Name: ____ Date:_________________

Review Unit 1 AP Calculus BC

NO CALCULATOR IS ALLOWED ON THE REVIEW

Multiple Choice

1. The table below gives several measurements of the velocity of a particle moving along a straight line. What is the smallest possible number of times where $v(t)$ is exactly 5 meters/second $(0 \leq t \leq 14)$?

***t* (sec)**	0	5	7	9	14
***v*(*t*) (m/sec)**	3.35	4.25	2.75	6.55	4.70

(A) 4

(B) 3

(C) 2

(D) 1

(E) 0

2. $\lim_{x\to\infty} \sqrt{\frac{8x^2-5}{2x^2+1}}$

(A) 4

(B) 2

(C) $\sqrt{2}$

(D) 1

(E) The limit does not exist

3. $\lim_{x\to 0} \frac{3x^4+5x^2}{2x^5-4x^2}$

(A) 0

(B) -5/4

(C) 5/2

(D) 1

(E) DNE

4. $\lim_{x\to\infty} 3\sin\left(\frac{1}{x}\right)$

(A) 0

(B) 3

(C) 1/3

(D) -3

(E) DNE

5. $\lim_{h\to 0} \frac{\tan(2(x+h))-\tan(2x)}{h}$ is

(A) 0

(B) $2\cot(2x)$

(C) $sec^2(2x)$

(D) $2sec^2(2x)$

(E) DNE

6. $f(x) = \begin{cases} x+2 & if\ x \le 3 \\ 4x-7 & if\ x > 3 \end{cases}$

Let f be the function defined above. Which of the following statements are true about f?

I. $\lim_{x \to 3} f(x)$ exists

II. f is continuous at $x = 3$

III. f is differentiable at $x = 3$

(A) None

(B) I only

(C) II only

(D) I and II only

(E) I, II and III

7. The graph of a function f is shown below. At which value of x is f continuous but not differentiable?

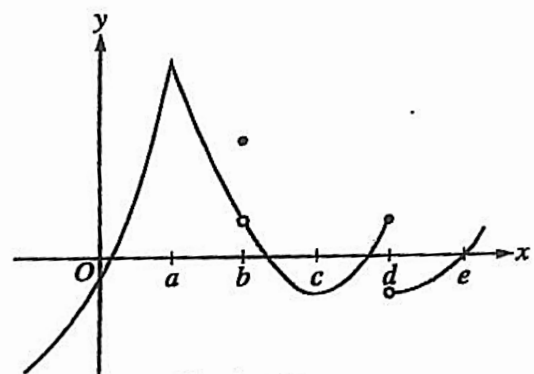
(A) a

(B) b

(C) c

(D) d

(E) e



Graph of f

8. If $y = x^2 \sin(2x)$, then $\frac{dy}{dx} =$

(A) $2x\cos(2x)$

(B) $4x\cos(2x)$

(C) $2x(\sin(2x) + \cos(2x))$

(D) $2x(\sin(2x) - x\cos(2x))$

(E) $2x(\sin(2x) + x\cos(2x))$

9. If f is a function such that $\lim_{x \to a} \frac{f(x)-f(a)}{x-a} = 0$, which of the following must be true?

(A) $\lim_{x \to a} f(x)$ does not exist

(B) $f(a)$ does not exist

(C) $f'(a) = 0$

(D) $f(a) = 0$

(E) $f(x)$ is continuous at $x = 0$

10. If $y = \frac{2x+3}{3x+2}$, then $\frac{dy}{dx}$=

(A) $\frac{12x+13}{(3x+2)^2}$

(B) $\frac{12x-13}{(3x+2)^2}$

(C) $\frac{5}{(3x+2)^2}$

(D) $\frac{-5}{(3x+2)^2}$

(E) $\frac{2}{3}$

11. If $y = (x^3+1)^2$, then $\frac{dy}{dx} =$

(A) $(3x^2)^2$

(B) $2(x^3+1)$

(C) $2(3x^2+1)$

(D) $3x^2(x^3+1)$

(E) $6x^2(x^3+1)$

12. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$

(B) $2\sqrt{3}$

(C) 4

(D) $4\sqrt{3}$

(E) 8

13. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2-2}{x-1}$?

(A) -2

(B) $1/6$

(C) $1/2$

(D) 2

(E) 6

14. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at $x = 0$
II. f is differentiable at $x = 0$
III. f has an absolute minimum at $x = 0$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

15. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = 3x$

(B) $y - 3 = -5(x - 2)$

(C) $y - 6 = -5(x - 2)$

(D) $y - 6 = -7(x - 2)$

(E) $y - 6 = -10(x - 2)$

16. The equation of the line tangent to the curve $y = \frac{kx+8}{k+x}$ at $x = -2$ is $y = x + 4$.

What is the value of k?

(A) -3

(B) -1

(C) 1

(D) 3

(E) 4

17. Let $f(x)$ be defined by $f(x) = \begin{cases} x^3, & if\ x \leq 0 \\ x^2, & if\ x > 0 \end{cases}$ Then $f''(0) =$

(A) 0

(B) 1

(C) 2

(D) 6

(E) DNE

18. If $y = cos^2(2x)$, then $\frac{dy}{dx} =$

(A) $2\cos(2x)\sin(2x)$

(B) $-4\sin(2x)\cos(2x)$

(C) $2\cos(2x)$

(D) $-2\cos(2x)$

(E) $4\cos(2x)$

19. If $y = 2x^{\frac{5}{2}}\tan x$, then $\frac{dy}{dx} =$

(A) $x^{\frac{3}{2}}(5\tan x + 2xsec^2x)$

(B) $5x^{\frac{3}{2}}\tan x - xcsc^2x$

(C) $5x^{\frac{5}{2}}\tan x + 2x^{\frac{3}{2}}sec^2x$

(D) $x^{\frac{3}{2}}(5\tan x + xsec^2x)$

(E) $x^2\tan x - 5xcsc^2x$

20. If $f(x) = x^2cos^2x$, then $f'(\pi) =$

(A) -1

(B) 2π

(C) 0

(D) π

(E) -2π

21. If $f(x) = \frac{\sqrt{x+1}}{x^3+2}$, then $f'(0) =$

(A) $-\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{8}$

(D) $\frac{1}{4}$

(E) DNE

22. Suppose $g(x) = \begin{cases} x^3 + 3x^2 + 2x, & if\ x \leq 0 \\ -x^2, & if\ x > 0 \end{cases}$ Then:

(A) g is continuous and differentiable at $x = 0$

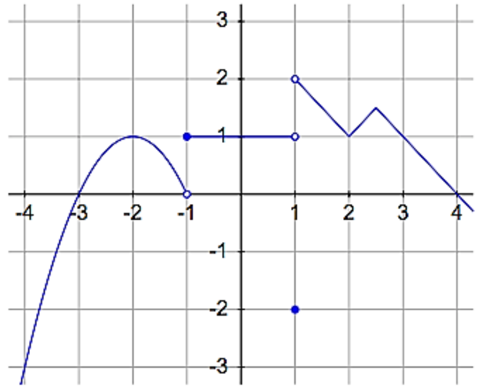
(B) g is continuous at $x = 0$, and g is not differentiable at $x = 0$

(C) g is not continuous at $x = 0$, but g is differentiable at $x = 0$

(D) g is not continuous at $x = 0$, and g is not differentiable at $x = 0$

(E) Nothing can be said about the differentiability of g at $x = 0$

23. Consider the function $g(x)$ whose graph is below:



For what values of x is the function clearly NOT-differentiable?

(A) $x = -2$ only

(B) $x = -1, 1$ only

(C) $x = 2, 2.5$ only

(D) $x = -1, 1, 2, 2.5$

(E) $x = -3, 4$

24. Assuming $f(x), g(x) and\ h(x)$ are continuous and twice differentiable, use the values in the table above to evaluate $h'(1)$ if $h(x) = f^3(g(x))$.

x	0	1	2	3
f(x)	1	3	2	2
g(x)	0	3	1	1
f'(x)	3	1	3	1
g'(x)	4	5	0	4

(A) 60

(B) 45

(C) 42

(D) 30

(E) 18

25. $\lim_{x \to \frac{\pi}{4}} \frac{\sin x - \sin\frac{\pi}{4}}{x - \frac{\pi}{4}} =$

(A) π

(B) $\sqrt{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{\sqrt{2}}{2}$

(E) $\frac{1}{2}$

Show all work.

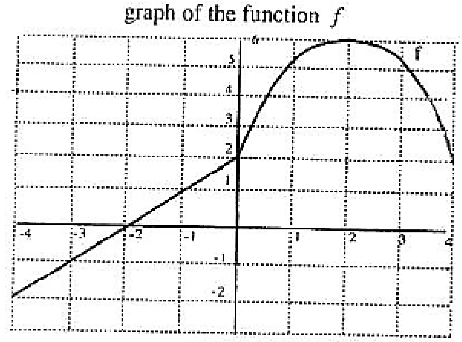
26. If $y = 2sin^2(2x)$, write the equation of the line tangent at $x = \frac{\pi}{8}$.

27. Find the values of a and b to make $f(x)$ differentiable at $x = 1$:

$$f(x) = \begin{cases} 5 + x, & if\ x < 1 \\ ax^2 + bx, & if\ x \geq 1 \end{cases}$$

28. Answer the following questions about the function f, whose graph is shown.

graph of the function f



a) Find $\lim_{x \to 0} f(x)$

b) Find $\lim_{h \to 0^-} \frac{f(0+h)-f(0)}{h}$

c) Find $\lim_{x \to 0} f'(x)$

29. Evaluate the following limits:

a) $\lim_{x \to 3} \left(\frac{x^2+3x-18}{x-3} \right)$

b) $\lim_{x \to \infty} \frac{2x^3-3x^2+2}{8x^3+6}$

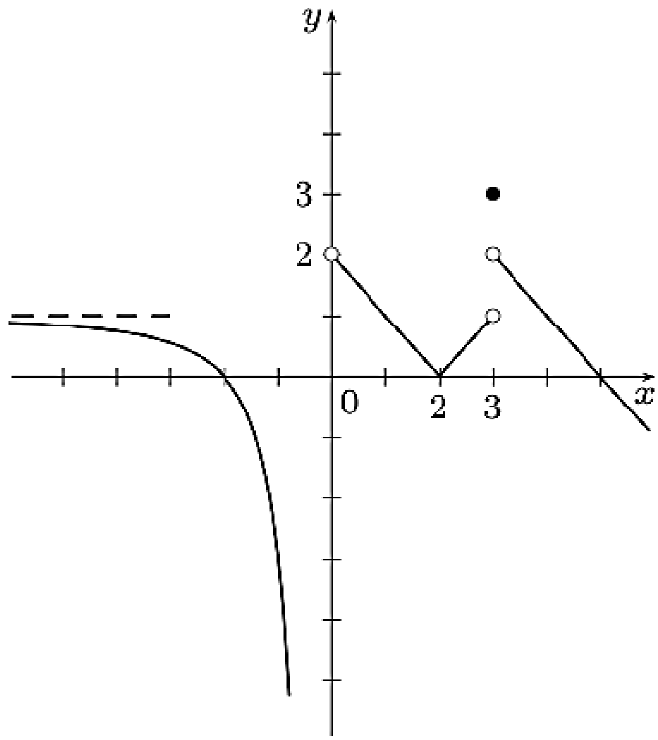
c) $\lim_{x \to 0^+} \frac{|x|}{x}$

d) $\lim_{x \to 0} \frac{\tan x}{5x}$

e) $\lim_{x \to 9} \frac{3-\sqrt{x}}{9-x}$

f) $\lim_{x \to 0} \frac{1-\cos x}{1+x-e^x}$

30. Use the graph of the function $f(x)$ to answer each question.
Use ∞, $-\infty$ or DNE where appropriate.



(a) $f(0) =$

(b) $f(2) =$

(c) $f(3) =$

(d) $\lim\limits_{x \to 0^-} f(x) =$

(e) $\lim\limits_{x \to 0} f(x) =$

(f) $\lim\limits_{x \to 3^+} f(x) =$

(g) $\lim\limits_{x \to 3} f(x) =$

(h) $\lim\limits_{x \to -\infty} f(x) =$

31. Answer the following questions for the piecewise defined function $f(x)$ described on the right hand side.

(a) $f(1) =$

(b) $\lim\limits_{x \to 0} f(x) =$

(c) $\lim\limits_{x \to 1} f(x) =$

$$f(x) = \begin{cases} \sin(\pi x) & \text{for } x < 1, \\ 2^{x^2} & \text{for } x > 1. \end{cases}$$

32. Answer the following questions for the piecewise defined function $f(t)$ described on the right hand side.

(a) $f(-3/2) =$

(b) $f(2) =$

(c) $f(3/2) =$

(d) $\lim\limits_{t \to -2} f(t) =$

(e) $\lim\limits_{t \to -1^+} f(t) =$

(f) $\lim\limits_{t \to 2} f(t) =$

(g) $\lim\limits_{t \to 0} f(t) =$

$$f(t) = \begin{cases} t^2 & \text{for } t < -2 \\ \dfrac{t+6}{t^2 - t} & \text{for } -1 < t < 2 \\ 3t - 2 & \text{for } t \geq 2 \end{cases}$$