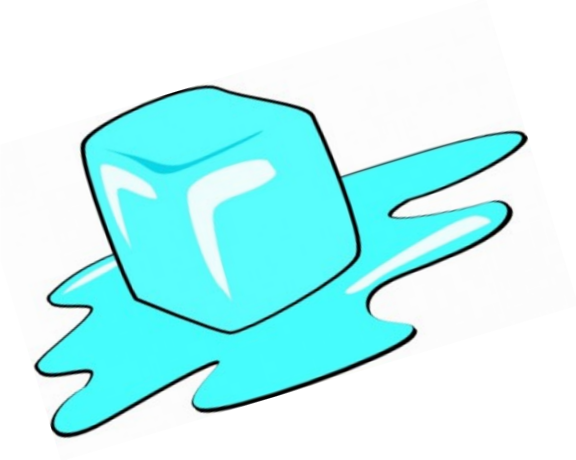
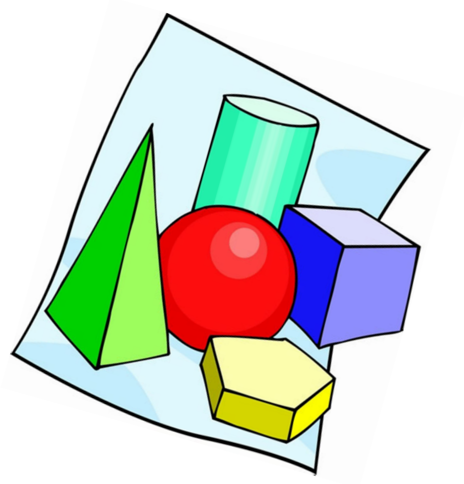
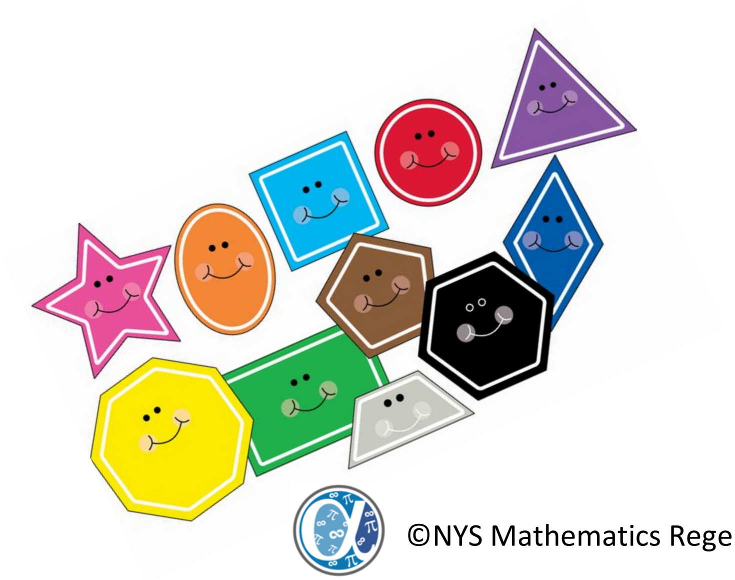
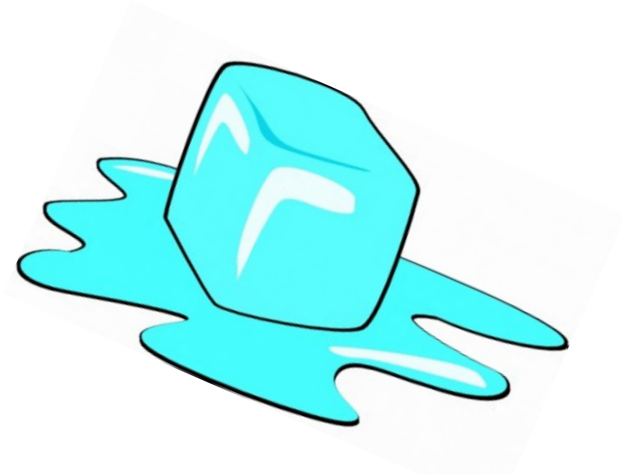




GEOMETRY (COMMON CORE)

FACTS YOU MUST KNOW COLD FOR THE REGENTS EXAM





Geometry [Common Core] Regents Exam Study Guide

Polygons – Interior/Exterior Angles

Sum of Interior Angles: 180 (n - 2)

Each Interior Angle of a Regular Polygon: $\frac{180\,(n-2)}{n}$

Sum of Exterior Angles: 360°

Each Exterior Angle: $\frac{360}{n}$

Triangles

Classifying Triangles

Sides:

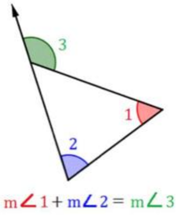
Scalene: No congruent sides
Isosceles: 2 congruent sides
Equilateral: 3 congruent sides

Angles:

Acute: All angles are < 90°
Right: One right angle that is 90°
Obtuse: One angle that is > 90°
Equiangular: 3 congruent angles (60°)

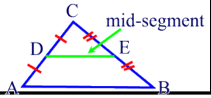
All triangles have 180°

Exterior Angle Theorem: The exterior angle is equal to the sum of the two non-adjacent interior angles.



Midsegment: segment joining the midpoints

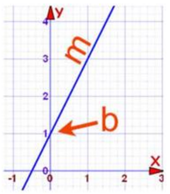
- Always parallel to the third side
- $\frac{1}{2}$ the length of the third side
- Splits the triangle into two similar triangles.



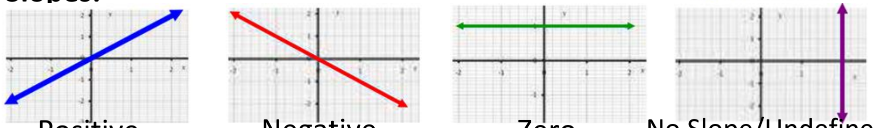
Coordinate Geometry

Standard Form of a Line: $y = mx + b$,where m is the slope and b is the y-intercept.

Slope Formula: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$



Slopes:



Positive Negative Zero No Slope/Undefined

Parallel Lines have the **SAME** slope

Perpendicular Lines have **NEGATIVE RECIPROCAL** slopes (flip & change the sign)

Collinear Points are points that lie of the **same** line.

Midpoint Formula: $M = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Segment Ratios:

$$\frac{x - x_1}{x_2 - x} = Given\ Ratio \qquad \frac{y - y_1}{y_2 - y} = Given\ Ratio$$

Pythagorean Theorem

To find the missing side of any **right** triangle, use:

$$a^2 + b^2 = c^2$$

where *a* and *b* are the legs, and *c* is the hypotenuse

Isosceles Triangle

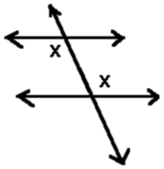
- 2 ≅ sides and 2 ≅ base angles
- The altitude drawn from the vertex is also the median and angle bisector
- If two sides of a triangle are ≅, then the angles opposite those ≅ sides are ≅.

Triangle Inequality Theorems

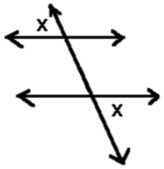
- The sum of 2 sides must be greater than the third side
- The difference of 2 sides must be less than the third side
- The longest side is opposite the largest angle
- The shortest side is opposite the smallest angle

Parallel Lines

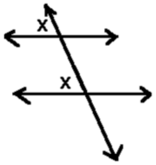
Alternate Interior angles are **congruent**



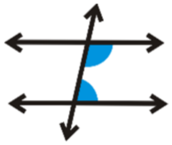
Alternate Exterior angles are **congruent**



Corresponding angles are **congruent**

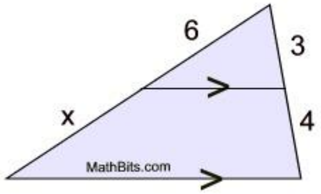


Same-Side Interior angles are **supplementary**



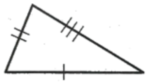
Side – Splitter Theorem

If a line is parallel to a side of a triangle and intersects the other two sides, then this line divides those two sides proportionally.

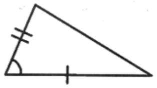


Triangle Congruence Theorems

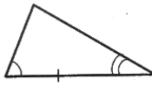
Side-Side-Side **(SSS)**



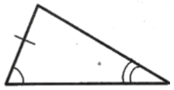
Side-Angle-Side **(SAS)**



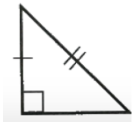
Angle-Side-Angle **(ASA)**



Angle-Angle-Side **(AAS)**



Hypotenuse-Leg **(HL)**

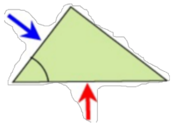


CPCTC – Corresponding Parts of Congruent Triangles are Congruent

Similar Triangle Theorems

Angle-Angle **(aa)**

Side-Angle-Side **(SAS)**



Side-Side-Side **(SSS)**

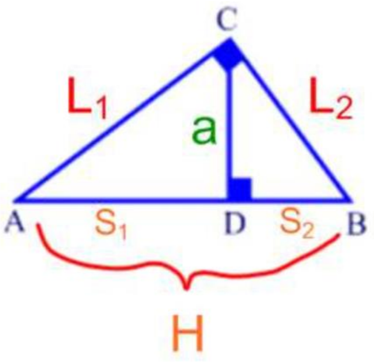
- Similar figures have congruent angles and proportional sides
- **CSSTP-**Corresponding Sides of Similar Triangles are in Proportion
- In a proportion, the product of the means equals the product of the extremes

Geometric Mean Theorems

Altitude Theorem (SAAS / Heartbeat Method):

The altitude is the geometric mean between the 2 segments of the hypotenuse.

$$\frac{S_1}{a} = \frac{a}{S_2}$$

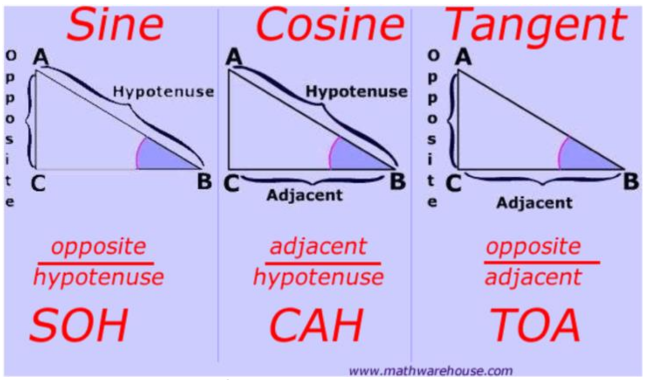


Leg Theorem (HYLLS / PSSW):

The leg is the geometric mean between the segment It touches and the whole hypotenuse.

$$\frac{S_1}{L_1} = \frac{L_1}{H} \quad \text{and} \quad \frac{S_2}{L_1} = \frac{L_2}{H}$$

Trigonometry



- When solving for a side, use the *sin, cos*, and *tan* buttons
- When solving for an angle, use the *sin*$^{-1}$, *cos*$^{-1}$, and *tan*$^{-1}$ buttons

Cofunctions:

- Sine and Cosine are cofunctions, which are complementary

$$\sin\theta = \cos(90° - \theta)$$
$$\cos\theta = \sin(90° - \theta)$$

- If $\angle A$ and $\angle B$ are the acute angles of a right triangle, then $\sin A = \cos B$

Factoring

The order of Factoring:

Greatest Common Factor (GCF)

↓

Difference of Two Perfect Squares (DOTS)

↓

Trinomial (TRI)

GCF:

$$ab + ac = a(b + c)$$

DOTS:

$$x^2 - y^2 = (x + y)(x - y)$$

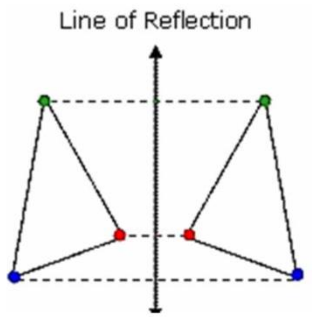
TRI:

$x^2 - x + 6$ » $(x + 2)(x - 3)$

Transformational Geometry

Rigid Motion: transformations that preserve distance, congruency, angle measure, and shape.

Reflection – FLIP



$$r_{x-axis}(x,y) = (x,-y)$$

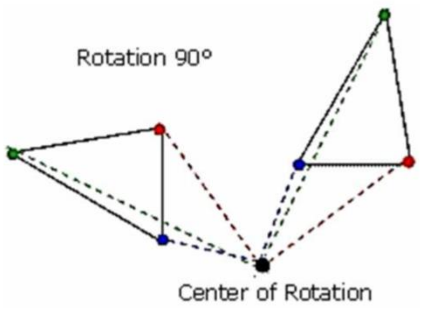
$$r_{y-axis}(x,y) = (-x,y)$$

$$r_{y=x}(x,y) = (y,x)$$

$$r_{y=-x}(x,y) = (-y,-x)$$

$$r_{(o,o)}(x,y) = (-x,-y)$$

Rotation – TURN

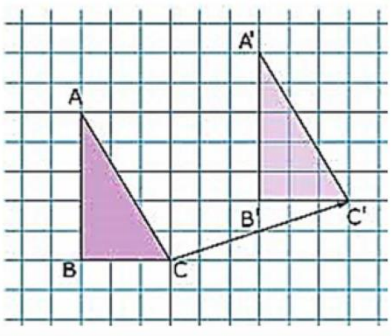


$$R_{90^\circ}(x,y) = (-y,x)$$

$$R_{180^\circ}(x,y) = (-x,-y)$$

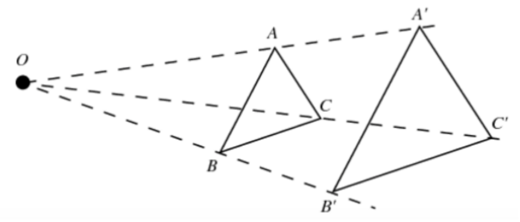
$$R_{270^\circ}(x,y) = (y,-x)$$

Translation – SHIFT/MOVE



$$T_{a,b}(x,y) = (x+a, y+b)$$

Dilation – ENLARGEMENT/REDUCTION



$$D_k(x,y) = (k \cdot x, k \cdot y)$$

- **Dilations create similar figures**
- **Dilations are NOT rigid motions, since they do NOT preserve distance.**

Composition of Transformations

When you see "∘", work from right to left.

$$R_{90^\circ} \circ T_{3,-4}$$

Do this Second!

Do this First!

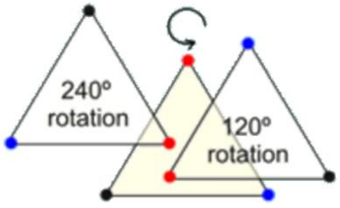
Translation, followed by a Rotation.

Types of Composition Transformations

- **A composition of 2 reflections over 2 parallel lines is equivalent to a TRANSLATION.**
- **A composition of 2 reflections over 2 intersecting lines is equivalent to a ROTATION.**

Rotational Symmetry Theorem

A regular polygon with n sides always has rotational symmetry, with rotations in increments equal to its central angle of $\frac{360^\circ}{n}$



equilateral triangle

Circles

Equations

General/Standard Equation of a Circle:

$$x^2 + y^2 + Cx + Dy + E = 0$$

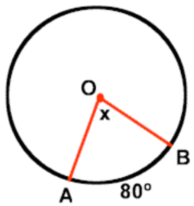
where *C, D*, and *E* are constants

Center – Radius Equation of a Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

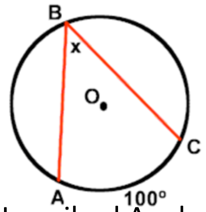
where *(h, k)* is the center and *r* is the radius.

Angles



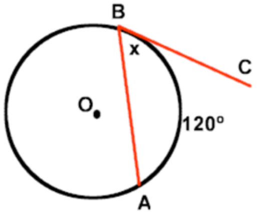
Central Angle:

$$\measuredangle x = \widehat{AB}$$



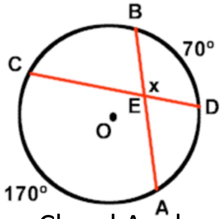
Inscribed Angle:

$$\measuredangle x = \frac{1}{2}\widehat{AC}$$



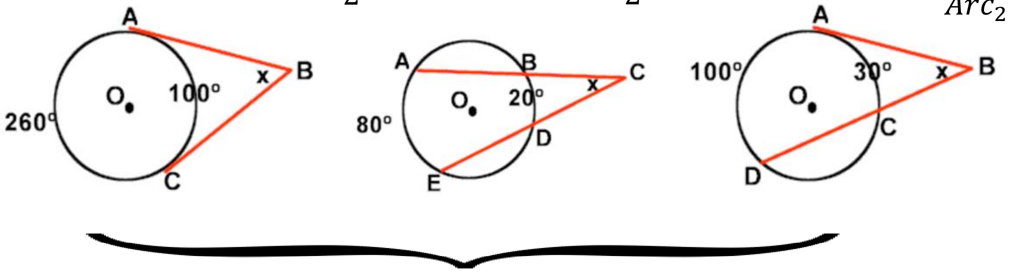
Tangent-Chord Angle:

$$\measuredangle x = \frac{1}{2}\widehat{AC}$$

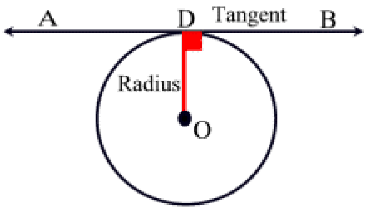


Two Chord Angles:

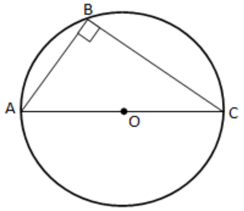
$$\measuredangle x = \frac{\widehat{Arc_1} + \widehat{Arc_2}}{2}$$



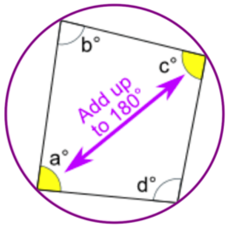
$$\frac{\widehat{Big} - \widehat{Little}}{2} = \measuredangle x$$



Tangent-Radius Angle = 90°

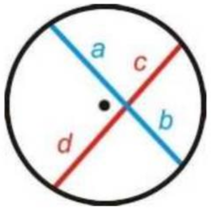


Angle Inscribed in a semicircle = 90°

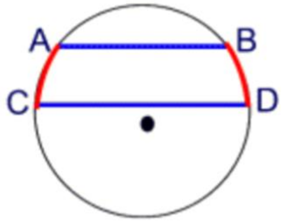


Opposite Angles in a Quad. = 180°

Segments

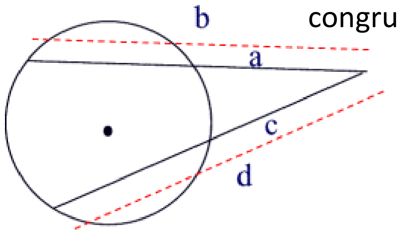


$$(a)(b) = (c)(d)$$



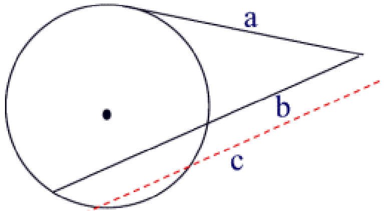
$\overline{AB} \parallel \overline{CD}$; $\widehat{AC} \cong \widehat{BD}$

Parallel chords intercept congruent arcs



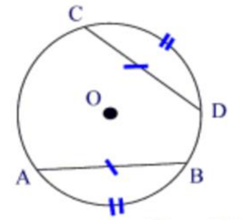
$$(Whole)(External) = (Whole)(External)$$

(W)(E) = (W)(E)

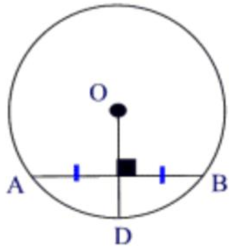


$$(Whole)(External) = (Tangent)^2$$

(W)(E) = (T)^2

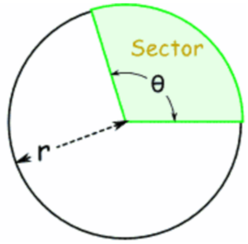


$\overline{CD} \cong \overline{AB}$; $\widehat{CD} \cong \widehat{AB}$



Circles (Con't)

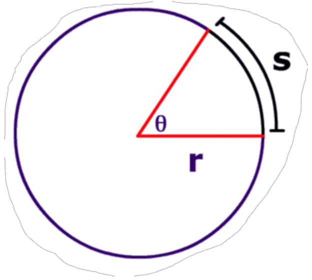
Area of a Sector



$$A = \frac{1}{2}r^2\theta$$

where A is the area of the sector, r is the radius, and θ is an angle in radians.

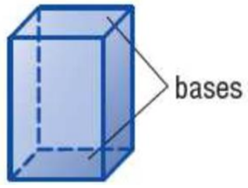
Sector Length



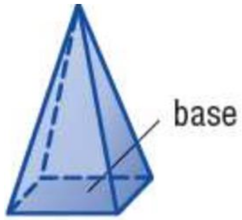
$$s = r\theta$$

where s is the sector length, r is the radius, and θ is an angle in radians.

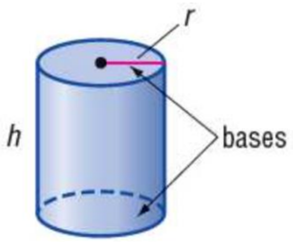
3D Figures



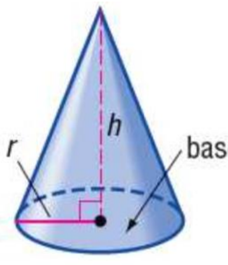
Prism



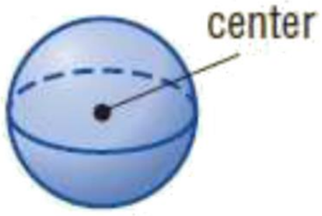
Pyramid



Cylinder

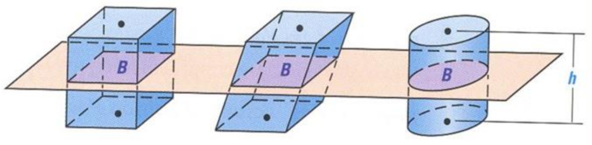


Cone

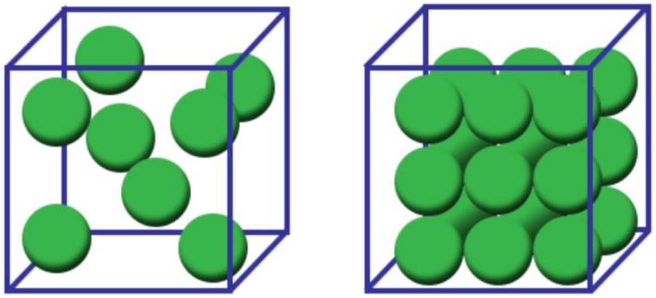


Sphere

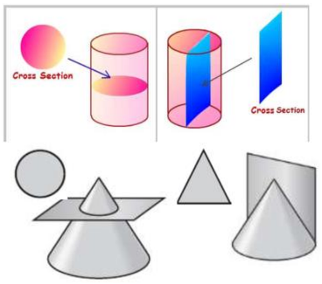
Cavalieri's Principle: If two solids have the same height and the same cross-sectional area at every level, then the solids have the same volume.



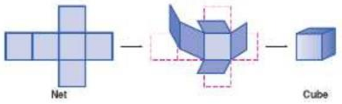
$$\boldsymbol{Mass} = (\boldsymbol{Density}) \cdot (\boldsymbol{Volume})$$



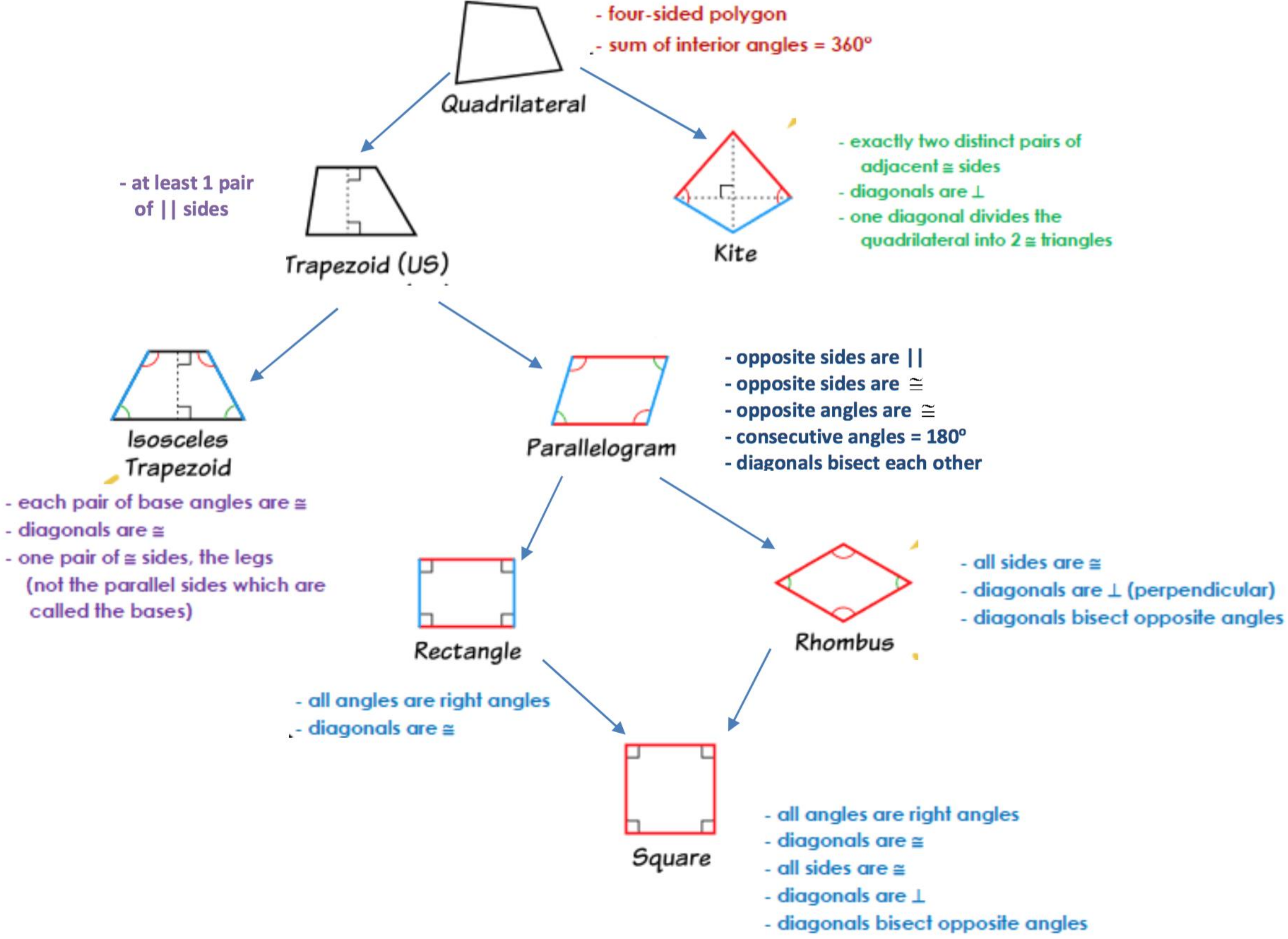
Cross Sections



Nets



The Quadrilateral Family Tree



Quadrilateral

- four-sided polygon
- sum of interior angles = 360°

Trapezoid (US)

- at least 1 pair of || sides

Kite

- exactly two distinct pairs of adjacent ≅ sides
- diagonals are ⊥
- one diagonal divides the quadrilateral into 2 ≅ triangles

Isosceles Trapezoid

- each pair of base angles are ≅
- diagonals are ≅
- one pair of ≅ sides, the legs (not the parallel sides which are called the bases)

Parallelogram

- opposite sides are ||
- opposite sides are ≅
- opposite angles are ≅
- consecutive angles = 180°
- diagonals bisect each other

Rectangle

- all angles are right angles
- diagonals are ≅

Rhombus

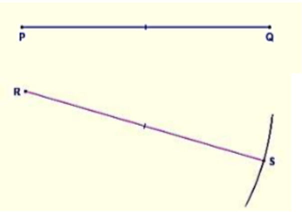
- all sides are ≅
- diagonals are ⊥ (perpendicular)
- diagonals bisect opposite angles

Square

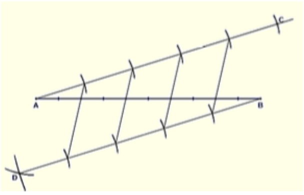
- all angles are right angles
- diagonals are ≅
- all sides are ≅
- diagonals are ⊥
- diagonals bisect opposite angles

Basic Constructions

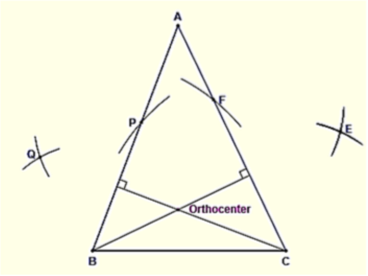
Copy a line segment



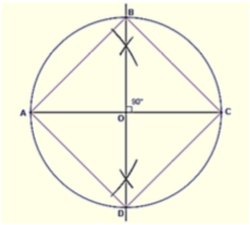
Dividing a segment into equal parts



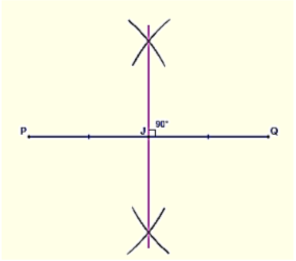
Orthocenter - altitudes



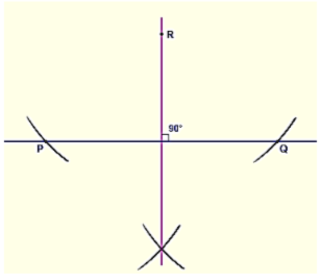
Inscribed square in a circle



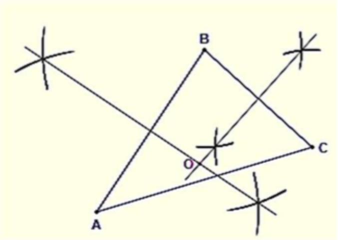
Perpendicular Bisector



Perpendicular line thru an external point

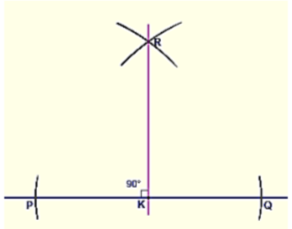


Circumcenter – perpendicular bisectors

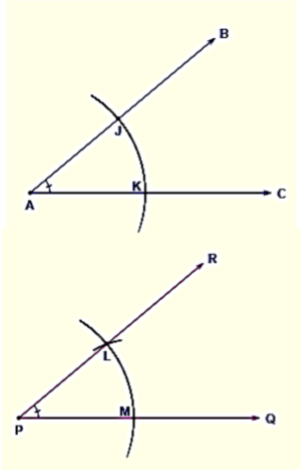


- Equidistant to each vertex of the triangle
- Used to circumscribe a circle

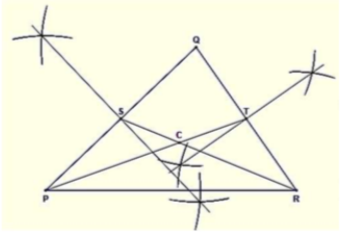
Perpendicular Line thru a point on a line



Copy an angle



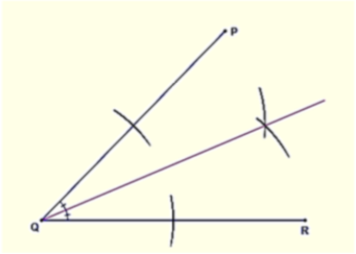
Centroid - medians



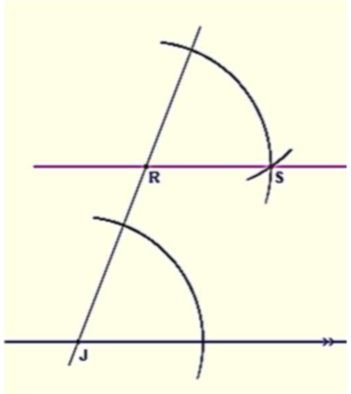
- 2:1 ratio

$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

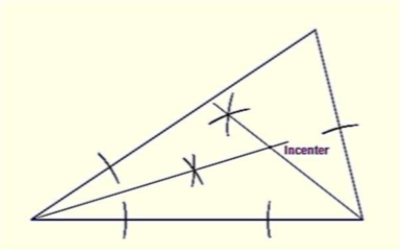
Bisect an angle



Parallel line

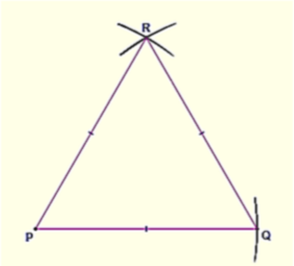


Incenter – angle bisectors

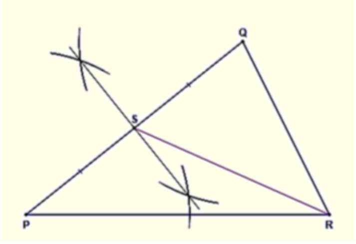


- equidistant to each side of the triangle
- Used to inscribe a circle

Equilateral Triangle



Median – vertex to midpoint



Altitude – vertex perpendicular to opposite side