The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY (Common Core)

Wednesday, August 12, 2015 — 8:30 to 11:30 a.m., only

Student Name: _____

School Name: _____ *Key* _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 36 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. [48]

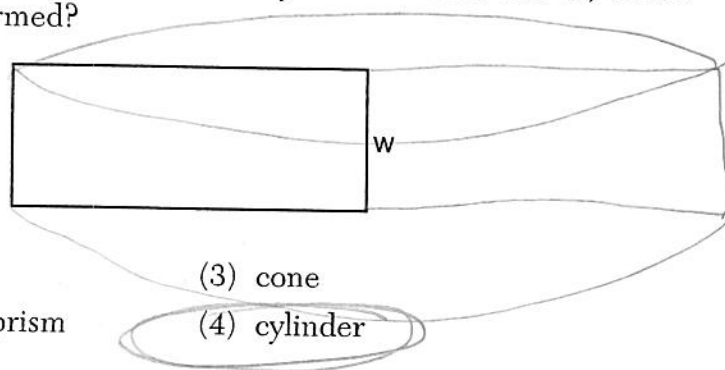**Use this space for computations.**

1 A parallelogram must be a rectangle when its
   (1) diagonals are perpendicular ← *Rhombus*
   (2) diagonals are congruent ← *Rectangles*
   (3) opposite sides are parallel ←
   (4) opposite sides are congruent ← *Parallelograms*

2 If $\triangle A'B'C'$ is the image of $\triangle ABC$, under which transformation will the triangles *not* be congruent? *Only dilations change size.*
   (1) reflection over the x-axis
   (2) translation to the left 5 and down 4
   (3) dilation centered at the origin with scale factor 2
   (4) rotation of 270° counterclockwise about the origin

3 If the rectangle below is continuously rotated about side w, which solid figure is formed?

   (1) pyramid   (3) cone
   (2) rectangular prism   (4) cylinder

4 Which expression is always equivalent to sin x when $0° < x < 90°$?
   (1) $\cos(90° - x)$ (3) $\cos(2x)$ $\sin x = \cos y, \quad IF \quad x+y = 90$
   (2) $\cos(45° - x)$ (4) $\cos x$

$$y = 90 - x$$

$$\therefore \sin x = \cos(90 - x)$$

Geometry (Common Core) – Aug. '15   [2]

5 In the diagram below, a square is graphed in the coordinate plane.

x = 5
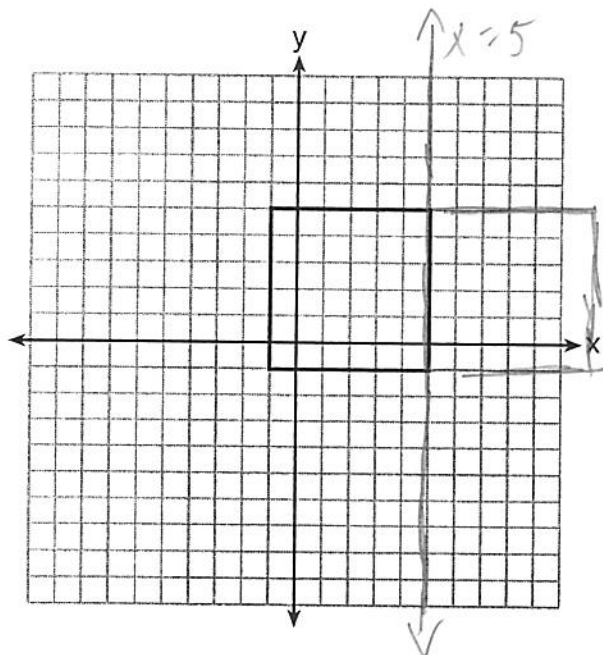
Not on itself.

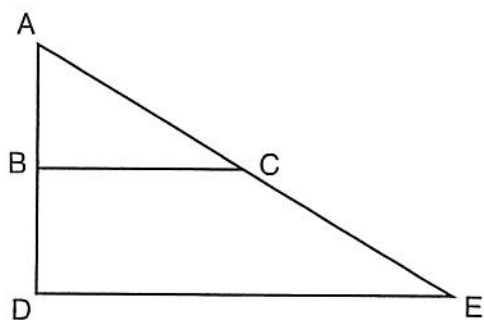A reflection over which line does *not* carry the square onto itself?

(1) $x = 5$

(2) $y = 2$

(3) $y = x$

(4) $x + y = 4$

6 The image of $\triangle ABC$ after a dilation of scale factor k centered at point A is $\triangle ADE$, as shown in the diagram below.

Side splitter Thm

① $\triangle ABC \sim \triangle ADE$

② $\overline{BC} \parallel \overline{DE}$

Which statement is always true?

(1) $2AB = AD$

(2) $\overline{AD} \perp \overline{DE}$

(3) $AC = CE$

(4) $\overline{BC} \parallel \overline{DE}$

7 A sequence of transformations maps rectangle $ABCD$ onto rectangle $A''B''C''D''$, as shown in the diagram below.
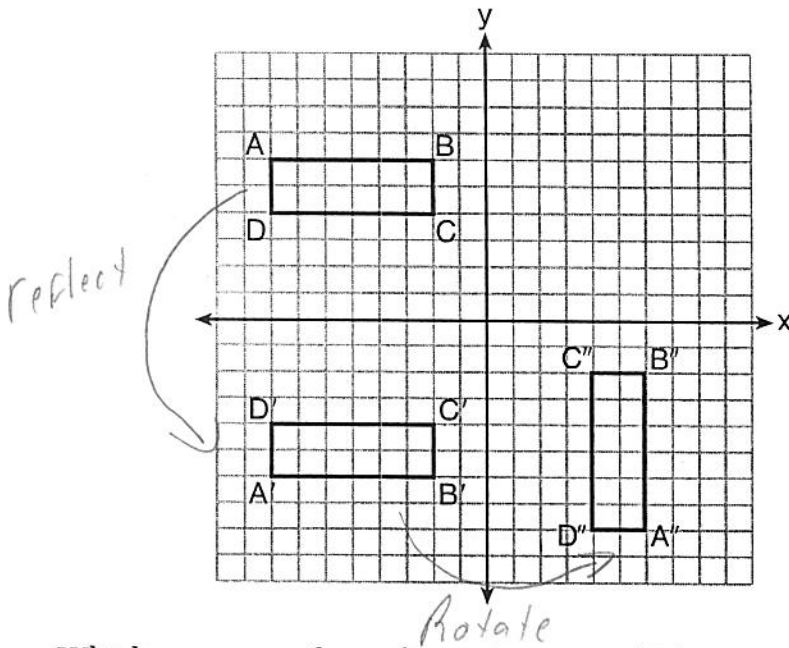
reflect

Rotate

Which sequence of transformations maps $ABCD$ onto $A'B'C'D'$ and then maps $A'B'C'D'$ onto $A''B''C''D''$?

(1) a reflection followed by a rotation

(2) a reflection followed by a translation

(3) a translation followed by a rotation

(4) a translation followed by a reflection

8 In the diagram of parallelogram $FRED$ shown below, $\overline{ED}$ is extended to A, and $\overline{AF}$ is drawn such that $\overline{AF} \cong \overline{DF}$.
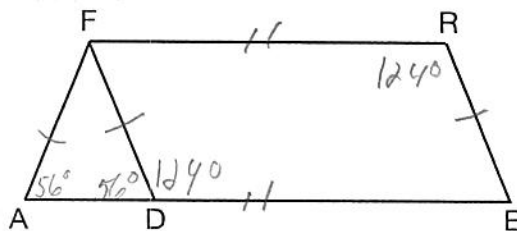
If $m\angle R = 124°$, what is $m\angle AFD$?

(1) 124°   (3) 68°

(2) 112°   (4) 56°

$\angle FDE = 124°$ Opp. $\angle$s = in a ▱.

$\angle FDA = 56°$ Linear pairs add to 180°

$\angle FAD = 56°$ Isos. △

$\angle AFD = 68°$ △s Sum to 180°

$\frac{1}{2}$ it square it"

Use this space for computations.

9 If $x^2 + 4x + y^2 - 6y - 12 = 0$ is the equation of a circle, the length of the radius is
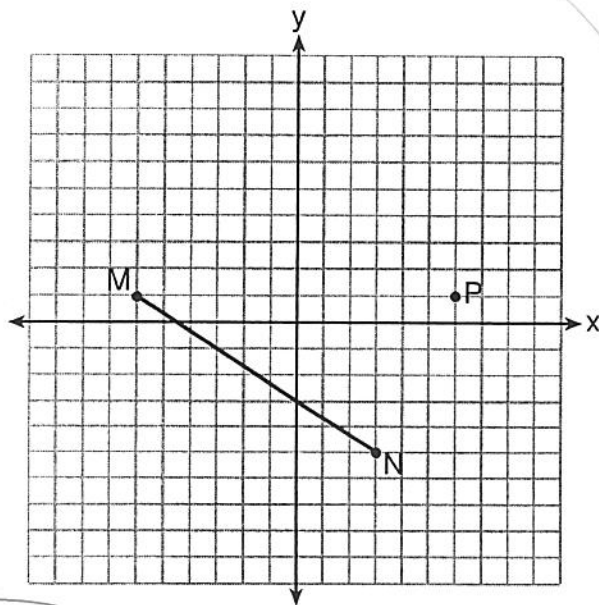
(1) 25

(3) 5 ⟵ circled

(2) 16

(4) 4

$x^2 + 4x + 4 + y^2 - 6y + 9 = 12 + 4 + 9$

$(x+2)^2 + (y-3)^2 = 25$

$r = \sqrt{25} = 5$

10 Given $\overline{MN}$ shown below, with $M(-6,1)$ and $N(3,-5)$, what is an equation of the line that passes through point $P(6,1)$ and is parallel to $\overline{MN}$?

Same slope as MN

$m\ \overline{MN} = \frac{-6}{9} = \frac{-2}{3}$

$y = \frac{-2}{3}x + b$

$1 = -\frac{2}{3}(6) + b$

$1 = -4 + b$

$+5 = b$

$y = -\frac{2}{3}x + 5$

(1) $y = -\frac{2}{3}x + 5$

(3) $y = \frac{3}{2}x + 7$

(2) $y = -\frac{2}{3}x - 3$

(4) $y = \frac{3}{2}x - 8$

11 Linda is designing a circular piece of stained glass with a diameter of 7 inches. She is going to sketch a square inside the circular region.

To the *nearest tenth of an inch*, the largest possible length of a side of the square is
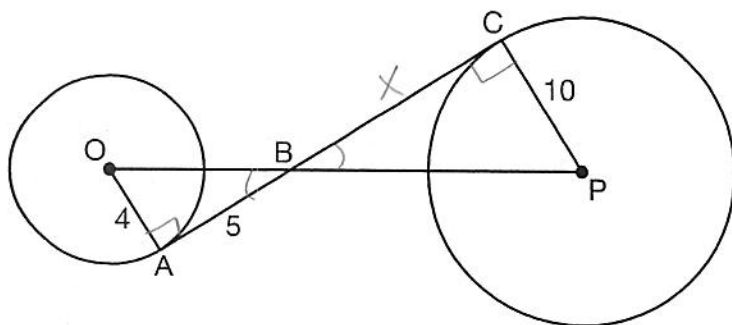
(1) 3.5

(3) 5.0

(2) 4.9

(4) 6.9
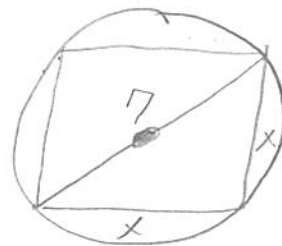
$x^2 + x^2 = 7^2$

$2x^2 = 49$

$x^2 = 24.5$

$x = \sqrt{24.5}$

12 In the diagram shown below, $\overline{AC}$ is tangent to circle O at A and to circle P at C, $\overline{OP}$ intersects $\overline{AC}$ at B, $OA = 4$, $AB = 5$, and $PC = 10$.

① $\triangle OAB \sim \triangle PCB$
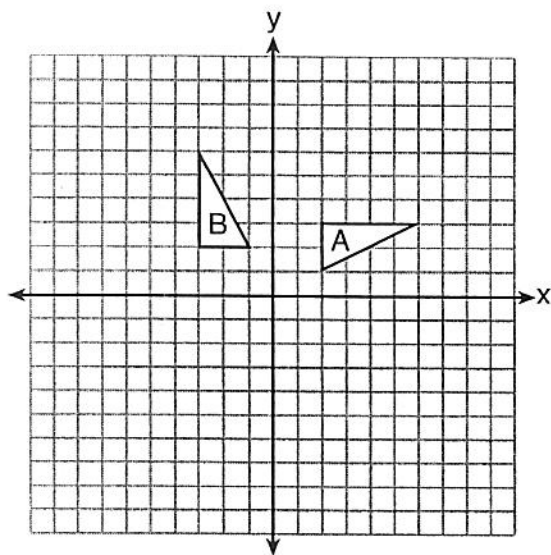b/c AA

② $\dfrac{4}{10} = \dfrac{5}{x}$

$4x = 50$

$x = 12.5$

What is the length of $\overline{BC}$?

(1) 6.4

(3) 12.5

(2) 8

(4) 16

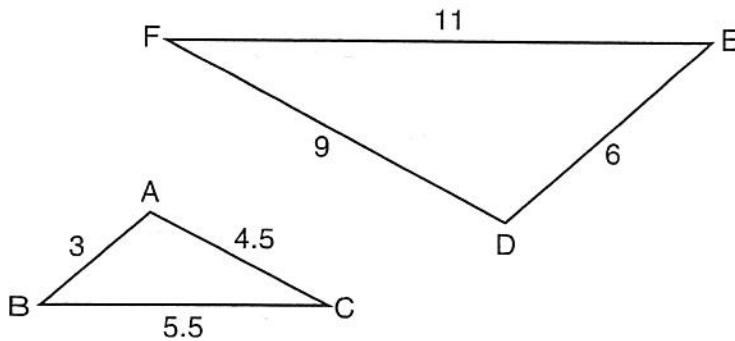13 In the diagram below, which single transformation was used to map triangle A onto triangle B?

(1) line reflection

(3) dilation

(2) rotation

(4) translation

14 In the diagram below, $\triangle DEF$ is the image of $\triangle ABC$ after a clockwise rotation of 180° and a dilation where $AB = 3$, $BC = 5.5$, $AC = 4.5$, $DE = 6$, $FD = 9$, and $EF = 11$.

(handwritten notes): △ ABC ~ △ DEF ∴ Corr. ∠'s are =. 1 & 2 wrong
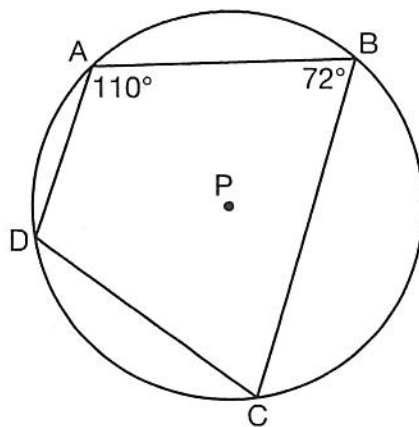
Which relationship must always be true?

(1) $\dfrac{m\angle A}{m\angle D} = \dfrac{1}{2}$ *(crossed out)*

(3) $\dfrac{m\angle A}{m\angle C} = \dfrac{m\angle F}{m\angle D}$

(2) $\dfrac{m\angle C}{m\angle F} = \dfrac{2}{1}$ *(crossed out)*

(4) $\dfrac{m\angle B}{m\angle E} = \dfrac{m\angle C}{m\angle F}$ *(circled)*

15 In the diagram below, quadrilateral $ABCD$ is inscribed in circle P.

(handwritten notes):
Cyclic Quad
have opp. ∠'s Supplementary.
∠B + ∠D = 180   ∠A + ∠C = 180
72 + ∠D = 180
∠D = 108   *(circled)*

What is $m\angle ADC$?

(1) 70°

(3) 108°   *(circled)*

(2) 72°

(4) 110°

16 A hemispherical tank is filled with water and has a diameter of 10 feet. If water weighs 62.4 pounds per cubic foot, what is the total weight of the water in a full tank, to the *nearest pound*?

(1) 16,336   (3) 130,690

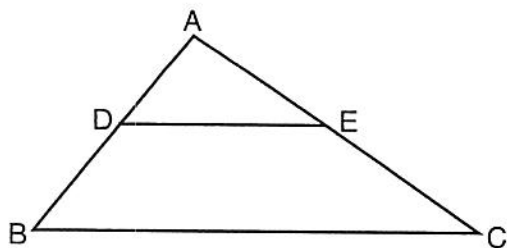(2) 32,673   (4) 261,381

handwritten: ½ sphere

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(5)^3$

$V = 523.5987756$

$V_{\ominus} = \frac{523.5987756}{2} = 261.7993 \times 62.4$

17 In the diagram below, △ABC ~ △ADE.

Which measurements are justified by this similarity?

(1) AD = 3, AB = 6, AE = 4, and AC = 12

(2) AD = 5, AB = 8, AE = 7, and AC = 10

(3) AD = 3, AB = 9, AE = 5, and AC = 10

(4) AD = 2, AB = 6, AE = 5, and AC = 15

handwritten: $\frac{2}{6} \overset{?}{=} \frac{5}{15}$ $\frac{1}{3} = \frac{1}{3}$ ✓

18 Triangle FGH is inscribed in circle O, the length of radius $\overline{OH}$ is 6, and $\overline{FH} \cong \overline{OG}$.

What is the area of the sector formed by angle FOH?

(1) 2π   (3) 6π

(2) $\frac{3}{2}\pi$ (4) 24π
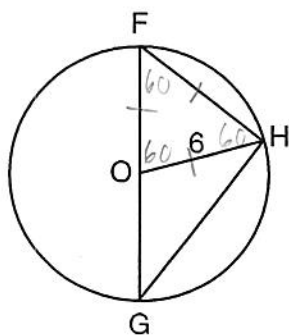
handwritten: △FOH is Equilateral b/c radii are =.

∠FOH = 60° b/c Equilateral △

$\frac{60}{360} = \frac{x}{\pi 6^2}$

$x = 6\pi$

Geometry (Common Core) – Aug. '15   [8]

19 As shown in the diagram below, $\overline{AB}$ and $\overline{CD}$ intersect at E, and $\overline{AC} \parallel \overline{BD}$.
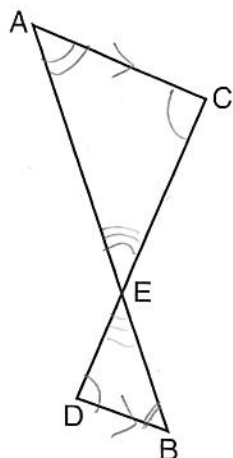
In ~ Δ's
Corresponding Sides
are in proportion.

Given $\triangle AEC \sim \triangle BED$, which equation is true?

(1) $\dfrac{CE}{DE} = \dfrac{EB}{EA}$

(3) $\dfrac{EC}{AE} = \dfrac{BE}{ED}$

(2) $\dfrac{AE}{BE} = \dfrac{AC}{BD}$

(4) $\dfrac{ED}{EC} = \dfrac{AC}{BD}$

20 A triangle is dilated by a scale factor of 3 with the center of dilation at the origin. Which statement is true?

(1) The area of the image is nine times the area of the original triangle.

(2) The perimeter of the image is nine times the perimeter of the original triangle.

(3) The slope of any side of the image is three times the slope of the corresponding side of the original triangle.

(4) The measure of each angle in the image is three times the measure of the corresponding angle of the original triangle.

ic

$A = \dfrac{2 \cdot 1}{2}$

$A = 1$

$A = \dfrac{6 \cdot 3}{2}$

$A = 9$

$B = x^2$
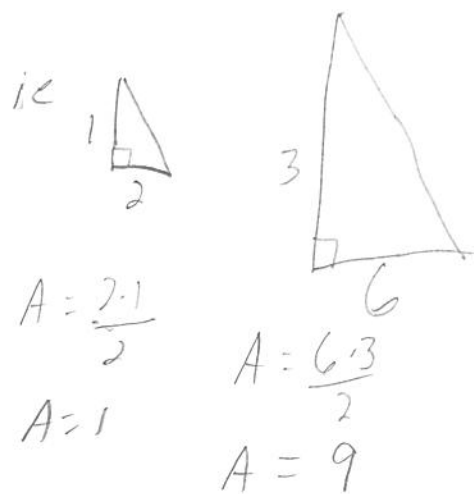
21 The Great Pyramid of Giza was constructed as a regular pyramid with a square base. It was built with an approximate volume of 2,592,276 cubic meters and a height of 146.5 meters. What was the length of one side of its base, to the *nearest meter*?

(1) 73

(3) 133

(2) 77

(4) 230

$V = \dfrac{1}{3} B \cdot h$
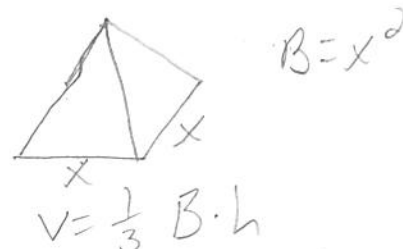
$2592276 = \dfrac{1}{3} B \cdot (146.5)$

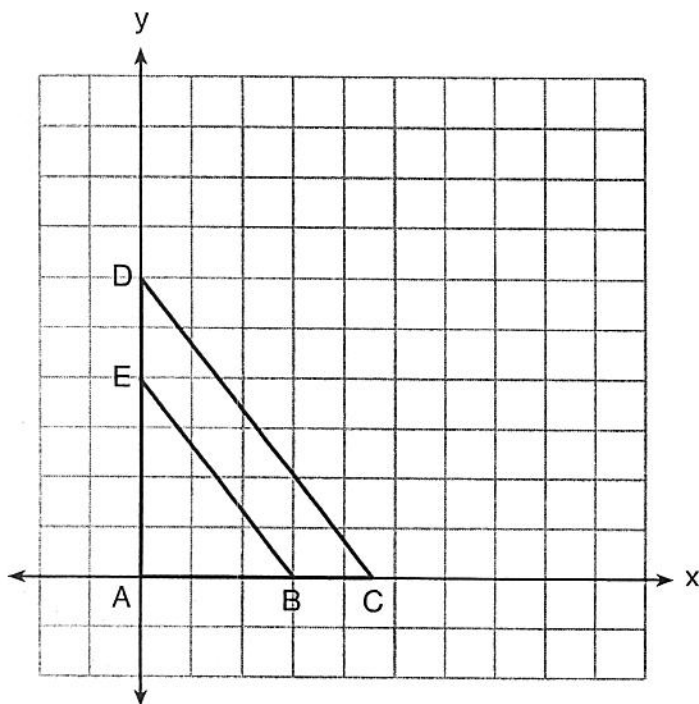*$53084.15 = B$*

$53084.15 = x^2$

$\sqrt{53084.15} = X$

22 A quadrilateral has vertices with coordinates $(-3,1)$, $(0,3)$, $(5,2)$, and $(-1,-2)$. Which type of quadrilateral is this?

(1) rhombus

(3) square

(2) rectangle

(4) trapezoid

use graph paper!
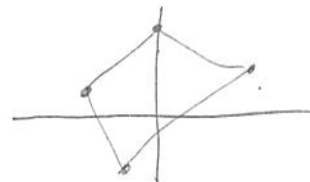
23 In the diagram below, $\triangle ABE$ is the image of $\triangle ACD$ after a dilation centered at the origin. The coordinates of the vertices are $A(0,0)$, $B(3,0)$, $C(4.5,0)$, $D(0,6)$, and $E(0,4)$.

The ratio of the lengths of $\overline{BE}$ to $\overline{CD}$ is

(1) $\frac{2}{3}$

(3) $\frac{3}{4}$

(2) $\frac{3}{2}$

(4) $\frac{4}{3}$

$\dfrac{BE}{CD} = \dfrac{EA}{DA} = \dfrac{4}{6} = \dfrac{2}{3}$

24 Line $y = 3x - 1$ is transformed by a dilation with a scale factor of 2 and centered at $(3,8)$. The line's image is

(1) $y = 3x - 8$

(3) $y = 3x - 2$

(2) $y = 3x - 4$

(4) $y = 3x - 1$

1) Dilations preserve parallelism. So same slope.

2) Since point of Dilation is on the line the lines stays the same.

___

Geometry (Common Core) – Aug. '15

[10]

25 A wooden cube has an edge length of 6 centimeters and a mass of 137.8 grams. Determine the density of the cube, to the *nearest thousandth*.

State which type of wood the cube is made of, using the density table below.

Type of Wood	Density (g/cm³)
Pine	0.373
Hemlock	0.431
Elm	0.554
Birch	0.601
Ash	0.638
Maple	0.676
Oak	0.711

$d = \dfrac{mass}{Volume}$ density .

$6 \cdot 6 \cdot 6 = \frac{96}{216}$

$d = \dfrac{m}{V}$

6cm
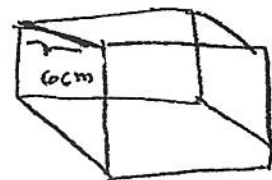
mass = 137.8

0.637962963
round up

$= \dfrac{137.8g}{216cm}$

0.638

Score 2: The student has a complete and correct response.

26 Construct an equilateral triangle inscribed in circle *T* shown below.

[Leave all construction marks.]
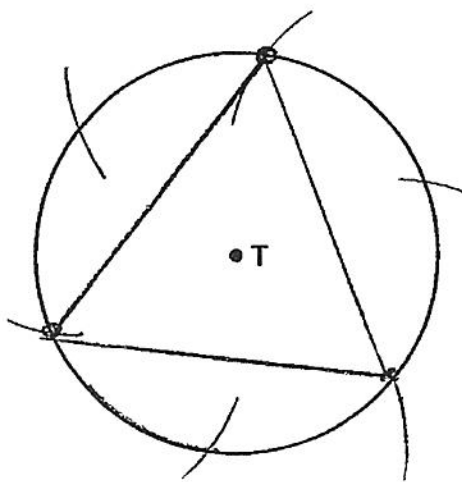
Score 2: The student drew a correct construction showing all appropriate construction marks.

27 To find the distance across a pond from point B to point C, a surveyor drew the diagram below. The measurements he made are indicated on his diagram.

$$\frac{120}{230} = \frac{x}{315}$$

$$37800 = 230x$$

$$164.3478261$$
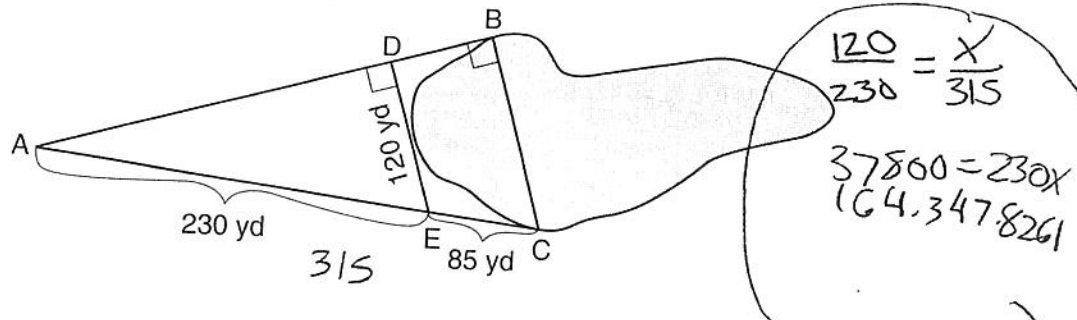
Use the surveyor's information to determine and state the distance from point B to point C, to the nearest yard.

B to C ~ 164 yards

Score 2:   The student has a complete and correct response.

Question 28

28 In parallelogram *ABCD* shown below, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at *E*.
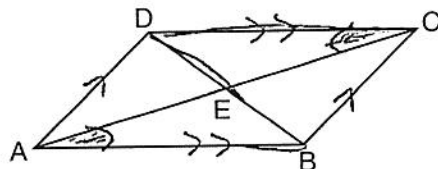
Prove: $\angle ACD \cong \angle CAB$

statements	reasons
1) ABCD is a parallelogram, diag. $\overline{AC}$ and $\overline{BD}$ intersect @ E	1) Given
2) $\overline{DC} \parallel \overline{AB}, \overline{DA} \parallel \overline{CB}$	2) Opp. sides of a parallelogram are $\parallel$.
3) $\angle ACD \cong \angle BAC$	3) If 2 $\parallel$ lines are cut by a transversal, then alt. int. $\angle$'s are $\cong$.

Score 2: The student has written a complete and correct proof.

29 Triangles RST and XYZ are drawn below. If $RS = 6$, $ST = 14$, $XY = 9$, $YZ = 21$, and $\angle S \cong \angle Y$, is $\triangle RST$ similar to $\triangle XYZ$? Justify your answer.
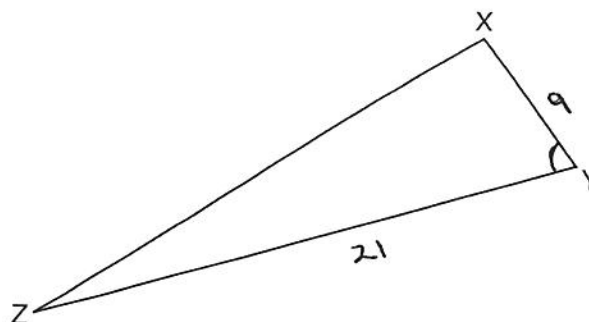
$$\frac{6}{14} = \frac{9}{21}$$

$$126 = 126$$

Yes, because of SAS similarity

Score 2: The student has a complete and correct response.

30 In the diagram below, $\triangle ABC$ and $\triangle XYZ$ are graphed.
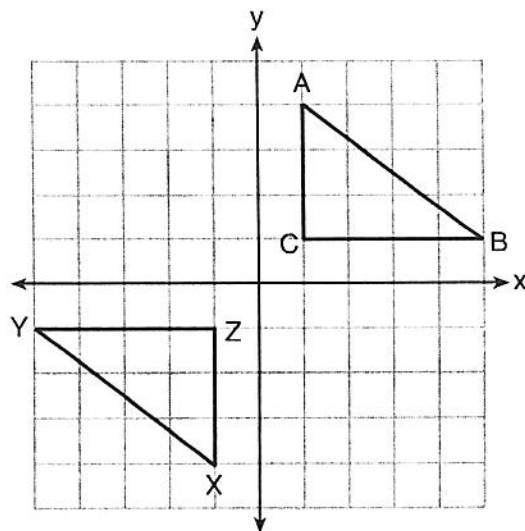
Use the properties of rigid motions to explain why $\triangle ABC \cong \triangle XYZ$.

$\triangle XYZ$ is the image of $\triangle ABC$ after a rotation of $180°$ about the origin which means $\triangle ABC$ can be mapped onto $\triangle XYZ$ and distance is preserved in any rotation. Rotations are also rigid motions and then the triangles are congruent.

Score 2: The student has a complete and correct response.

31 The endpoints of $\overline{DEF}$ are $D(1,4)$ and $F(16,14)$. Determine and state the coordinates of point E, if $DE:EF = 2:3$.

$$\frac{DE}{DF} = \frac{2}{5}$$

$$16 - 1 \qquad 14 - 4$$
$$15 \qquad 10$$

$$\frac{2}{5} \cdot 15 \qquad \frac{2}{5} \cdot 10$$

$$\frac{30}{5} \qquad \frac{20}{5}$$

$$6 \qquad 4$$

$$1 + 6 \qquad 4 + 4$$

$$(7, \quad 8)$$

Score 2: The student has a complete and correct response.

32 As shown in the diagram below, a ship is heading directly toward a lighthouse whose beacon is 125 feet above sea level. At the first sighting, point A, the angle of elevation from the ship to the light was 7°. A short time later, at point D, the angle of elevation was 16°.
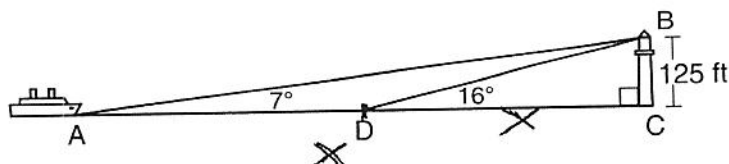
To the *nearest foot*, determine and state how far the ship traveled from point A to point D.

1)
$$\text{Tan } 7° = \frac{125}{X}$$

$$\frac{X \text{ Tan } 7°}{\text{Tan } 7} = \frac{125}{\text{Tan } 7}$$

$$X = 1018.043303$$

2) $$\text{Tan } 16° = \frac{125}{X}$$

$$\frac{X \text{ Tan } 16°}{\text{Tan } 16°} = \frac{125}{\text{Tan } 16°}$$

$$X = 435.9268055$$

3) 1018.043303
   $- 435.9268055$
   _____

$$\boxed{\text{Distance } A \longrightarrow D = 582 ft}$$

Score 4: The student has a complete and correct response.

33 Triangle *ABC* has vertices with $A(x,3)$, $B(-3,-1)$, and $C(-1,-4)$.

Determine and state a value of x that would make triangle *ABC* a right triangle. Justify why △*ABC* is a right triangle.

[The use of the set of axes below is optional.]

$$\text{slope of } BC = \frac{\Delta y}{\Delta x}$$

$$BC = \frac{-3}{2}$$

$$y = \frac{2}{3}x + b$$

$$x = 3$$
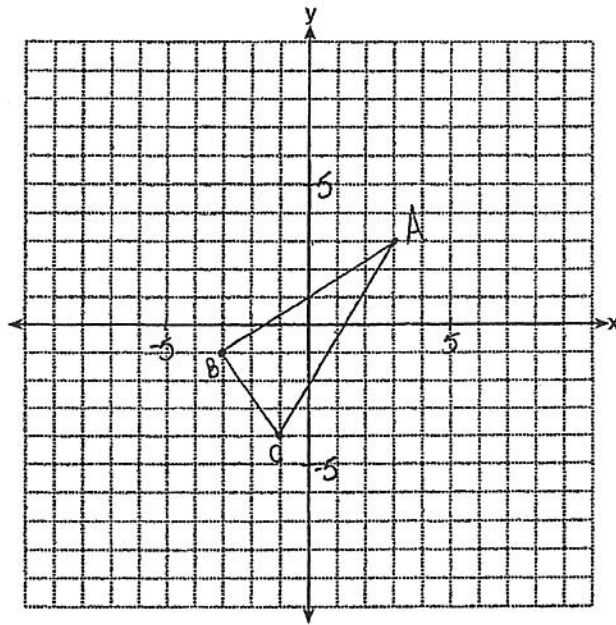
Score 4: The student has a complete and correct response.

34 In the diagram below, $\overline{AC} \cong \overline{DF}$ and points A, C, D, and F are collinear on line ℓ.
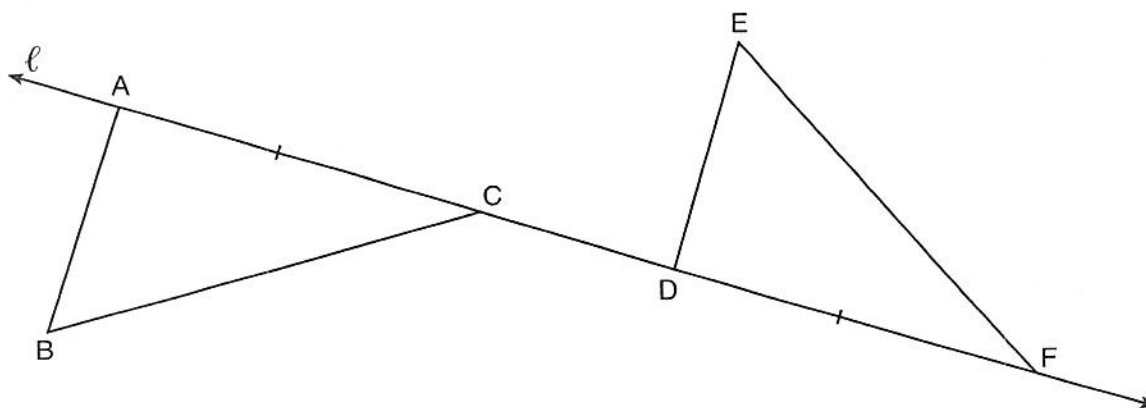
Let $\triangle D'E'F'$ be the image of $\triangle DEF$ after a translation along ℓ, such that point D is mapped onto point A. Determine and state the location of F'. Explain your answer.

Point F' would be at point C after translation
because $\overline{AC} \cong \overline{DF}$ and if D' meets A then F' will
touch C.

Let $\triangle D''E''F''$ be the image of $\triangle D'E'F'$ after a reflection across line ℓ. Suppose that E'' is located at B. Is $\triangle DEF$ congruent to $\triangle ABC$? Explain your answer.

△ DEF is congruent to △ABC because
from the previous answer, $\overline{DF} \cong \overline{AC}$ So if
Point E'' lays on point B the traingles
are Congruent. This is proven by $\overline{SSS} \cong \overline{SSS}$

Score 4: The student has a complete and correct response.

35 In the diagram of parallelogram *ABCD* below, $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, and $\overline{CE} \cong \overline{CF}$.

Prove *ABCD* is a rhombus.

S	R
① ABCD is a ▱, BE ⊥ CED, DF ⊥ BFC, CE ≅ CF	① given
② ∠1, ∠2, ∠3, ∠4 are Rt ∠s	② ⊥ lines intersect to (1) form Rt ∠s
③ ∠2 ≅ ∠3	③ If 2 ∠s are Rt, then they are ≅ (2)
④ ∠C ≅ ∠C	④ Reflexive
⑤ △DFC ≅ △BEC	⑤ ASA ≅ ASA (1, 3, 4)
⑥ DC ≅ BC	⑥ if 2 △'s are ≅ corresp parts are ≅ (5)
⑦ ABCD is a rhombus	⑦ a rhombus is a ▱ w/ 2 consec. sides ≅ (1, 7)

Score 6: The student has a complete and correct proof.

36 Walter wants to make 100 candles in the shape of a cone for his new candle business. The mold shown below will be used to make the candles. Each mold will have a height of 8 inches and a diameter of 3 inches. To the *nearest cubic inch*, what will be the total volume of 100 candles?

$$V = \frac{1}{3}\pi(1.5)^2 \cdot 8$$

$$V_{100} = \frac{800}{3}\pi(2.25)$$

$$= 1885 \text{ in}^3$$

↑
Volume for 100 candles
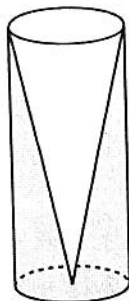
Walter goes to a hobby store to buy the wax for his candles. The wax costs $0.10 per ounce. If the weight of the wax is 0.52 ounce per cubic inch, how much will it cost Walter to buy the wax for 100 candles?

$$(1885)(\text{in}^3)(0.52)\left(\frac{oz}{in^3}\right)(0.10)\left(\frac{\$}{oz}\right) = \$98.02$$

If Walter spent a total of $37.83 for the molds and charges $1.95 for each candle, what is Walter's profit after selling 100 candles?

Revenue = $195

Cost = $37.83 + $98.02 = $135.85

Profit = $195 − $135.85 = $59.15

Score 6: The student has a complete and correct response.