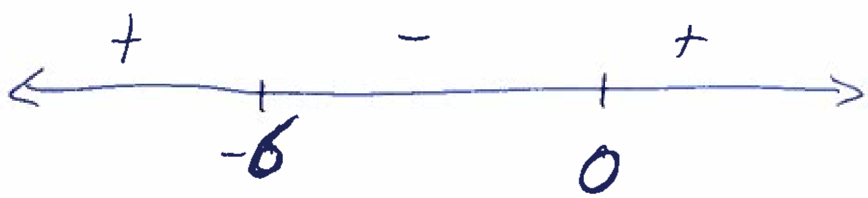
$$f'(x) = x^3 + 9x^2$$

$$f''(x) = 3x^2 + 18x$$

$$3x(x+6) = 0$$

$x = 0$ | $x = -6$

Inflection values at $x=0$ and $x=-6$



$$f''(-7) = 3(-7)^2 + 18(7) = 147 + 126 = 273$$

$$f''(-1) = 3(-1)^2 + 18(-1) = -15$$

$$f''(1) = 3(1)^2 + 18(1) = 21$$

Concave up $(-\infty, -6) \cup (0, \infty)$

Concave Down $(-6, 0)$