

GEOMETRY (COMMON CORE)

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# GEOMETRY (Common Core)

Friday, June 17, 2016 — 1:15 to 4:15 p.m., only

Student Name: Key

School Name: \_\_\_\_\_

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 36 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

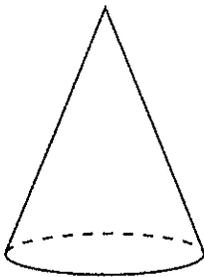
**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

Part I

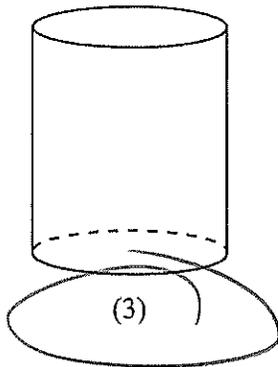
Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

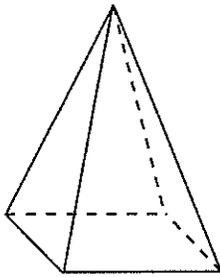
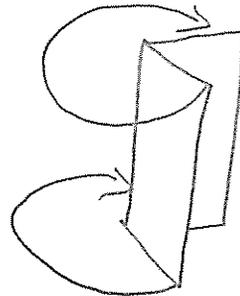
1 A student has a rectangular postcard that he folds in half lengthwise. Next, he rotates it continuously about the folded edge. Which three-dimensional object below is generated by this rotation?



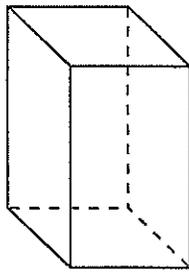
(1)



(3)



(2)

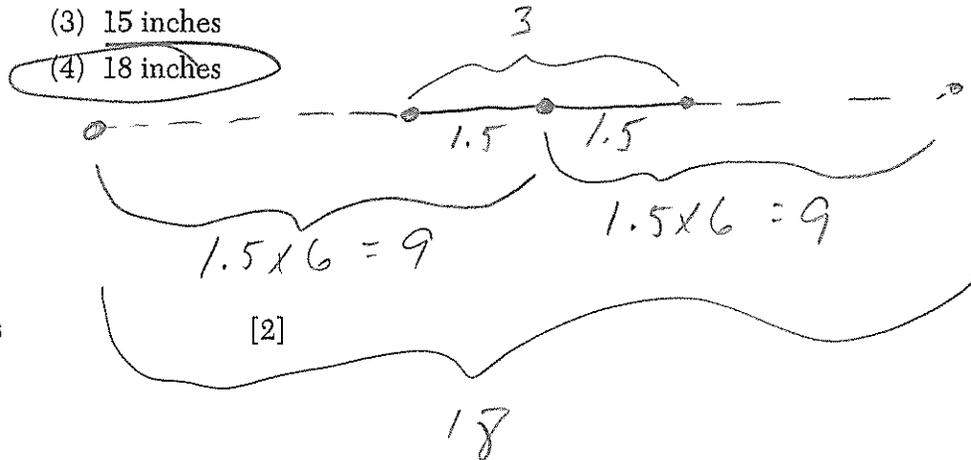


(4)

2 A three-inch line segment is dilated by a scale factor of 6 and centered at its midpoint. What is the length of its image?

- (1) 9 inches
- (2) 2 inches

- (3) 15 inches
- (4) 18 inches



Use this space for computations.

3 Kevin's work for deriving the equation of a circle is shown below.

$$x^2 + 4x = -(y^2 - 20)$$

STEP 1  $x^2 + 4x = -y^2 + 20$

STEP 2  $x^2 + 4x + 4 = -y^2 + 20 - 4$

STEP 3  $(x + 2)^2 = -y^2 + 20 - 4$

STEP 4  $(x + 2)^2 + y^2 = 16$

*Distributed the negative*  
*1/2 it, squared it but*  
*Should be +4 on*  
*both sides,*

In which step did he make an error in his work?

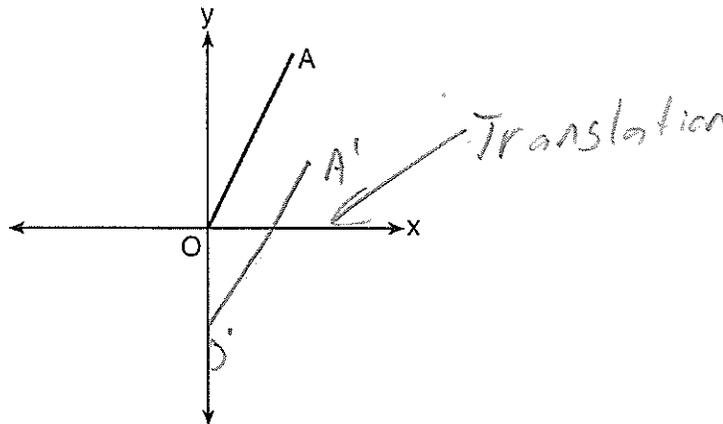
(1) Step 1

(3) Step 3

(2) Step 2

(4) Step 4

4 Which transformation of  $\overline{OA}$  would result in an image parallel to  $\overline{OA}$ ?



(1) a translation of two units down

(2) a reflection over the x-axis

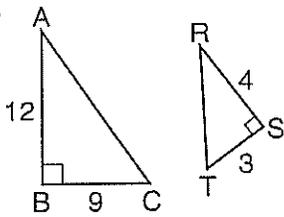
(3) a reflection over the y-axis

(4) a clockwise rotation of  $90^\circ$  about the origin

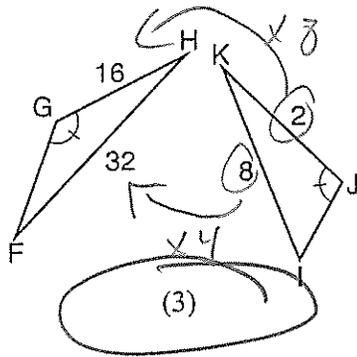
5 Using the information given below, which set of triangles can not be proven similar?

Use this space for computations.

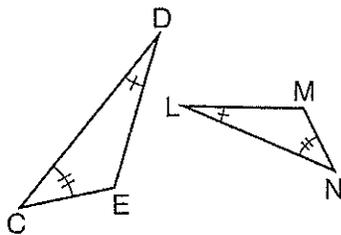
SAS ~



~~(1)~~

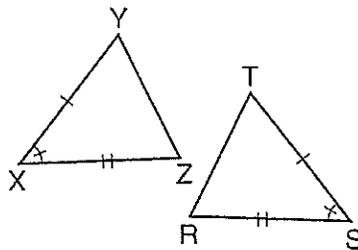


Not the same  
Scale Factor!



~~(2)~~

AA



~~(4)~~

SAS

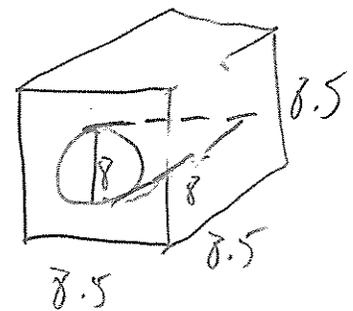
6 A company is creating an object from a wooden cube with an edge length of 8.5 cm. A right circular cone with a diameter of 8 cm and an altitude of 8 cm will be cut out of the cube. Which expression represents the volume of the remaining wood?

(1)  $(8.5)^3 - \pi(8)^2(8)$

(3)  $(8.5)^3 - \frac{1}{3}\pi(8)^2(8)$

(2)  $(8.5)^3 - \pi(4)^2(8)$

(4)  $(8.5)^3 - \frac{1}{3}\pi(4)^2(8)$



$V_{\text{cube}} = l \cdot w \cdot h = (8.5)^3$

$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi 4^2 \cdot 8$

7 Two right triangles must be congruent if

(1) an acute angle in each triangle is congruent *Similar*

(2) the lengths of the hypotenuses are equal ~~X~~

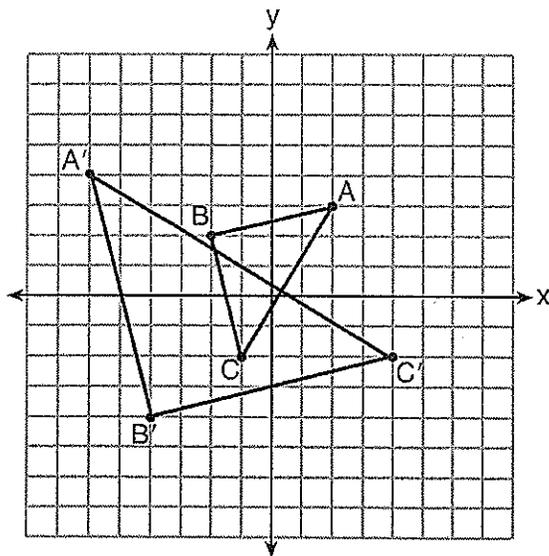
(3) the corresponding legs are congruent SAS

(4) the areas are equal ~~X~~



8 Which sequence of transformations will map  $\triangle ABC$  onto  $\triangle A'B'C'$ ?

Use this space for computations.



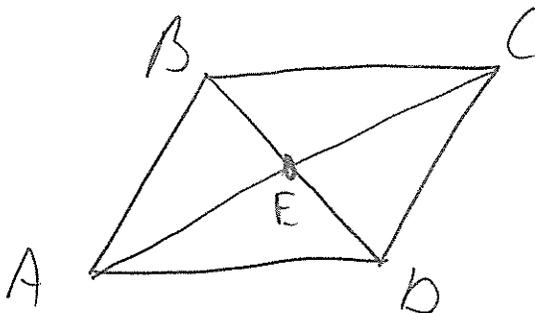
$\triangle ABC$  Got bigger  
 So Dilation  
 and it rotated

- (1) reflection and translation
- (2) rotation and reflection
- (3) translation and dilation
- (4) dilation and rotation

9 In parallelogram  $ABCD$ , diagonals  $\overline{AC}$  and  $\overline{BD}$  intersect at  $E$ . Which statement does not prove parallelogram  $ABCD$  is a rhombus?

- (1)  $\overline{AC} \cong \overline{DB}$
- (2)  $\overline{AB} \cong \overline{BC}$
- (3)  $\overline{AC} \perp \overline{DB}$
- (4)  $\overline{AC}$  bisects  $\angle DCB$ .

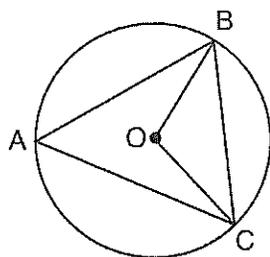
- ① All sides =
- ② Diagonals  $\perp$
- ③ Diagonals bisect the  $\angle$ 's.



True of a rectangle

Use this space for computations.

- 10 In the diagram below of circle  $O$ ,  $\overline{OB}$  and  $\overline{OC}$  are radii, and chords  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$  are drawn.



Which statement must always be true?

- (1)  $\angle BAC \cong \angle BOC$   
 (2)  $m\angle BAC = \frac{1}{2}m\angle BOC$   
 (3)  $\triangle BAC$  and  $\triangle BOC$  are isosceles.  
 (4) The area of  $\triangle BAC$  is twice the area of  $\triangle BOC$ .

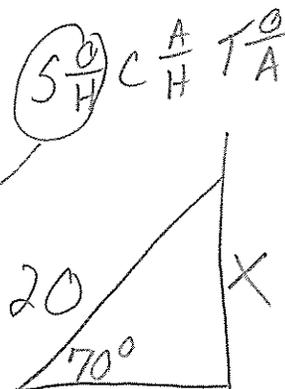
Inscribed  $\angle$ 's are  $\frac{1}{2}$  of intercepted Arc

- 11 A 20-foot support post leans against a wall, making a  $70^\circ$  angle with the ground. To the nearest tenth of a foot, how far up the wall will the support post reach?

- (1) 6.8  
 (2) 6.9  
 (3) 18.7  
 (4) 18.8

$$\sin 70^\circ = \frac{x}{20}$$

$$20 \sin 70^\circ = x$$



- 12 Line segment  $\overline{NY}$  has endpoints  $N(-11, 5)$  and  $Y(5, -7)$ . What is the equation of the perpendicular bisector of  $\overline{NY}$ ?

(1)  $y + 1 = \frac{4}{3}(x + 3)$       (3)  $y - 6 = \frac{4}{3}(x - 8)$

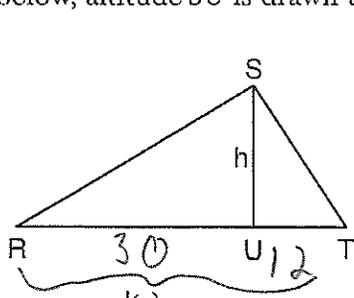
(2)  $y + 1 = -\frac{3}{4}(x + 3)$       (4)  $y - 6 = -\frac{3}{4}(x - 8)$

$$m_{\overline{NY}} = \frac{5 - (-7)}{-11 - 5} = \frac{12}{-16} = -\frac{3}{4}$$

$$m_{\perp} = \frac{4}{3}$$

$$\begin{aligned} \text{Midpt } \overline{NY} &= \left( \frac{-11 + 5}{2}, \frac{5 + (-7)}{2} \right) \\ &= \left( \frac{-6}{2}, \frac{-2}{2} \right) \\ &= (-3, -1) \end{aligned}$$

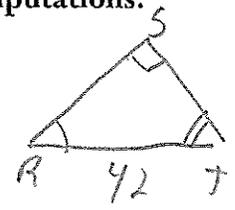
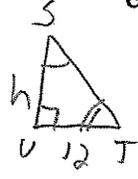
13 In  $\triangle RST$  shown below, altitude  $\overline{SU}$  is drawn to  $\overline{RT}$  at  $U$ .



If  $SU = h$ ,  $UT = 12$ , and  $RT = 42$ , which value of  $h$  will make  $\triangle RST$  a right triangle with  $\angle RST$  as a right angle?

- (1)  $6\sqrt{3}$                       (3)  $6\sqrt{14}$   
 (2)  $6\sqrt{10}$                     (4)  $6\sqrt{35}$

Use this space for computations.

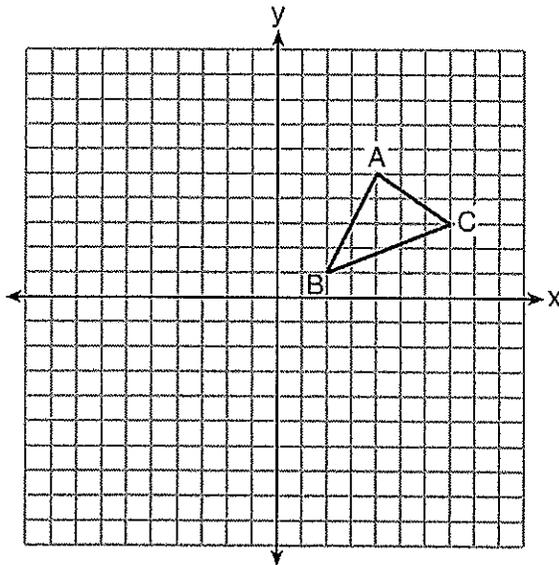


$$\begin{array}{r} 30 \quad h \\ \hline h \quad 12 \\ \hline 360 = h^2 \end{array}$$

$$\begin{array}{l} \sqrt{360} = h \\ \sqrt{36 \cdot 10} = h \end{array}$$

$$6\sqrt{10} = h$$

14 In the diagram below,  $\triangle ABC$  has vertices  $A(4,5)$ ,  $B(2,1)$ , and  $C(7,3)$ .



$$m_{\overline{BC}} = \frac{1-3}{2-7} = \frac{-2}{-5} = \frac{2}{5}$$

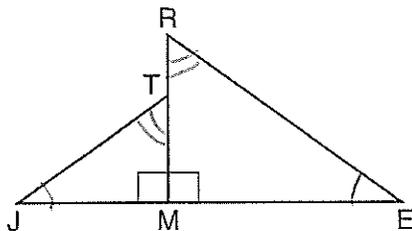
$$m_{\perp} = -\frac{5}{2}$$

What is the slope of the altitude drawn from  $A$  to  $\overline{BC}$ ?

- (1)  $\frac{2}{5}$                               (3)  $-\frac{1}{2}$   
 (2)  $\frac{3}{2}$                               (4)  $-\frac{5}{2}$

Use this space for computations.

15 In the diagram below,  $\triangle ERM \sim \triangle JTM$ .



Which statement is always true?

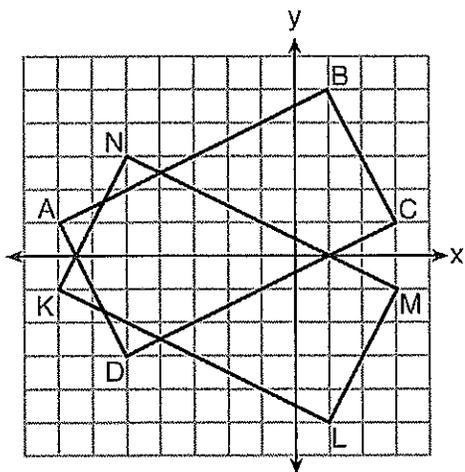
~~(1)~~  $\cos J = \frac{RM}{RE} = \frac{0}{H}$

~~(3)~~  $\tan T = \frac{RM}{EM} = \frac{A}{0}$

~~(2)~~  $\cos R = \frac{JM}{JT} = \frac{0}{H}$

(4)  $\tan E = \frac{TM}{JM} = \frac{0}{A}$

16 On the set of axes below, rectangle  $ABCD$  can be proven congruent to rectangle  $KLMN$  using which transformation?



(1) rotation

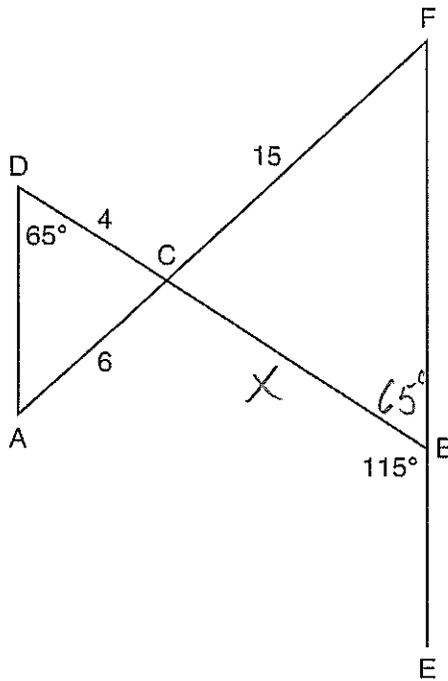
(2) translation

(3) reflection over the  $x$ -axis

(4) reflection over the  $y$ -axis

17 In the diagram below,  $\overline{DB}$  and  $\overline{AF}$  intersect at point C, and  $\overline{AD}$  and  $\overline{FBE}$  are drawn.

Use this space for computations.



$\Delta's \sim$

$$\frac{4}{x} = \frac{6}{15}$$

$$\frac{60}{6} = \frac{6x}{6}$$

$$10 = x$$

If  $AC = 6$ ,  $DC = 4$ ,  $FC = 15$ ,  $m\angle D = 65^\circ$ , and  $m\angle CBE = 115^\circ$ , what is the length of  $\overline{CB}$ ?

(1) 10

(3) 17

(2) 12

(4) 22.5

18 Seawater contains approximately 1.2 ounces of salt per liter on average. How many gallons of seawater, to the nearest tenth of a gallon, would contain 1 pound of salt?

(1) 3.3

(3) 4.7

(2) 3.5

(4) 13.3

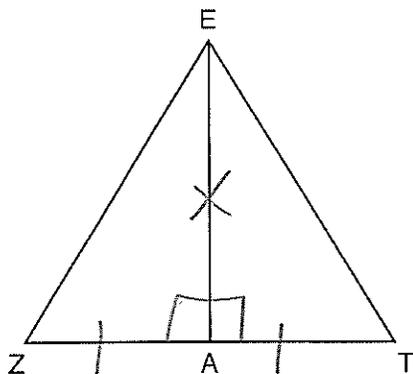
$$\frac{.075}{.264} = \frac{1}{x}$$

$$.075x = .264$$

$$x = 3.52$$

19 Line segment  $EA$  is the perpendicular bisector of  $\overline{ZT}$ , and  $\overline{ZE}$  and  $\overline{TE}$  are drawn.

Use this space for computations.



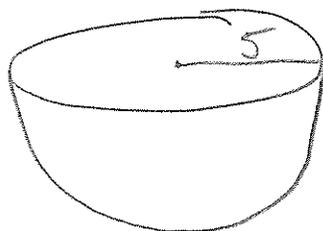
$$\triangle ZEA \cong \triangle TEA \quad \underline{SAS}$$

Which conclusion can not be proven?

- (1)  $\overline{EA}$  bisects angle  $ZET$ .
- (2) Triangle  $EZT$  is equilateral.
- (3)  $\overline{EA}$  is a median of triangle  $EZT$ .
- (4) Angle  $Z$  is congruent to angle  $T$ .

20 A hemispherical water tank has an inside diameter of 10 feet. If water has a density of 62.4 pounds per cubic foot, what is the weight of the water in a full tank, to the nearest pound?

- (1) 16,336
- (2) 32,673
- (3) 130,690
- (4) 261,381

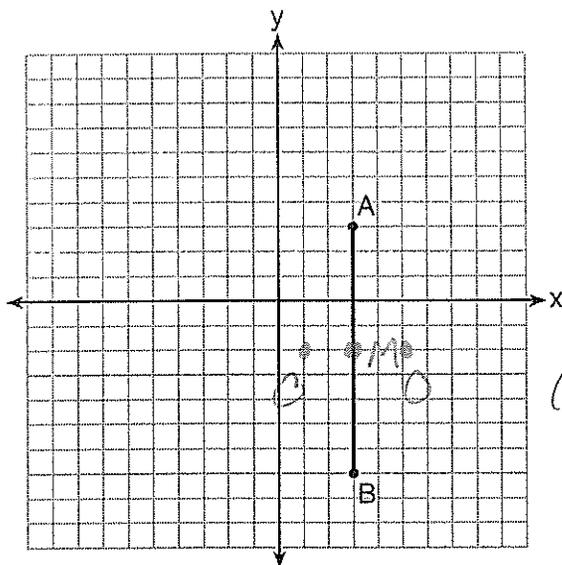


$$\begin{aligned}
 V &= \frac{1}{2} \left( \frac{4}{3} \pi r^3 \right) \\
 &= \frac{1}{2} \left( \frac{4}{3} \pi 5^3 \right) \\
 &= 261.7993878 \\
 &\quad \times 62.4 \\
 \hline
 &= 16336.2818
 \end{aligned}$$



- 23 The graph below shows  $\overline{AB}$ , which is a chord of circle  $O$ . The coordinates of the endpoints of  $\overline{AB}$  are  $A(3,3)$  and  $B(3,-7)$ . The distance from the midpoint of  $\overline{AB}$  to the center of circle  $O$  is 2 units.

Use this space for computations.



Center can be  
 $(1, -2)$  or  $(5, -2)$

$$(x-1)^2 + (y+2)^2 = r^2 \text{ or } (x-5)^2 + (y+2)^2 = r^2$$

$r =$  distance from  $O$  to  $A$

$$\begin{aligned} r &= \sqrt{(1-3)^2 + (-2-3)^2} \\ &= \sqrt{4 + 25} \\ &= \sqrt{29} \end{aligned}$$

What could be a correct equation for circle  $O$ ?

(1)  $(x-1)^2 + (y+2)^2 = 29$

~~(2)  $(x+5)^2 + (y-2)^2 = 29$~~

~~(3)  $(x-1)^2 + (y-2)^2 = 25$~~

~~(4)  $(x-5)^2 + (y+2)^2 = 25$~~

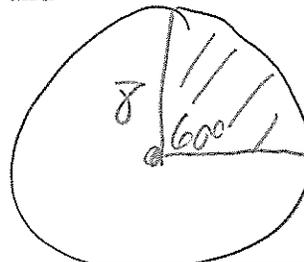
- 24 What is the area of a sector of a circle with a radius of 8 inches and formed by a central angle that measures  $60^\circ$ ?

(1)  $\frac{8\pi}{3}$

(3)  $\frac{32\pi}{3}$

(2)  $\frac{16\pi}{3}$

(4)  $\frac{64\pi}{3}$

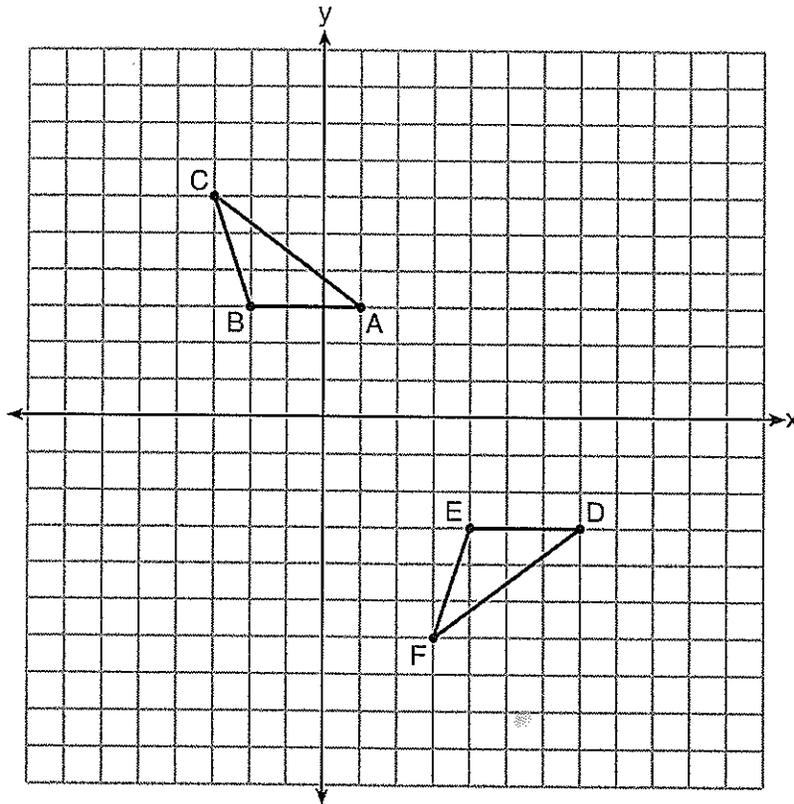


$$\begin{aligned} A_{\text{sector}} &= \frac{60}{360} \pi r^2 = \frac{1}{6} \pi 64 \\ &= \frac{32\pi}{3} \end{aligned}$$

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 Describe a sequence of transformations that will map  $\triangle ABC$  onto  $\triangle DEF$  as shown below.

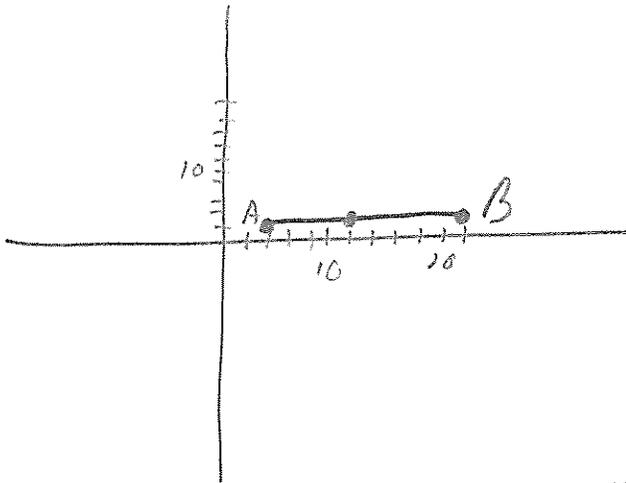


A reflection over the x-axis followed by a translation 6 units to the right.

26 Point  $P$  is on segment  $AB$  such that  $AP:PB$  is  $4:5$ . If  $A$  has coordinates  $(4,2)$ , and  $B$  has coordinates  $(22,2)$ , determine and state the coordinates of  $P$ .

→ 9 parts

∴  $\frac{4}{9}$  up the line

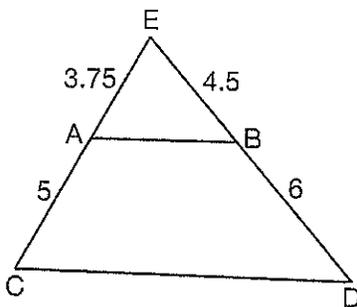


$$\left(4 + \frac{4}{9}(18), 2 + \frac{4}{9}(0)\right)$$

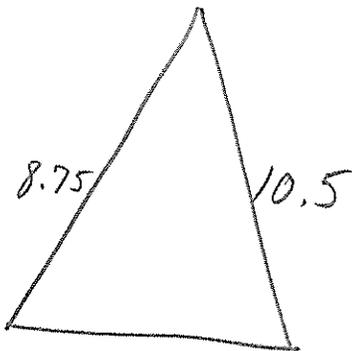
$$(4 + 8, 2 + 0)$$

$$P(12, 2)$$

27 In  $\triangle CED$  as shown below, points  $A$  and  $B$  are located on sides  $\overline{CE}$  and  $\overline{ED}$ , respectively. Line segment  $\overline{AB}$  is drawn such that  $AE = 3.75$ ,  $AC = 5$ ,  $EB = 4.5$ , and  $BD = 6$ .



Explain why  $\overline{AB}$  is parallel to  $\overline{CD}$ .



$$\frac{3.75}{8.75} \stackrel{?}{=} \frac{4.5}{10.5}$$

$$.42857142 = .42857142 \checkmark$$

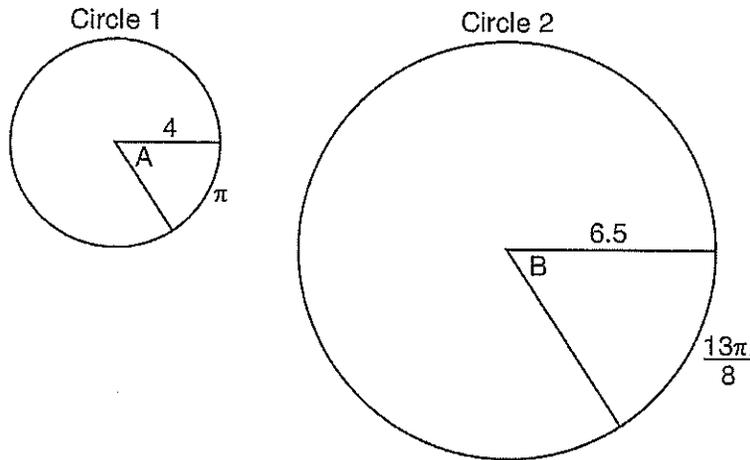
By the Side Splitter  
thm IF corresponding  
sides are proportional  
then  $AB \parallel CD$ .

- 28 Find the value of  $R$  that will make the equation  $\sin 73^\circ = \cos R$  true when  $0^\circ < R < 90^\circ$ .  
Explain your answer.

$$\sin A = \cos B \quad \text{only if } A + B = 90$$

$$\begin{array}{r} \therefore \quad 73 + R = 90 \\ \quad -73 \quad \quad -73 \\ \hline \quad \quad R = 17^\circ \end{array}$$

29 In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle A intercepts an arc of length  $\pi$ , and angle B intercepts an arc of length  $\frac{13\pi}{8}$ .



Dominic thinks that angles A and B have the same radian measure. State whether Dominic is correct or not. Explain why.

Find  $\angle A$

$$\text{Arc} = \frac{A}{360} \pi d$$

$$9\pi = \frac{A}{360} \pi \cdot 8$$

$$45^\circ = A$$

Radians

$$A = \frac{45}{180} \cdot \pi = \boxed{\frac{\pi}{4}}$$

Find  $\angle B$

$$\text{Arc} = \frac{B}{360} \pi d$$

$$\frac{13\pi}{8} = \frac{B}{360} \pi \cdot 13$$

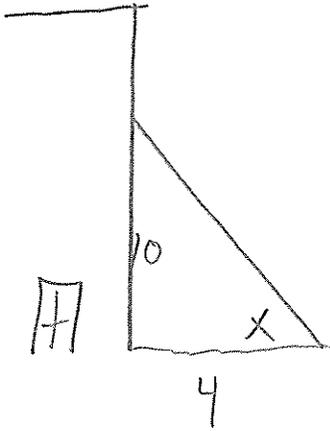
$$45^\circ = B$$

Radians

$$B = \frac{45}{180} \cdot \pi = \boxed{\frac{\pi}{4}}$$

Dominic is correct. A & B are both  $\pi/4$  radians

- 30 A ladder leans against a building. The top of the ladder touches the building 10 feet above the ground. The foot of the ladder is 4 feet from the building. Find, to the *nearest degree*, the angle that the ladder makes with the level ground.



$$\frac{SO}{H} \quad \frac{A}{H} \quad \left| \frac{TO}{A} \right.$$

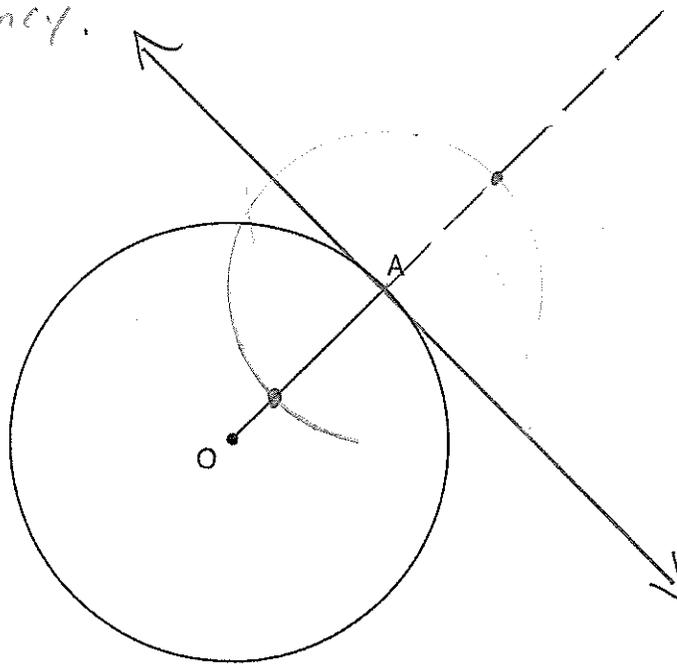
$$\tan x = \frac{10}{4}$$

$$\tan^{-1}\left(\frac{10}{4}\right) = 68^{\circ}$$

31 In the diagram below, radius  $\overline{OA}$  is drawn in circle  $O$ . Using a compass and a straightedge, construct a line tangent to circle  $O$  at point  $A$ . [Leave all construction marks.]

Tangents are  $\perp$  to the radii at the  
Point of tangency.

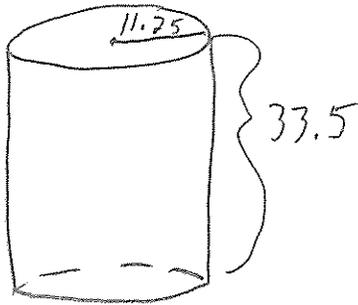
Draw a  $\perp$  bisector



Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

- 32 A barrel of fuel oil is a right circular cylinder where the inside measurements of the barrel are a diameter of 22.5 inches and a height of 33.5 inches. There are 231 cubic inches in a liquid gallon. Determine and state, to the *nearest tenth*, the gallons of fuel that are in a barrel of fuel oil.

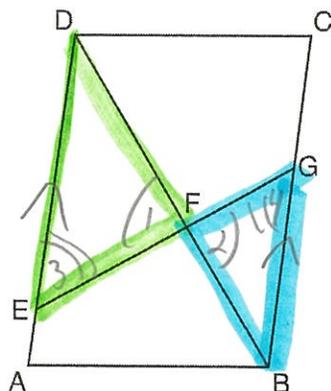


$$\begin{aligned}V &= \pi r^2 \cdot h \\ &= \pi (11.25)^2 \cdot (33.5) \\ &= 13,319.86198 \text{ in}^3\end{aligned}$$

$$\frac{13,319.86198}{231} = 57.6617$$

57.7 gallons

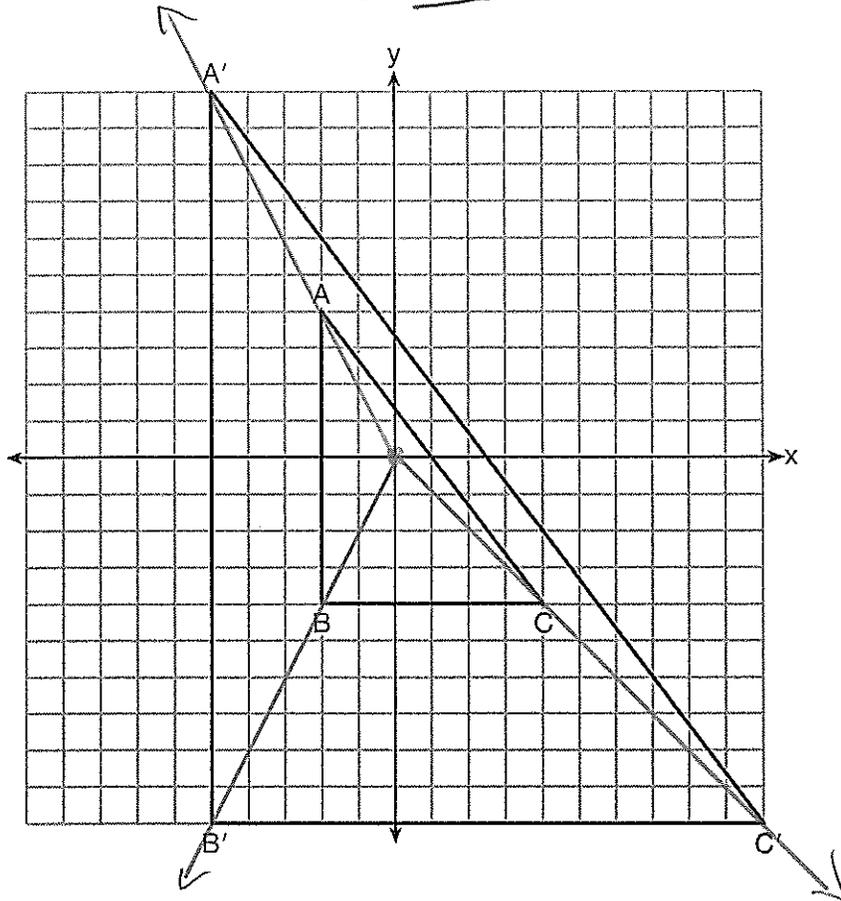
33 Given: Parallelogram  $ABCD$ ,  $\overline{EFG}$ , and diagonal  $\overline{DFB}$



Prove:  $\triangle DEF \sim \triangle BGF$

S	R
1) Parallelogram $ABCD$ $\overline{EFG}$ , diag. $\overline{DFB}$	1) Given
2) $\angle 1 = \angle 2$	2) Vertical $\angle$ s are $=$ .
3) $\overline{AD} \parallel \overline{BC}$	3) Opposite sides in a $\square$ are $\parallel$ .
4) $\angle 3 = \angle 4$	4) Alternate interior $\angle$ s are $=$ when lines $\parallel$ .
5) $\triangle DEF \sim \triangle BGF$	5) AA = AA

34 In the diagram below,  $\triangle A'B'C'$  is the image of  $\triangle ABC$  after a transformation.



Describe the transformation that was performed.

$$r = \frac{A'B'}{AB} = \frac{20}{8} = \frac{5}{2}$$

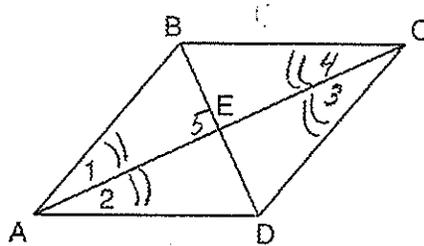
A dilation with scale factor of  $\frac{5}{2}$  from the origin

Explain why  $\triangle A'B'C' \sim \triangle ABC$ .

Dilations preserve  $\angle$  measure so all corresponding angles are  $\cong$ .  $\therefore \triangle A'B'C' \sim \triangle ABC$  by AA  $\sim$ .

Question 35

35 Given: Quadrilateral  $ABCD$  with diagonals  $\overline{AC}$  and  $\overline{BD}$  that bisect each other, and  $\angle 1 \cong \angle 2$



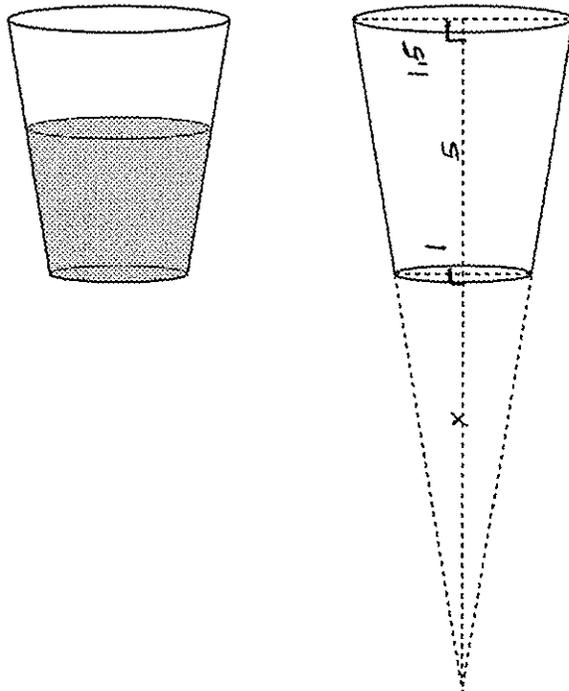
Prove:  $\triangle ACD$  is an isosceles triangle and  $\triangle AEB$  is a right triangle

- |   |   |
|---|---|
| ① Quad $ABCD$ , Diagonal $\overline{AC}$ & $\overline{BD}$ Bisect | ① Given   |
| ② Quad $ABCD$ is a parallelogram                                  | ② a parallelogram has diagonal that bisect each other   |
| ③ $\overline{AB} \parallel \overline{CD}$                         | ③ opposite sides of a parallelogram are parallel  |
| ④ $\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$             | ④ $\angle 1 + \angle 3$ and $\angle 2 + \angle 4$ are alt interior $\angle$ 's and ALT interior $\angle$ 's are $\cong$ when lines are parallel |
| ⑤ $\angle 2 \cong \angle 3$ and $\angle 3 \cong \angle 4$         | ⑤ Substitution  |
| ⑥ $\triangle ACD$ is isosceles                                    | ⑥ An isosceles $\triangle$ has $2 \cong$ base $\angle$ 's   |
| ⑦ $\overline{AD} \cong \overline{DC}$                             | ⑦ The sides of an isosceles $\triangle$ are $\cong$   |
| ⑧ Quad $ABCD$ is a Rhombus  | ⑧ If two consecutive sides of a parallelogram are congruent it is a Rhombus   |
| ⑨ $\overline{AE} \perp \overline{BE}$                             | ⑨ THE Diagonals of a Rhombus form $\perp$ lines   |
| ⑩ $\angle 5$ is a Right $\angle$                                  | ⑩ $\perp$ segments form Right $\angle$ 's   |
| ⑪ $\triangle AEB$ is a Right $\triangle$                          | ⑪ A Right $\triangle$ has a Right angle   |

Score 6: The student had a complete and correct proof.

Question 36

36 A water glass can be modeled by a truncated right cone (a cone which is cut parallel to its base) as shown below.



The diameter of the top of the glass is 3 inches, the diameter at the bottom of the glass is 2 inches, and the height of the glass is 5 inches.

The base with a diameter of 2 inches must be parallel to the base with a diameter of 3 inches in order to find the height of the cone. Explain why.

Because you need similar triangles in order to set up and solve a proportion.

Question 36 is continued on the next page.

Question 36

Question 36 continued

Determine and state, in inches, the height of the larger cone.

From diagram above

$$\frac{x+5}{1.5} = \frac{x}{1}$$

$$x+5 = 1.5x$$

$$5 = 0.5x$$

$$x = 10$$

The height of the cone is 15 inches.

Determine and state, to the nearest tenth of a cubic inch, the volume of the water glass.

$$\text{Volume of large cone} = \frac{1}{3} \pi (15)^2 (15) = 35.343 \text{ in}^3$$

$$\text{Volume of small cone} = \frac{1}{3} \pi (1)^2 (10) = 10.472 \text{ in}^3$$

$$\begin{aligned} \text{Volume of glass} &= (\text{Volume of large cone}) - (\text{Volume of small cone}) \\ &= 24.9 \text{ in}^3 \end{aligned}$$

Score 6: The student had a complete and correct response.

