The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY (Common Core)

Thursday, January 26, 2017 — 9:15 a.m. to 12:15 p.m., only

Student Name: Key

School Name: Mr. Barnes

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 36 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

GEOMETRY (COMMON CORE)

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 Which equation represents the line that passes through the point $(-2,2)$ and is parallel to $y = \frac{1}{2}x + 8$?

(1) $y = \frac{1}{2}x$

(2) $y = -2x - 3$

(3) $y = \frac{1}{2}x + 3$

(4) $y = -2x + 3$

same slope

(1) $y = \frac{1}{2}x$

$2 = \frac{1}{2}(-2)$

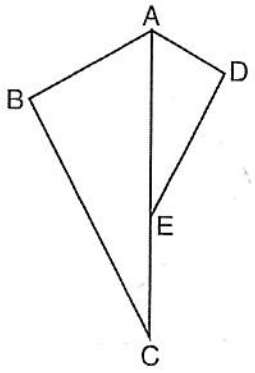
$2 \neq -1$

(3) $y = \frac{1}{2}x + 3$

$2 = \frac{1}{2}(-2) + 3$

$2 = 2$ ✓

2 In the diagram below, $\triangle ADE$ is the image of $\triangle ABC$ after a reflection over the line AC followed by a dilation of scale factor $\frac{AE}{AC}$ centered at point A.



Corresponding $\angle$s of similar $\triangle$s are $=$.

Which statement must be true?

(1) $m\angle BAC = m\angle AED$

(2) $m\angle ABC = m\angle ADE$

(3) $m\angle DAE = \frac{1}{2}m\angle BAC$

(4) $m\angle ACB = \frac{1}{2}m\angle DAB$

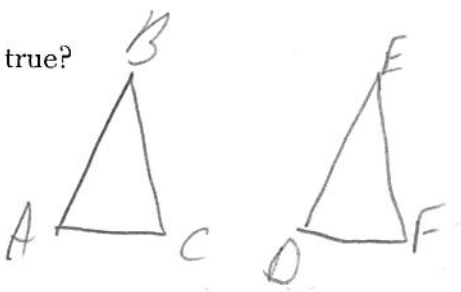
3 Given $\triangle ABC \cong \triangle DEF$, which statement is *not* always true?

(1) $\overline{BC} \cong \overline{DF}$

(2) $m\angle A = m\angle D$

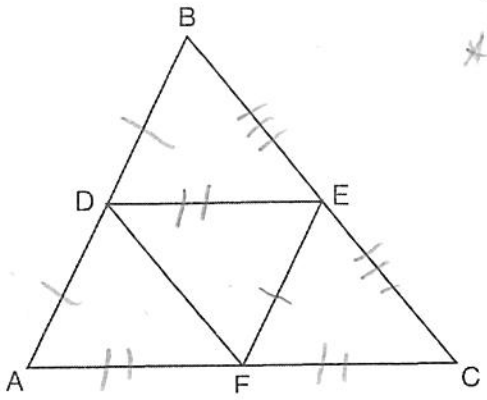
(3) area of $\triangle ABC$ = area of $\triangle DEF$

(4) perimeter of $\triangle ABC$ = perimeter of $\triangle DEF$



Use this space for computations.

4 In the diagram below, $\overline{DE}$, $\overline{DF}$, and $\overline{EF}$ are midsegments of $\triangle ABC$.

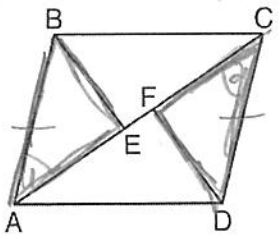


* Cut all sides of $\triangle ABC$ into $\cong$ segments
* half of // side.
∴ $DE = AF$, $EF = AD$

The perimeter of quadrilateral $ADEF$ is equivalent to

(1) $AB + BC + AC$
(2) $\frac{1}{2}AB + \frac{1}{2}AC$
(3) $2AB + 2AC$
(4) $AB + AC$

5 In the diagram below, if $\triangle ABE \cong \triangle CDF$ and $\overline{AEFC}$ is drawn, then it could be proven that quadrilateral $ABCD$ is a



$AB = CD$ CPCTC
$\angle 1 = \angle 2$ CPCTC
$AB \parallel CD$ Alt. int $\angle$s.

(1) square
(2) rhombus
(3) rectangle
(4) parallelogram

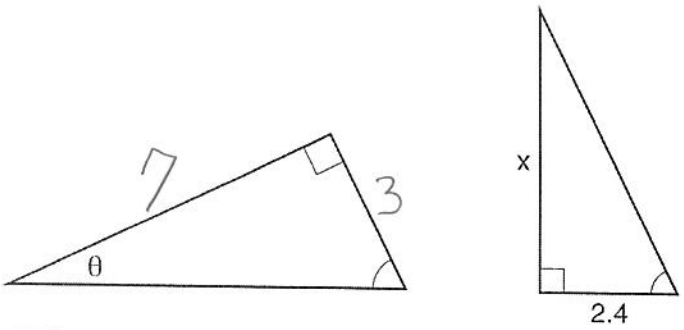
One pair of sides $\cong$ + // so

6 Under which transformation would $\triangle A'B'C'$, the image of $\triangle ABC$, *not* be congruent to $\triangle ABC$?

(1) reflection over the y-axis
(2) rotation of 90° clockwise about the origin
(3) translation of 3 units right and 2 units down
(4) dilation with a scale factor of 2 centered at the origin

Use this space for computations.

7 The diagram below shows two similar triangles.



If $\tan \theta = \frac{3}{7}$, what is the value of x, to the *nearest tenth*?

(1) 1.2
(2) 5.6
(3) 7.6
(4) 8.8

SOH CAH TOA

$\frac{3}{2.4} = \frac{7}{x}$

$3x = 16.8$

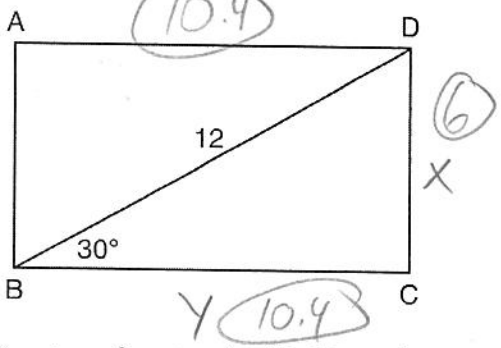
$x = 5.6$

8 A farmer has 64 feet of fence to enclose a rectangular vegetable garden. Which dimensions would result in the biggest area for this garden?

(1) the length and the width are equal
(2) the length is 2 more than the width
(3) the length is 4 more than the width
(4) the length is 6 more than the width

1) 16 × 16, A = 256
2) 17 × 15, 255
3) 18 × 14, A = 252
4) 19 × 13, 247

9 The diagram shows rectangle $ABCD$, with diagonal $\overline{BD}$.



What is the perimeter of rectangle $ABCD$, to the *nearest tenth*?

(1) 28.4
(2) 32.8
(3) 48.0
(4) 62.4

SOH CAH TOA

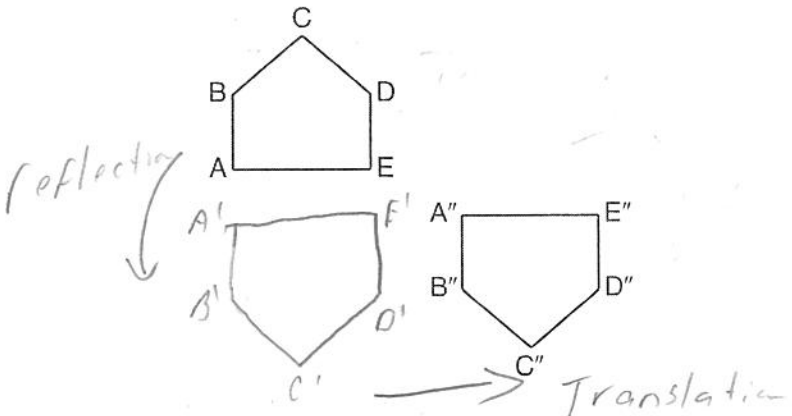
$\frac{\sin 30}{1} = \frac{x}{12}$ | $\cos 30 = \frac{y}{12}$

$x = 12 \cdot \sin 30$ | $y = 12 \cdot \cos 30$

$x = 6$ | $y = 10.4$

Use this space for computations.

10 Identify which sequence of transformations could map pentagon $ABCDE$ onto pentagon $A''B''C''D''E''$, as shown below.



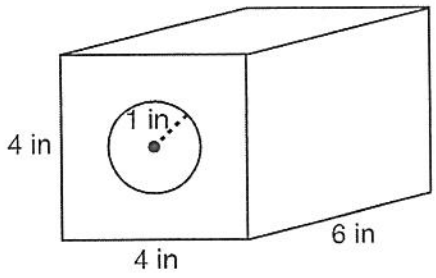
(1) dilation followed by a rotation

(2) translation followed by a rotation

(3) line reflection followed by a translation

(4) line reflection followed by a line reflection

11 A solid metal prism has a rectangular base with sides of 4 inches and 6 inches, and a height of 4 inches. A hole in the shape of a cylinder, with a radius of 1 inch, is drilled through the entire length of the rectangular prism.



$V_{\square} = l \cdot w \cdot h = 4 \cdot 4 \cdot 6 = 96$

$V_{\bigcirc} = \pi r^2 \cdot h = \pi (1)^2 \cdot 6 = 18.84956$

$V_{remaining} = 96 - 18.84956 = 77.15$

What is the approximate volume of the remaining solid, in cubic inches?

(1) 19

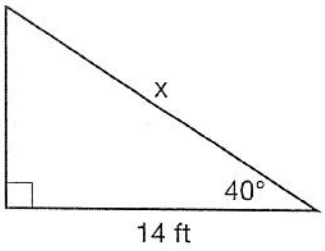
(2) 77

(3) 93

(4) 96

Use this space for computations.

12 Given the right triangle in the diagram below, what is the value of x, to the *nearest foot*?



SOH CAH TOA

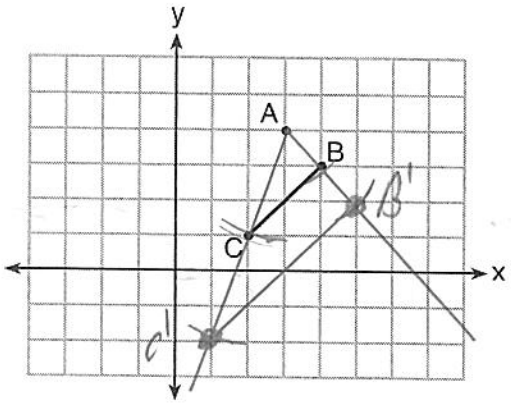
$\cos 40 = \frac{14}{x}$

$x \cdot \cos 40 = 14$

$x = \frac{14}{\cos 40} = 18.3$

(1) 11
(2) 17
(3) 18
(4) 22

13 On the graph below, point $A(3,4)$ and $\overline{BC}$ with coordinates $B(4,3)$ and $C(2,1)$ are graphed.



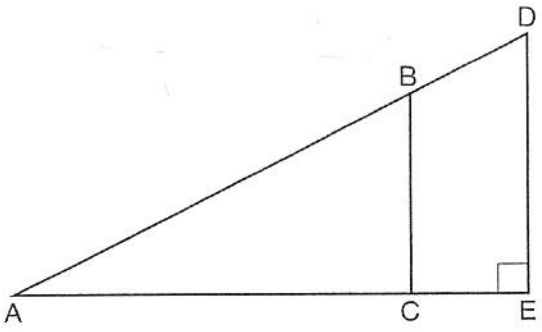
Use your compass!

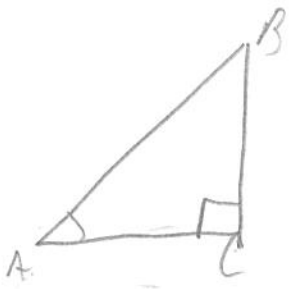
What are the coordinates of B' and C' after $\overline{BC}$ undergoes a dilation centered at point A with a scale factor of 2?

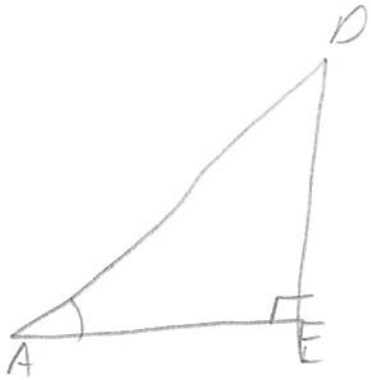
(1) $B'(5,2)$ and $C'(1,-2)$
(2) $B'(6,1)$ and $C'(0,-1)$
(3) $B'(5,0)$ and $C'(1,-2)$
(4) $B'(5,2)$ and $C'(3,0)$

Use this space for computations.

14 In the diagram of right triangle ADE below, $\overline{BC} \parallel \overline{DE}$.







Which ratio is always equivalent to the sine of $\angle A$?

(1) $\frac{AD}{DE}$

(2) $\frac{AE}{AD}$

(3) $\frac{BC}{AB}$

(4) $\frac{AB}{AC}$

$\sin A = \frac{BC}{AB}$ $\sin A = \frac{DE}{AD}$

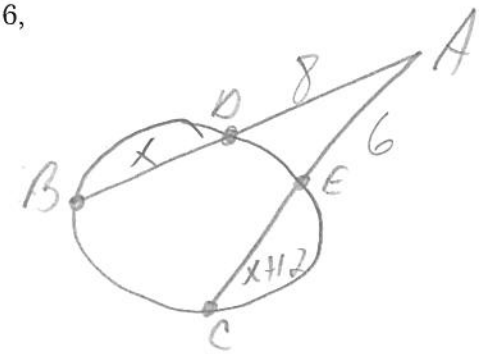
15 In circle O, secants $\overline{ADB}$ and $\overline{AEC}$ are drawn from external point A such that points D, B, E, and C are on circle O. If $AD = 8$, $AE = 6$, and EC is 12 more than BD, the length of $\overline{BD}$ is

(1) 6

(2) 22

(3) 36

(4) 48



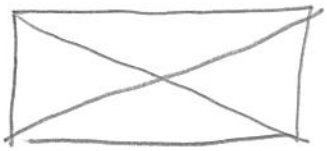
16 A parallelogram is always a rectangle if

(1) the diagonals are congruent

(2) the diagonals bisect each other

(3) the diagonals intersect at right angles

(4) the opposite angles are congruent



$WO = WO$

$(x+8)8 = (x+18)6$

$8x + 64 = 6x + 108$

$-6x \quad -6x$

$2x + 64 = 108$

$-64 \quad -64$

$2x = 44$

$x = 22$

17 Which rotation about its center will carry a regular decagon onto itself?

(1) 54°
(2) 162°
(3) 198°
(4) 252°

Use this space for computations.

$\frac{360}{n} = \frac{360}{10} = 36$

Any multiple of 36

$\therefore 252^\circ$

18 The equation of a circle is $x^2 + y^2 - 6y + 1 = 0$. What are the coordinates of the center and the length of the radius of this circle?

(1) center (0,3) and radius = $2\sqrt{2}$
(2) center (0,−3) and radius = $2\sqrt{2}$
(3) center (0,6) and radius = $\sqrt{35}$
(4) center (0,−6) and radius = $\sqrt{35}$

"Half it Square it"

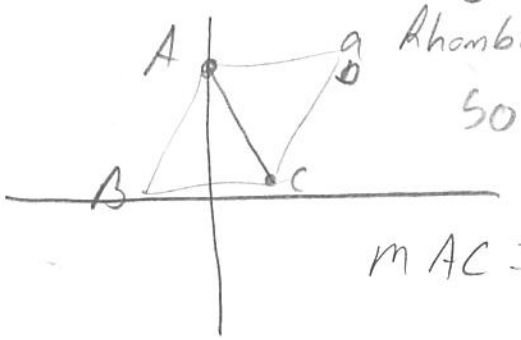
$x^2 + y^2 - 6y + 9 = -1 + 9$

$x^2 + y^2 - 6y + 9 = 8$

$x^2 + (y-3)^2 = 8$

19 Parallelogram $ABCD$ has coordinates $A(0,7)$ and $C(2,1)$. Which statement would prove that $ABCD$ is a rhombus?

(1) The midpoint of $\overline{AC}$ is (1,4).
(2) The length of $\overline{BD}$ is $\sqrt{40}$.
(3) The slope of $\overline{BD}$ is $\frac{1}{3}$.
(4) The slope of $\overline{AB}$ is $\frac{1}{3}$.



Diagonals of Rhombus are ⊥.
So negative recip. slopes.

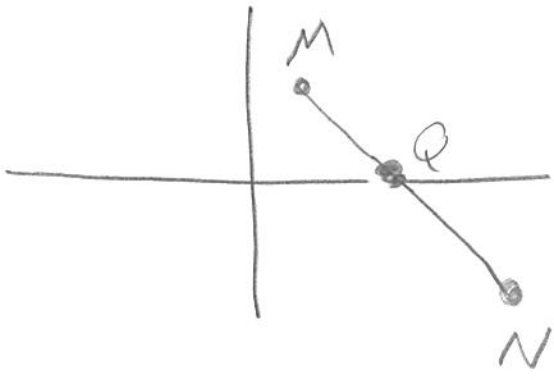
$m_{AC} = \frac{7-1}{0-1} = \frac{6}{-2} = \frac{-3}{1}$

20 Point Q is on $\overline{MN}$ such that $MQ:QN = 2:3$. If M has coordinates (3,5) and N has coordinates (8,−5), the coordinates of Q are

(1) (5,1)
(2) (5,0)
(3) (6,−1)
(4) (6,0)

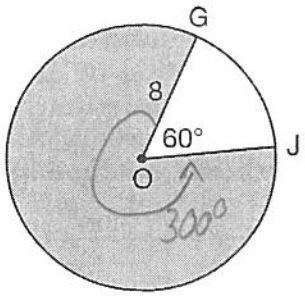
$\left(3 + \frac{2}{5}(5), 5 - \frac{2}{5}(10)\right)$

$(3+2, 5-4) = (5,1)$



Use this space for computations.

21 In the diagram below of circle O, $GO = 8$ and $m\angle GOJ = 60°$.



What is the area, in terms of π, of the shaded region?

(1) $\frac{4\pi}{3}$

(2) $\frac{20\pi}{3}$

(3) $\frac{32\pi}{3}$

(4) $\frac{160\pi}{3}$

$\text{Area} = \frac{X}{360}\pi r^2$

$= \frac{300}{360}\pi 8^2$

$= \frac{5}{6}\pi 64$

$= \frac{160}{3}\pi$

22 A circle whose center is the origin passes through the point $(-5,12)$. Which point also lies on this circle?

(1) (10,3)

(2) $(-12,13)$

(3) $(11, 2\sqrt{12})$

(4) $(-8, 5\sqrt{21})$

(0,0)

use graph!

$r = \sqrt{(-5-0)^2 + (12-0)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13$

$(x-0)^2 + (y-0)^2 = 13^2$

Check each answer

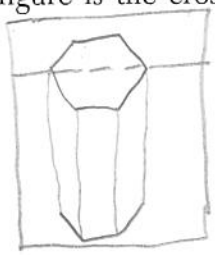
23 A plane intersects a hexagonal prism. The plane is perpendicular to the base of the prism. Which two-dimensional figure is the cross section of the plane intersecting the prism?

(1) triangle

(2) trapezoid

(3) hexagon

(4) rectangle



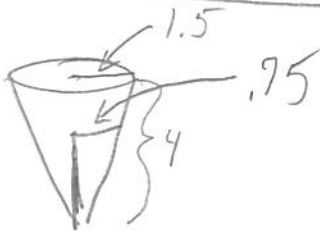
24 A water cup in the shape of a cone has a height of 4 inches and a maximum diameter of 3 inches. What is the volume of the water in the cup, to the *nearest tenth of a cubic inch*, when the cup is filled to half its height?

(1) 1.2

(2) 3.5

(3) 4.7

(4) 14.1



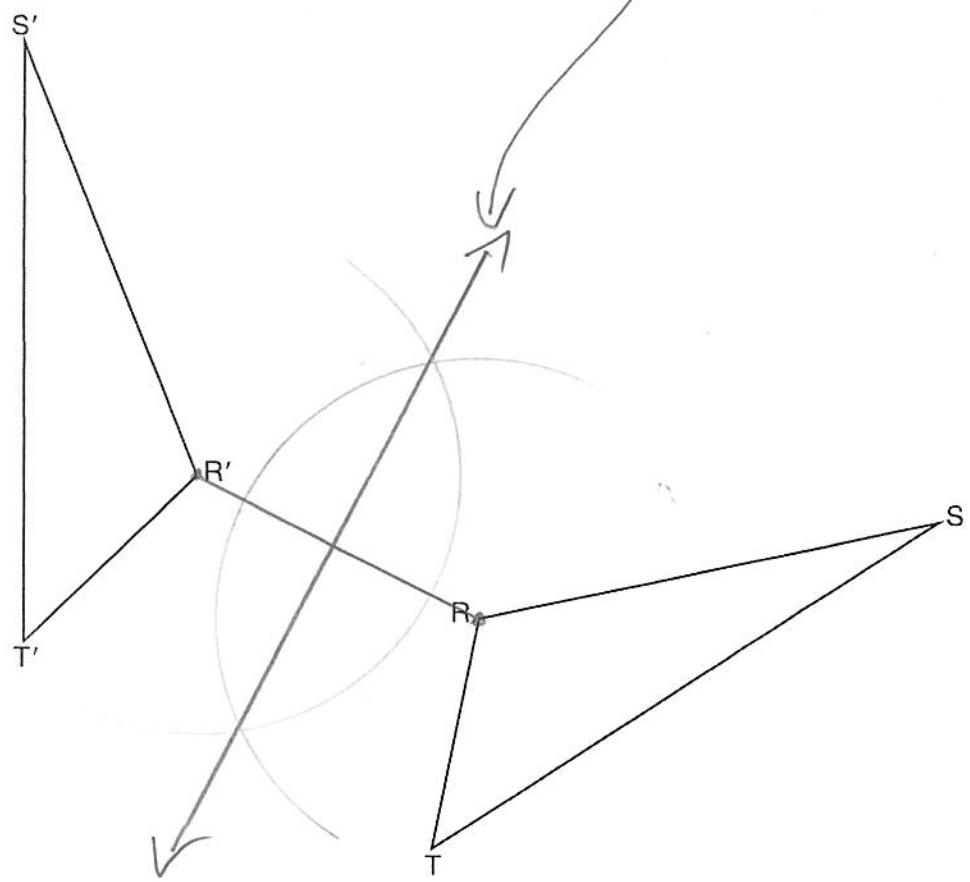
$V = \frac{1}{3}\pi r^2 \cdot h = \frac{1}{3}\pi (.75)^2 \cdot 2$

$= 1.178$

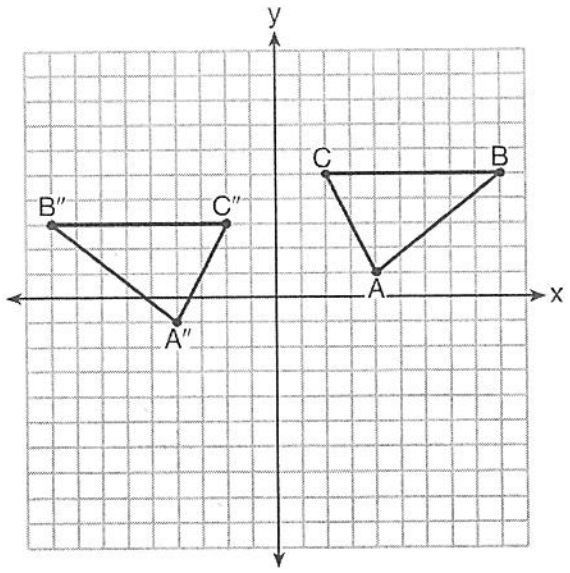
Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 Using a compass and straightedge, construct the line of reflection over which triangle RST reflects onto triangle $R'S'T'$. [Leave all construction marks.]



26 The graph below shows $\triangle ABC$ and its image, $\triangle A''B''C''$.

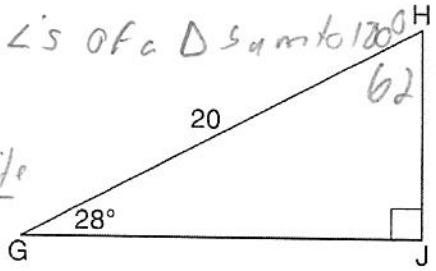


Describe a sequence of rigid motions which would map $\triangle ABC$ onto $\triangle A''B''C''$.

$\triangle ABC$ was translated down 2 units and then reflected over the y-axis to map onto $\triangle A''B''C''$.

27 When instructed to find the length of $\overline{HJ}$ in right triangle HJG, Alex wrote the equation $\sin 28° = \frac{HJ}{20}$ while Marlene wrote $\cos 62° = \frac{HJ}{20}$. Are both students' equations correct? Explain why.

$\angle H = 62$ b/c All $\angle$'s of a $\triangle$ sum to 180



$\sin 28 = \frac{\text{opposite}}{\text{hyp}}$

$\sin 28 = \frac{HJ}{20}$

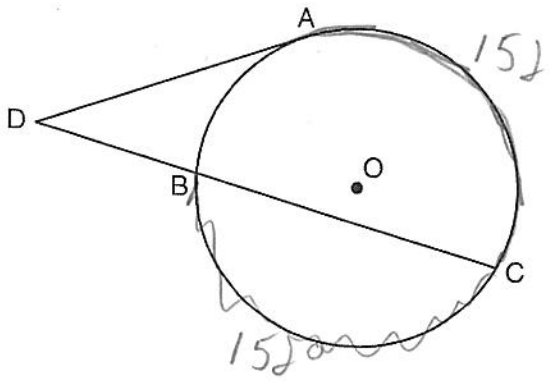
Alex is correct

$\cos 62 = \frac{\text{Adjacent}}{\text{hyp}}$

$\cos 62 = \frac{HJ}{20}$

Marlene is correct as well!

28 In the diagram below, tangent $\overline{DA}$ and secant $\overline{DBC}$ are drawn to circle O from external point D, such that $\overset{\frown}{AC} \cong \overset{\frown}{BC}$.



If $m\overset{\frown}{BC} = 152°$, determine and state $m\angle D$.

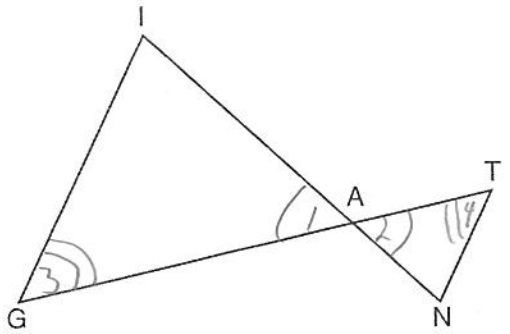
$\overset{\frown}{AB} + 152 + 152 = 360$

$\overset{\frown}{AB} = 56$

$\angle D = \frac{152 - 56}{2}$

$\angle D = 48°$

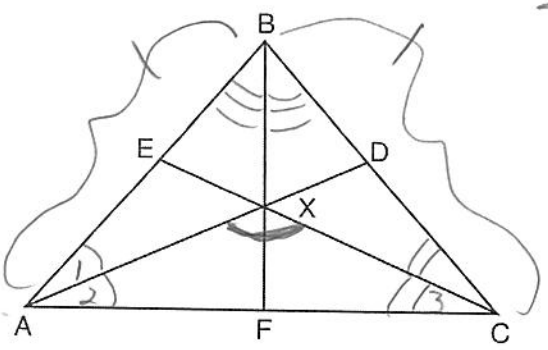
29 In the diagram below, $\overline{GI}$ is parallel to $\overline{NT}$, and $\overline{IN}$ intersects $\overline{GT}$ at A.



Prove: $\triangle GIA \sim \triangle TNA$

Statements	Reasons
1) $GI \parallel NT$, $\overline{IN}$ intersects GT at A.	1) Given
2) $\angle 1 = \angle 2$	2) Vertical $\angle$s are $=$.
3) $\angle 3 = \angle 4$	3) When lines $\parallel$, Alternate interior $\angle$s are $=$.
4) $\triangle GIA \sim \triangle TNA$	4) AA

30 In the diagram below of isosceles triangle ABC, $\overline{AB} \cong \overline{CB}$ and angle bisectors $\overline{AD}$, $\overline{BF}$, and $\overline{CE}$ are drawn and intersect at X.



If $m\angle BAC = 50°$, find $m\angle AXC$.

$\angle 2 = \frac{1}{2} 50^\circ = 25$

$\angle 3 = \angle 2 = 25$

$\angle 2 + \angle 3 + \angle AXC = 180$

$\angle AXC = 130$

31 In square $GEOM$, the coordinates of G are $(2,-2)$ and the coordinates of O are $(-4,2)$. Determine and state the coordinates of vertices E and M.

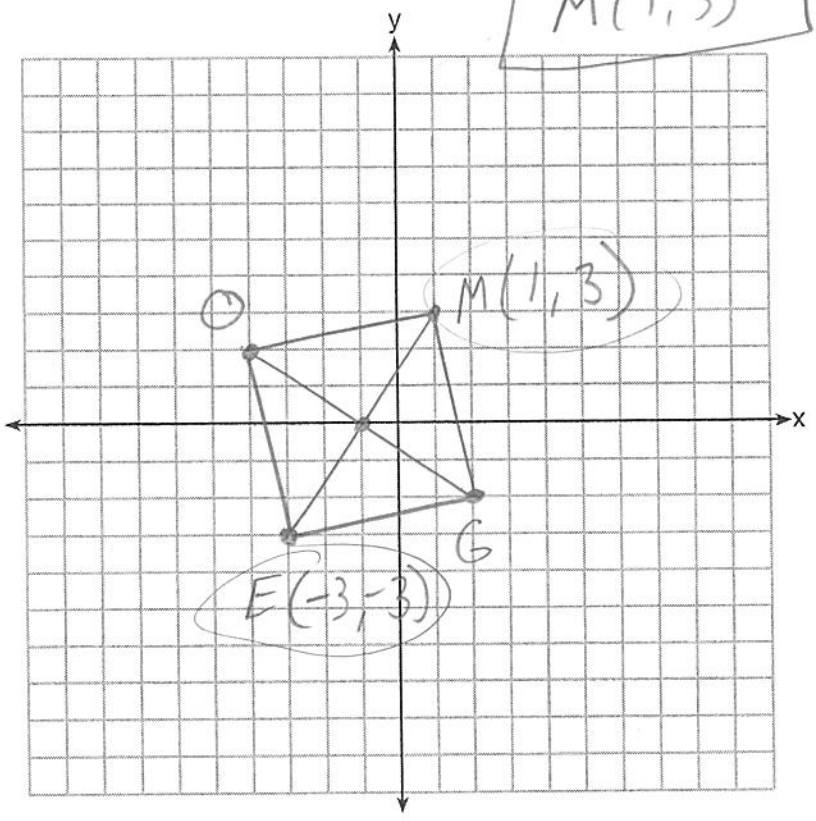
[The use of the set of axes below is optional.]

Diagonals of a square are ⊥ so they have negative reciprocal slopes.

$m\overline{OG} = \frac{-2-2}{2--4} = \frac{-4}{6} = -\frac{2}{3}$

$m\overline{EM} = \frac{3}{2}$

E(-3,-3)
M(1,3)



Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

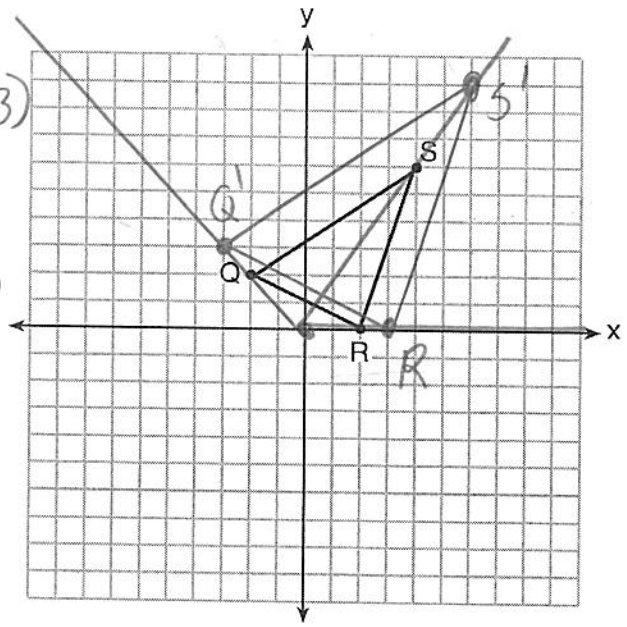
32 Triangle QRS is graphed on the set of axes below.

Times 3/2

$Q(-2,2) \rightarrow Q'(-3,3)$

$R(2,0) \rightarrow R'(3,0)$

$S(4,6) \rightarrow S'(6,9)$



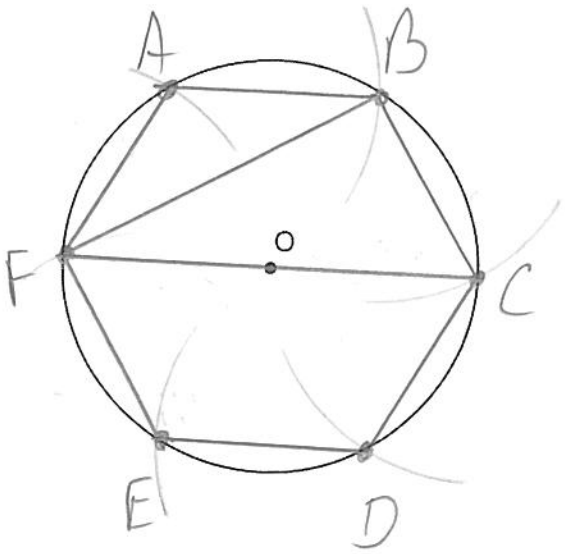
On the same set of axes, graph and label $\triangle Q'R'S'$, the image of $\triangle QRS$ after a dilation with a scale factor of $\frac{3}{2}$ centered at the origin.

Use slopes to explain why $\overline{Q'R'} \parallel \overline{QR}$.

$m\overline{Q'R'} = \frac{3-0}{-3-3} = \frac{3}{-6} = -\frac{1}{2}$ | $mQR = \frac{2-0}{-1-7} = \frac{2}{-4} = \frac{-1}{2}$

The slopes are $=$, $\therefore$ $\overline{Q'R'} \parallel \overline{QR}$

33 Using a compass and straightedge, construct a regular hexagon inscribed in circle O below. Label it $ABCDEF$. [Leave all construction marks.]

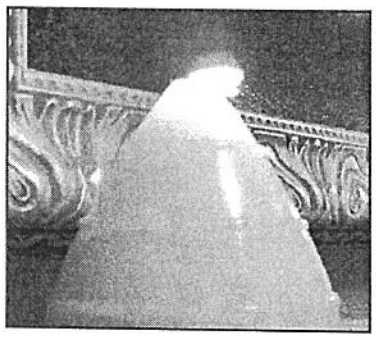


If chords $\overline{FB}$ and $\overline{FC}$ are drawn, which type of triangle, according to its angles, would $\triangle FBC$ be? Explain your answer.

∠B is right, b/c of Thales Thm,
so △FBC is a right △.

Inscribed ∠ to a diameter is 90°

34 A candle maker uses a mold to make candles like the one shown below.



The height of the candle is 13 cm and the circumference of the candle at its widest measure is 31.416 cm. Use modeling to approximate how much wax, to the *nearest cubic centimeter,* is needed to make this candle. Justify your answer.

$C = \pi d$

$\frac{31.416}{\pi} = \frac{\pi d}{\pi}$

$10 = d$

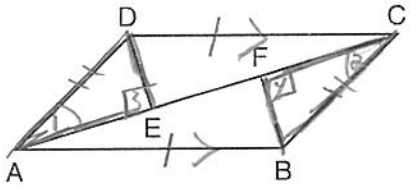
$5 = r$

$V_{\Delta} = \frac{1}{3}\pi r^2 \cdot h = \frac{1}{3}\pi (5)^2 \cdot (13) = 340\,cm^3$

Part IV

Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 In quadrilateral $ABCD$, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$, and $\overline{BF}$ and $\overline{DE}$ are perpendicular to diagonal $\overline{AC}$ at points F and E.



Prove: $\overline{AE} \cong \overline{CF}$

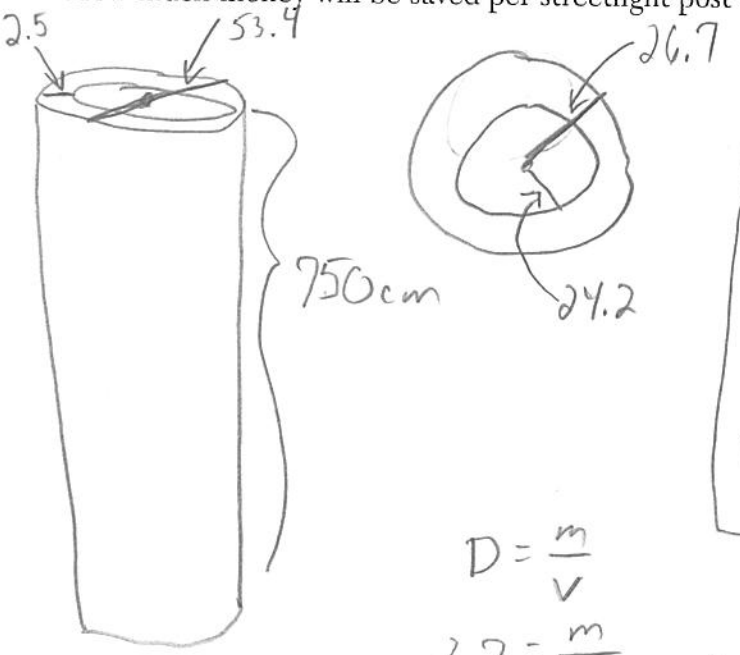
Statements	Reasons
1)	1) Given
2) $ABCD$ is a parallelogram	2) A Quad. w/ one pair of opp. sides = & $\parallel$ is a Parallelogram ($\square$)
3) $AD = BC$, $\overline{AD} \parallel \overline{BC}$	3) opp. sides = and $\parallel$ in a $\square$.
4) $\angle 1 = \angle 2$	4) When lines $\parallel$, Alt. int. $\angle$'s =.
5) $\angle 3 + \angle 4 = 90$	5) $\perp$ lines form 90° $\angle$s.
6) $\angle 3 = \angle 4$	6) Substitution.
7) $\triangle ADE \cong \triangle CBF$	7) AAS
8) $AE \cong CF$	8) CPCTC

36 New streetlights will be installed along a section of the highway. The posts for the streetlights will be 7.5 m tall and made of aluminum. The city can choose to buy the posts shaped like cylinders or the posts shaped like rectangular prisms. The cylindrical posts have a hollow core, with aluminum 2.5 cm thick, and an outer diameter of 53.4 cm. The rectangular-prism posts have a hollow core, with aluminum 2.5 cm thick, and a square base that measures 40 cm on each side.

The density of aluminum is 2.7 g/cm^3, and the cost of aluminum is \$0.38 per kilogram.

If all posts must be the same shape, which post design will cost the town less?

How much money will be saved per streetlight post with the less expensive design?



$V_{\text{cylinder}} = B \cdot h$

$= (\pi R^2 - \pi r^2) \cdot h$

$= (\pi(26.7)^2 - \pi(24.2)^2) \cdot 750$

$V_{\text{cylinder}} = 299{,}825.7489 \text{ cm}^3$

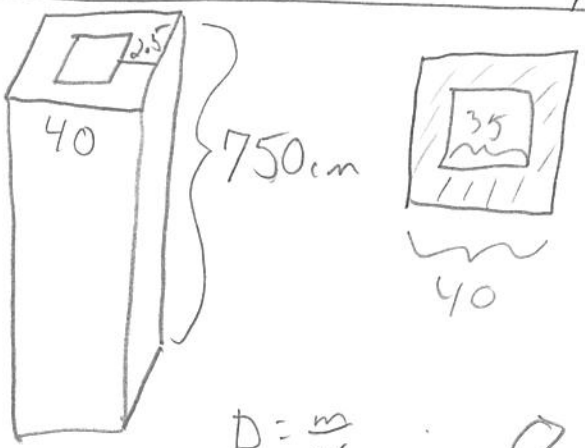
$D = \frac{m}{V}$

$2.7 = \frac{m}{299825.7489}$

$m = 809529.582$ grams

$\times .00038$

≈ 307.62



$V_{\text{prism}} = B \cdot h$

$= ((40)(40) - 35(35)) \cdot 750$

$V_{\text{prism}} = 281{,}250$

$D = \frac{m}{V}$

$2.7 = \frac{m}{281250}$

$m = 759375$

$\times .00038$

≈ 288.56

Work space for question 36 is continued on the next page.

Rectangular prism is cheaper by \$19.06 per post