The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY (Common Core)

Tuesday, June 2, 2015 — 1:15 to 4:15 p.m., only

Student Name: _____

School Name: Key

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 36 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

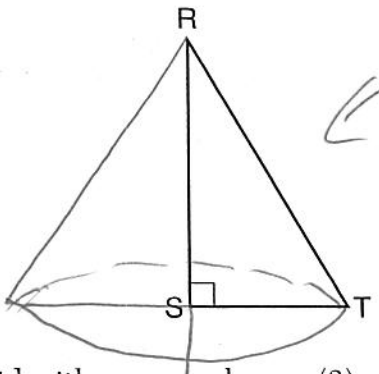
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. [48]

Use this space for computations.

1 Which object is formed when right triangle RST shown below is rotated around leg $\overline{RS}$?



think of it constantly spinning around RS. It makes a cone.

(1) a pyramid with a square base
(2) an isosceles triangle
(3) a right triangle
(4) a cone

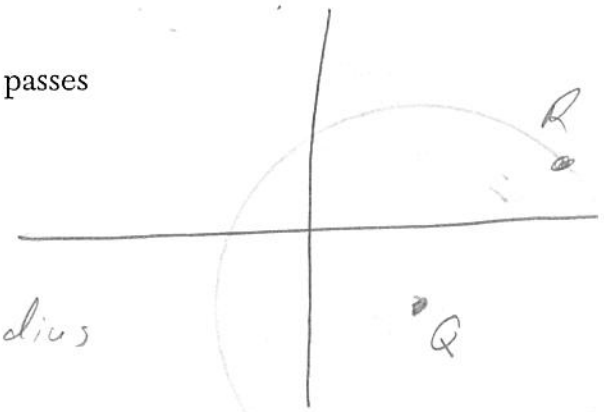
2 The vertices of $\triangle JKL$ have coordinates $J(5,1)$, $K(-2,-3)$, and $L(-4,1)$. Under which transformation is the image $\triangle J'K'L'$ not congruent to $\triangle JKL$?

Only dilations make shapes that are not ≅.

(1) a translation of two units to the right and two units down
(2) a counterclockwise rotation of 180 degrees around the origin
(3) a reflection over the x-axis
(4) a dilation with a scale factor of 2 and centered at the origin

3 The center of circle Q has coordinates $(3,-2)$. If circle Q passes through $R(7,1)$, what is the length of its diameter?

(1) 50
(3) 10
(2) 25
(4) 5



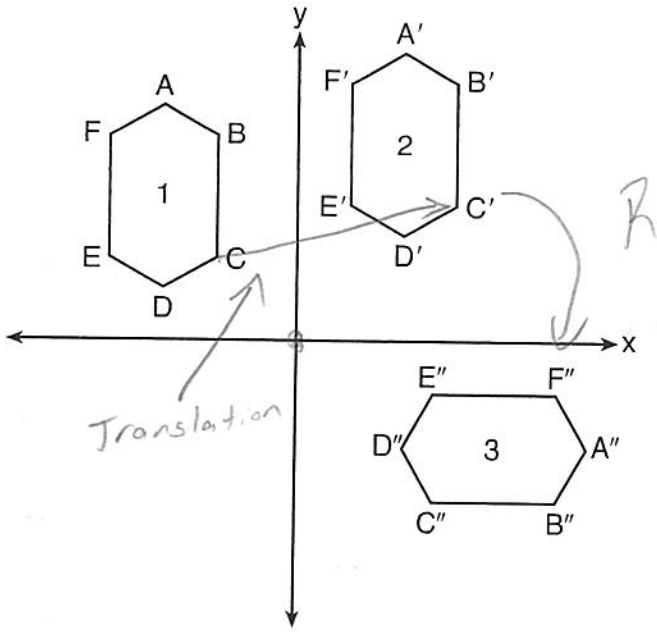
$RQ = \sqrt{(3-7)^2 + (-2-1)^2} = \sqrt{16+9}$ radius

$= 5$

diameter $= 10$

Use this space for computations.

4 In the diagram below, congruent figures 1, 2, and 3 are drawn.



Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

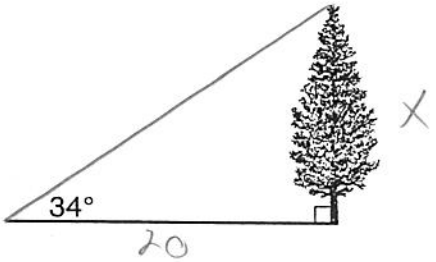
(1) a reflection followed by a translation

(2) a rotation followed by a translation

(3) a translation followed by a reflection

(4) a translation followed by a rotation

5 As shown in the diagram below, the angle of elevation from a point on the ground to the top of the tree is 34°.



$\tan 34° = \frac{x}{20}$

$x = 20 \cdot \tan 34°$

$= 13.5$ ft

If the point is 20 feet from the base of the tree, what is the height of the tree, to the *nearest tenth of a foot*?

(1) 29.7

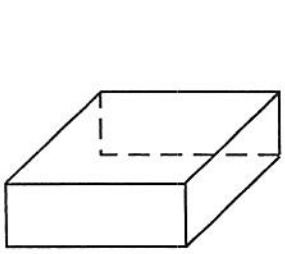
(2) 16.6

(3) 13.5

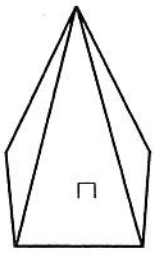
(4) 11.2

Use this space for computations.

6 Which figure can have the same cross section as a sphere?



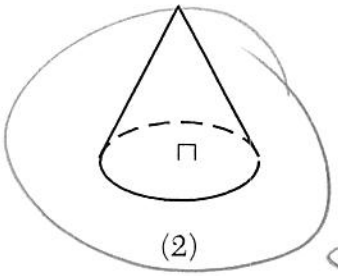
(1)



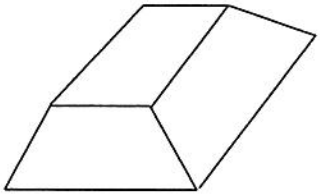
(3)

Sphere

Circle cross sections



(2)



(4)

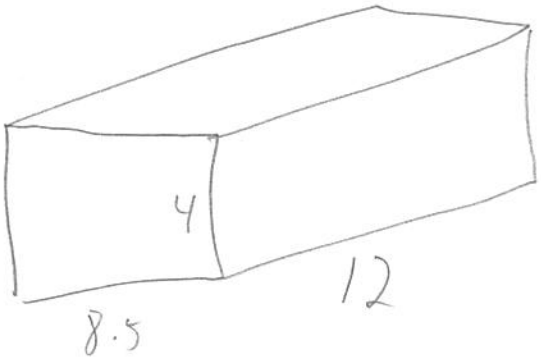
7 A shipping container is in the shape of a right rectangular prism with a length of 12 feet, a width of 8.5 feet, and a height of 4 feet. The container is completely filled with contents that weigh, on average, 0.25 pound per cubic foot. What is the weight, in pounds, of the contents in the container?

(1) 1,632

(2) 408

(3) 102

(4) 92



$V = l \cdot w \cdot h$

$= (12)(8.5)(4)$

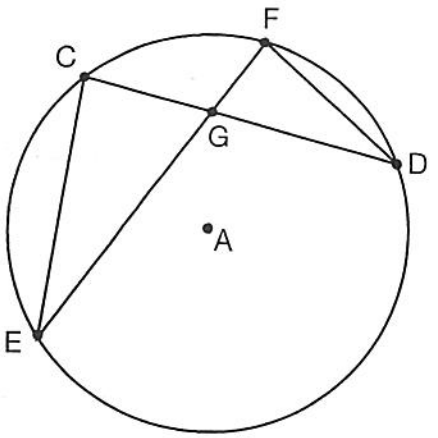
$= 408\ ft^3$

Weight $= .25(408)$

$= 102$ lbs

Use this space for computations.

8 In the diagram of circle A shown below, chords $\overline{CD}$ and $\overline{EF}$ intersect at G, and chords $\overline{CE}$ and $\overline{FD}$ are drawn.



Which statement is *not* always true?

(1) $\overline{CG} \cong \overline{FG}$

(2) $\angle CEG \cong \angle FDG$

(3) $\frac{CE}{EG} = \frac{FD}{DG}$

(4) $\triangle CEG \sim \triangle FDG$

CSSTP

△s are ~ b/c AA

Inscribed ∠s to the same arc.

9 Which equation represents a line that is perpendicular to the line represented by $2x - y = 7$?

(1) $y = -\frac{1}{2}x + 6$

(2) $y = \frac{1}{2}x + 6$

(3) $y = -2x + 6$

(4) $y = 2x + 6$

Negative reciprocal slopes

$2x - y = 7$

$+y \quad +y$

$2x = y + 7$

$-7 \quad -7$

$2x - 7 = y$

$m = 2$ so $\perp$ slope $= -\frac{1}{2}$

Use this space for computations.

10 Which regular polygon has a minimum rotation of 45° to carry the polygon onto itself?

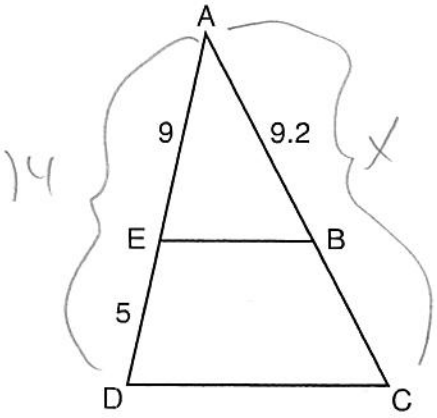
(1) octagon
(2) decagon
(3) hexagon
(4) pentagon

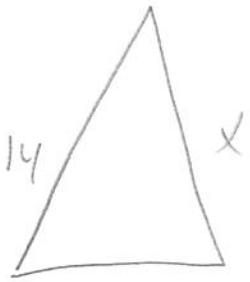
↓ rotational Symmetry

$\frac{360}{n} = 45$

$n = 8$

11 In the diagram of $\triangle ADC$ below, $\overline{EB} \parallel \overline{DC}$, $AE = 9$, $ED = 5$, and $AB = 9.2$.





Δ's ~ by the side splitter

$\frac{9}{14} = \frac{9.2}{X}$

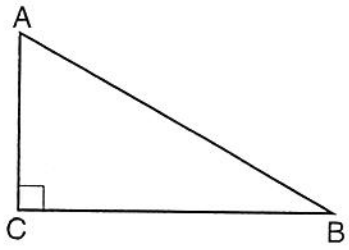
What is the length of $\overline{AC}$, to the *nearest tenth*?

(1) 5.1
(2) 5.2
(3) 14.3
(4) 14.4

$X = 14.3$

12 In scalene triangle ABC shown in the diagram below, $m\angle C = 90°$.

Use this space for computations.



Complimentary trig functions

$\cos A = \sin B$ if $A+B=90$

Which equation is always true?

(1) $\sin A = \sin B$

(2) $\cos A = \cos B$

(3) $\cos A = \sin C$

(4) $\sin A = \cos B$

13 Quadrilateral $ABCD$ has diagonals $\overline{AC}$ and $\overline{BD}$. Which information is *not* sufficient to prove $ABCD$ is a parallelogram?

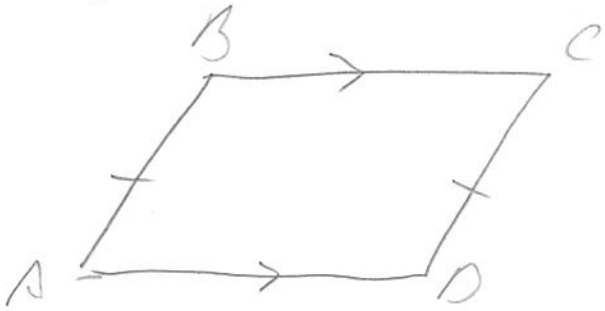
(1) $\overline{AC}$ and $\overline{BD}$ bisect each other.

(2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$

(3) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$

(4) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$

This can be a trapezoid.



14 The equation of a circle is $x^2 + y^2 + 6y = 7$. What are the coordinates of the center and the length of the radius of the circle?

(1) center (0,3) and radius 4

(2) center (0,−3) and radius 4

(3) center (0,3) and radius 16

(4) center (0,−3) and radius 16

$x^2 + y^2 + 6y = 7$

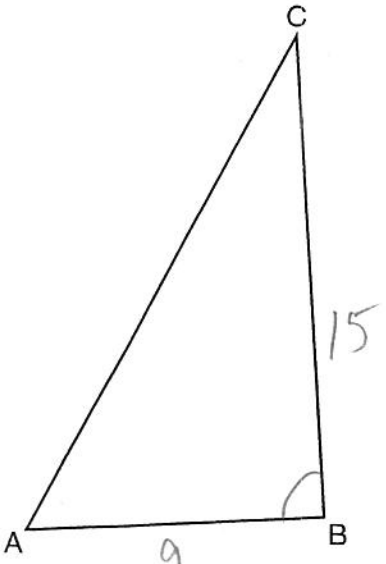
$x^2 + y^2 + 6y + 9 = 7 + 9$

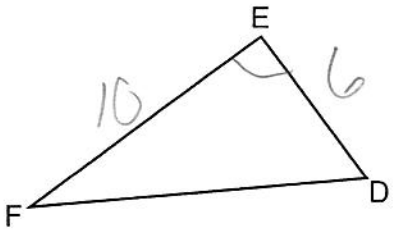
$x^2 + (y+3)(y+3) = 16$

$x^2 + (y+3)^2 = 16$

Use this space for computations.

15 Triangles ABC and DEF are drawn below.





$\frac{9}{6} = \frac{7}{10}$

$90 = 90$ ✓

So $\triangle$s ~ by SAS ~

Not ∠s (corr.)

If $AB = 9$, $BC = 15$, $DE = 6$, $EF = 10$, and $\angle B \cong \angle E$, which statement is true?

(1) $\angle CAB \cong \angle DEF$

(2) $\frac{AB}{CB} = \frac{FE}{DE}$

(3) $\triangle ABC \sim \triangle DEF$

(4) $\frac{AB}{DE} = \frac{FE}{CB}$

Not the right proportions

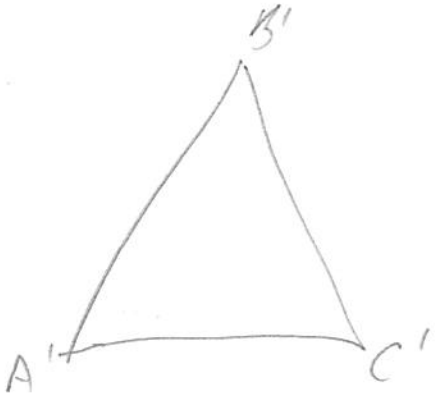
16 If $\triangle ABC$ is dilated by a scale factor of 3, which statement is true of the image $\triangle A'B'C'$?

(1) $3A'B' = AB$

(2) $B'C' = 3BC$

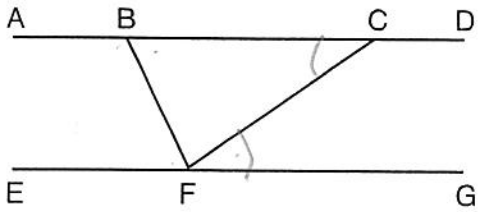
(3) $m\angle A' = 3(m\angle A)$ ← Corr. ∠s =

(4) $3(m\angle C') = m\angle C$



Use this space for computations.

17 Steve drew line segments $ABCD$, EFG, BF, and CF as shown in the diagram below. Scalene $\triangle BFC$ is formed.



Which statement will allow Steve to prove $\overline{ABCD} \parallel \overline{EFG}$?

(1) $\angle CFG \cong \angle FCB$

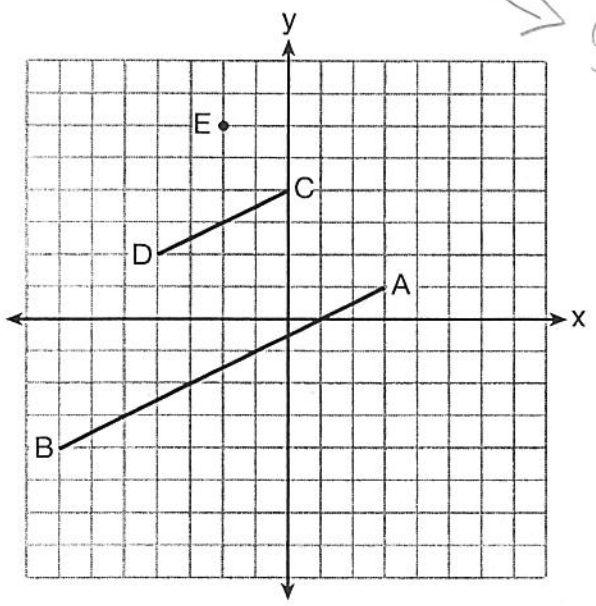
(2) $\angle ABF \cong \angle BFC$

(3) $\angle EFB \cong \angle CFB$

(4) $\angle CBF \cong \angle GFC$

Alt. int. $\angle$'s $=$.

18 In the diagram below, $\overline{CD}$ is the image of $\overline{AB}$ after a dilation of scale factor k with center E.



got smaller

Which ratio is equal to the scale factor k of the dilation?

(1) $\frac{EC}{EA}$

(2) $\frac{BA}{EA}$

(3) $\frac{EA}{BA}$

(4) $\frac{EA}{EC}$

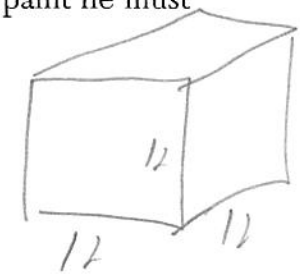
small corresponding side / big corr. side.

all faces =, all sides =

Use this space for computations.

19 A gallon of paint will cover approximately 450 square feet. An artist wants to paint all the outside surfaces of a cube measuring 12 feet on each edge. What is the *least* number of gallons of paint he must buy to paint the cube?

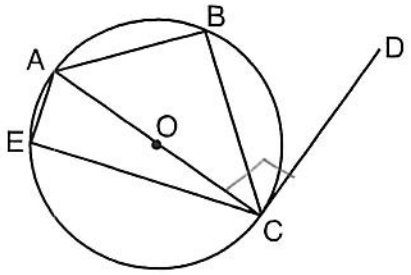
(1) 1
(2) 2
(3) 3
(4) 4



Surface Area =
= 144 (6)
= 864

$\frac{864}{450} = 1.92$

20 In circle O shown below, diameter $\overline{AC}$ is perpendicular to $\overline{CD}$ at point C, and chords $\overline{AB}$, $\overline{BC}$, $\overline{AE}$, and $\overline{CE}$ are drawn.



Inscribed ∠s to same arc

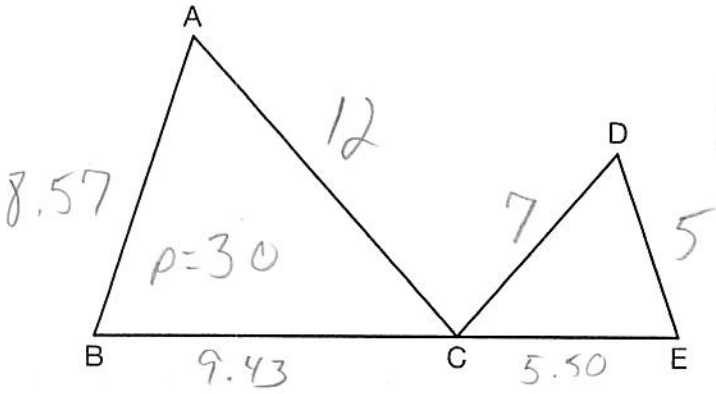
Which statement is *not* always true?

(1) $\angle ACB \cong \angle BCD$
(2) $\angle ABC \cong \angle ACD$
(3) $\angle BAC \cong \angle DCB$
(4) $\angle CBA \cong \angle AEC$

Thales Thm

Thales Thm
= 90°

21 In the diagram below, $\triangle ABC \sim \triangle DEC$.



① $\frac{AB}{5} = \frac{12}{7}$
$AB = 8.57$

② $8.57 + 12 + BC = 30$
$BC = 9.43$

③ $\frac{9.43}{CE} = \frac{12}{7}$
$CE = 5.50$

If $AC = 12$, $DC = 7$, $DE = 5$, and the perimeter of $\triangle ABC$ is 30, what is the perimeter of $\triangle DEC$?

(1) 12.5
(2) 14.0
(3) 14.8
(4) 17.5

Per. DEC = 7 + 5 + 5.50
= 17.5

22 The line $3y = -2x + 8$ is transformed by a dilation centered at the origin. Which linear equation could be its image?

Use this space for computations.

(1) $2x + 3y = 5$

(2) $2x - 3y = 5$

(3) $3x + 2y = 5$

(4) $3x - 2y = 5$

$\frac{3y}{3} = \frac{-2x}{3} + \frac{8}{3}$

$y = -\frac{2}{3}x + \frac{8}{3}$

$2x + 3y = 5$

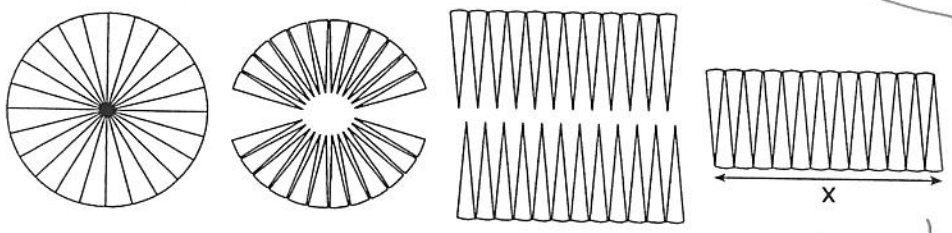
$-2x \quad -2x$

$3y = -2x + 5$

$y = -\frac{2}{3}x + \frac{5}{3}$

Dilations Preserve Parallelism so find same slope.

23 A circle with a radius of 5 was divided into 24 congruent sectors. The sectors were then rearranged, as shown in the diagram below.



To the *nearest integer*, the value of x is

(1) 31

(2) 16

(3) 12

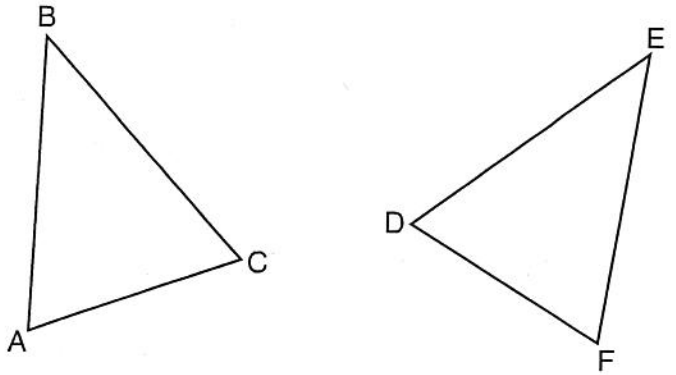
(4) 10

$x \approx \frac{1}{2}$ Circumference

$= \frac{1}{2}(10\pi) = 5\pi$

$= 16$

24 Which statement is sufficient evidence that $\triangle DEF$ is congruent to $\triangle ABC$?



SSS
SAS
AAS
ASA
HL

(1) $AB = DE$ and $BC = EF$ SS

(2) $\angle D \cong \angle A$, $\angle B \cong \angle E$, $\angle C \cong \angle F$ AAA

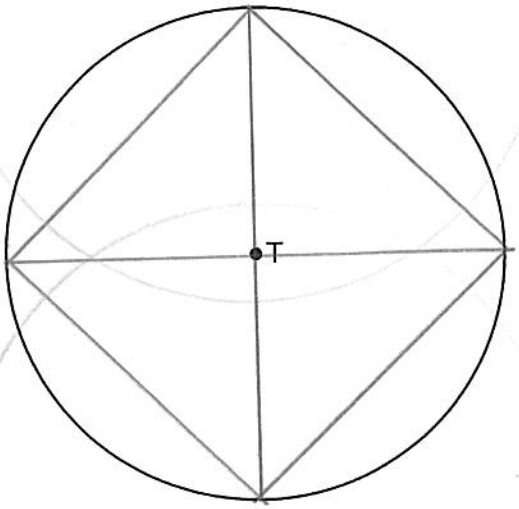
(3) There is a sequence of rigid motions that maps $\overline{AB}$ onto $\overline{DE}$, $\overline{BC}$ onto $\overline{EF}$, and $\overline{AC}$ onto $\overline{DF}$. SSS

(4) There is a sequence of rigid motions that maps point A onto point D, $\overline{AB}$ onto $\overline{DE}$, and $\angle B$ onto $\angle E$.

Part II

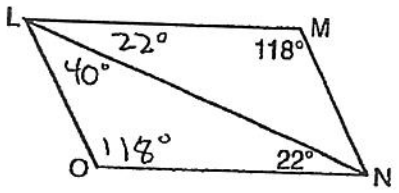
Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 Use a compass and straightedge to construct an inscribed square in circle T shown below. [Leave all construction marks.]



Question 26

26 The diagram below shows parallelogram $LMNO$ with diagonal $\overline{LN}$, $m\angle M = 118°$, and $m\angle LNO = 22°$.



Explain why $m\angle NLO$ is 40 degrees.

∠LON is 118° b/c opposite ∠'s of a ▱ are ≅.

& △'s ∠ measures add up to 180°.
118+22 = 140 so ∠NLO must be 40°.

Score 2: The student has a complete and correct response.

Question 27

27 The coordinates of the endpoints of $\overline{AB}$ are $A(-6,-5)$ and $B(4,0)$. Point P is on $\overline{AB}$. Determine and state the coordinates of point P, such that $AP:PB$ is 2:3.
[The use of the set of axes below is optional.]

X-value

$-6+\frac{2}{5}(4--6)$

$-6+\frac{2}{5}(10)$

$-6+4$

-2

y-value

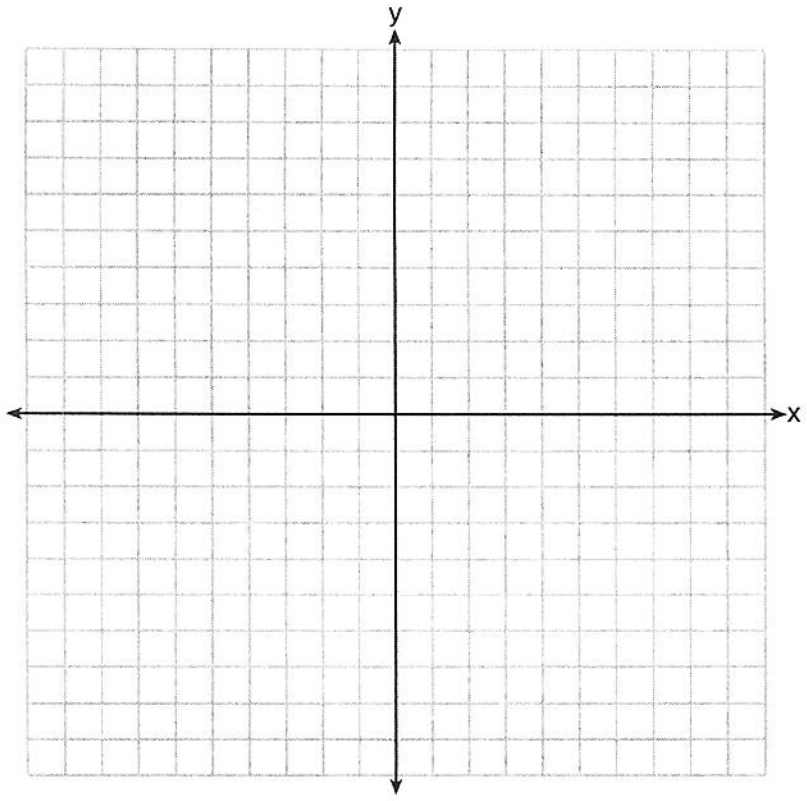
$-5+\frac{2}{5}(0--5)$

$-5+\frac{2}{5}(5)$

$-5+2$

-3

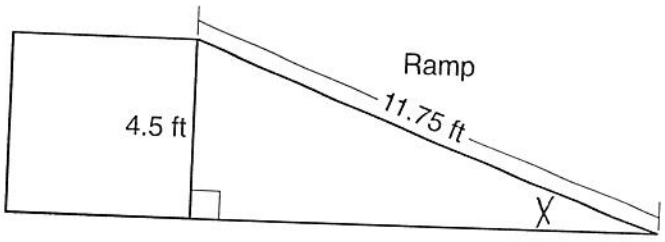
$(-2,-3)$



Score 2: The student has a complete and correct response.

Question 28

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

$X = \sin^{-1}\left(\frac{4.5}{11.75}\right)$

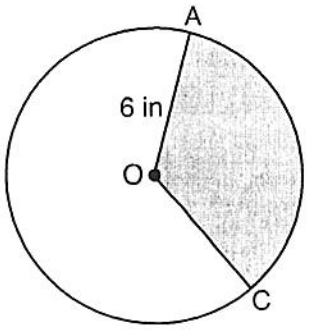
$X = 22.518$

$X = 23°$

Score 2: The student has a complete and correct response.

Question 29

29 In the diagram below of circle O, the area of the shaded sector AOC is 12π in^2 and the length of $\overline{OA}$ is 6 inches. Determine and state m$\angle AOC$.



$A = \pi r^2$

$= 6^2 \cdot \pi$

$= 36\pi$

$\frac{12\pi}{36\pi} = \frac{1}{3}$

$\frac{1}{3} \cdot 360$

$120°$

Score 2: The student has a complete and correct response.

Question 30

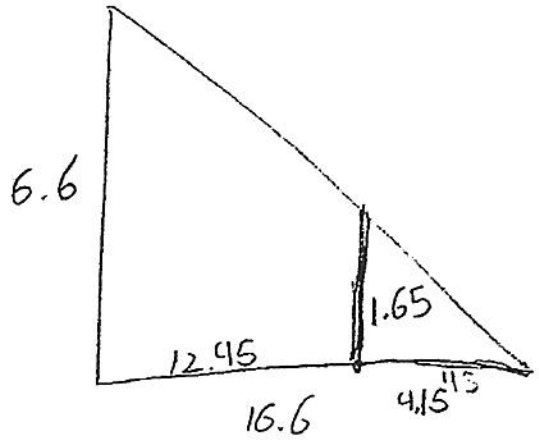
30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.

Reflections are rigid motions and Rigid Motions ~~are~~ Keep distances the same. So $\overline{AB} \cong \overline{A'B'}$ and $\overline{BC} \cong \overline{B'C'}$ and $\overline{AC} \cong \overline{A'C'}$, so $\triangle$'s $\cong$ SSS

Score 2: The student has a complete and correct response.

Question 31

31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter.*



$\frac{1.65}{4.15} = \frac{x}{16.6}$

$4.15x = 27.39$

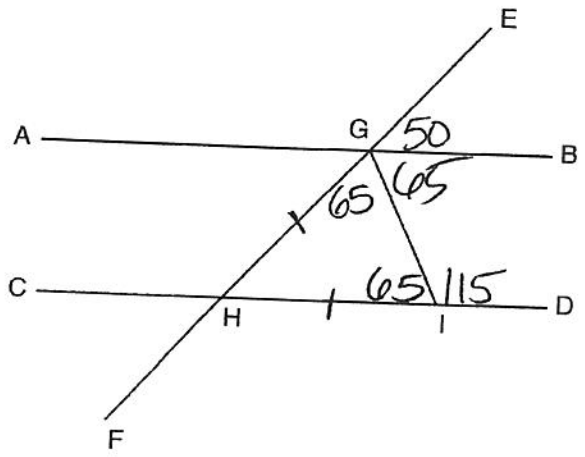
$x = 6.6$

6.6 m

Score 2: The student has a complete and correct response.

Question 32

32 In the diagram below, $\overline{EF}$ intersects $\overline{AB}$ and $\overline{CD}$ at G and H, respectively, and $\overline{GI}$ is drawn such that $\overline{GH} \cong \overline{IH}$.



If $m\angle EGB = 50°$ and $m\angle DIG = 115°$, explain why $\overline{AB} \parallel \overline{CD}$.

$m\angle GIH = 65$ linear pairs are supplementary

$m\angle HGI = 65$ - Base angles of an isosceles triangle are equal

$m\angle EGB + m\angle BGI + m\angle HGI = 180$

$50 + m\angle BGI + 65 = 180$

$115 + m\angle BGI = 180$

$-115 \qquad -115$

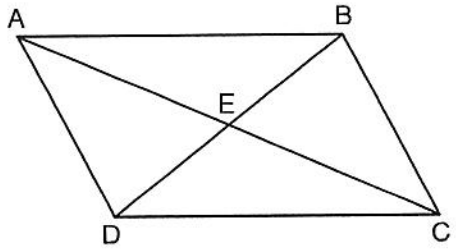
$m\angle BGI = 65$

$\angle BGI$ and $\angle DIG$ are same-side interior $\angle$'s, and since they are supplementary, $\overline{AB} \parallel \overline{CD}$.

Score 4: The student has a complete and correct response.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

Statement	Reason
1. Quad ABCD is a parallelogram	1. given
2. $\overline{AD} \cong \overline{CB}$	2. opposite sides of parallelogram are congruent
3. $\overline{AC}$ and $\overline{DB}$ intersect at E	3. given
4. $\angle AED \cong \angle CEB$	4. vertical angles are congruent
5. $\overline{BC} \parallel \overline{DA}$	5. def. of $\square$
6. $\angle DBC \cong \angle BDA$	6. alt. interior angles are $\cong$
7. $\triangle AED \cong \triangle CEB$	7. AAS $\cong$ AAS

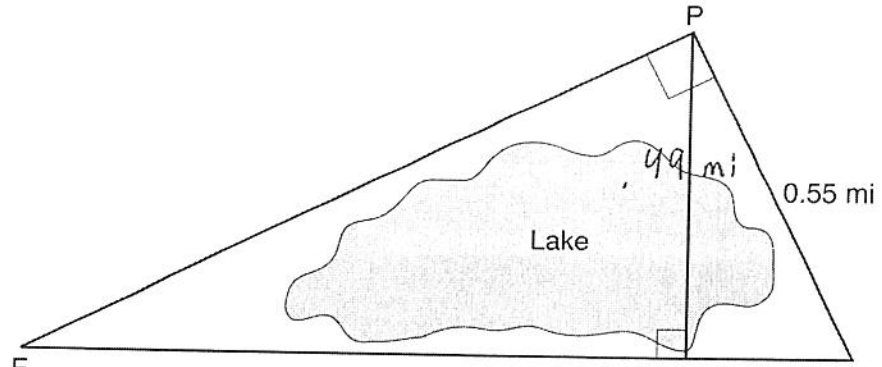
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Rotation of $\triangle AED$ around point E of 180°

Score 4: The student has a complete and correct proof, and a correct rigid motion is described.

Question 34

34 In the diagram below, the line of sight from the park ranger station, P, to the lifeguard chair, L, on the beach of a lake is perpendicular to the path joining the campground, C, and the first aid station, F. The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



0.9604 miles

If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$a^2+b^2=c^2$

$a^2+.0625=.3025$

$a^2=0.24$

$a=.489897...$

the distance is 0.49 miles

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

Altitude $= \frac{x}{h} = \frac{h}{y}$

$\frac{0.25}{0.49} = \frac{0.49}{y}$

$0.25y = 0.2401$

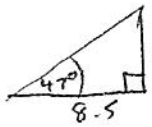
$y = 0.9604$

NO, the distance from F to L is 0.9604 miles. When added to the distance from L to C, it's only around 1.2 miles, not 1.5 miles.

Score 4: The student has a complete and correct response.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower.



$\tan 47° = \frac{x}{8.5}$

$x = 8.5 \tan 47°$

$x = 9.11513$

Volume cone

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi (8.5)^2 (9.11513)$

$V = 689.65125$

Volume cylinder

$V = \pi r^2 h$

$V = \pi (8.5)^2 (25)$

$V = 5674.50173$

Volume Hemisphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{2}{3}\pi (8.5)^3$

$= 1286.22039$

$V = 689.65125 + 5674.50173 + 1286.22039 = 7650.37337$

$= \boxed{7650 \text{ ft}^3}$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$7650 \times 62.4 = 477{,}360 \text{ lbs}$

$.85 \times 477{,}360 = \boxed{405{,}756 \text{ lbs}}$

No - the weight would exceed 400,000 lbs

Score 6: The student had a complete and correct response.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

$m_{\overline{RS}} = \frac{3}{5}$

$m_{\overline{ST}} = \frac{-10}{6} = \frac{-5}{3}$

Therefore the slopes of $\overline{RS}$ and $\overline{ST}$ are negative reciprocals and so $\overline{RS} \perp \overline{ST}$. Since the segments are $\perp$, $\angle S$ is a rt $\angle$.

$\therefore \triangle RST$ is a rt $\triangle$ because it has 1 rt $\angle$.

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

(0,9)

Question 36 is continued on the next page.

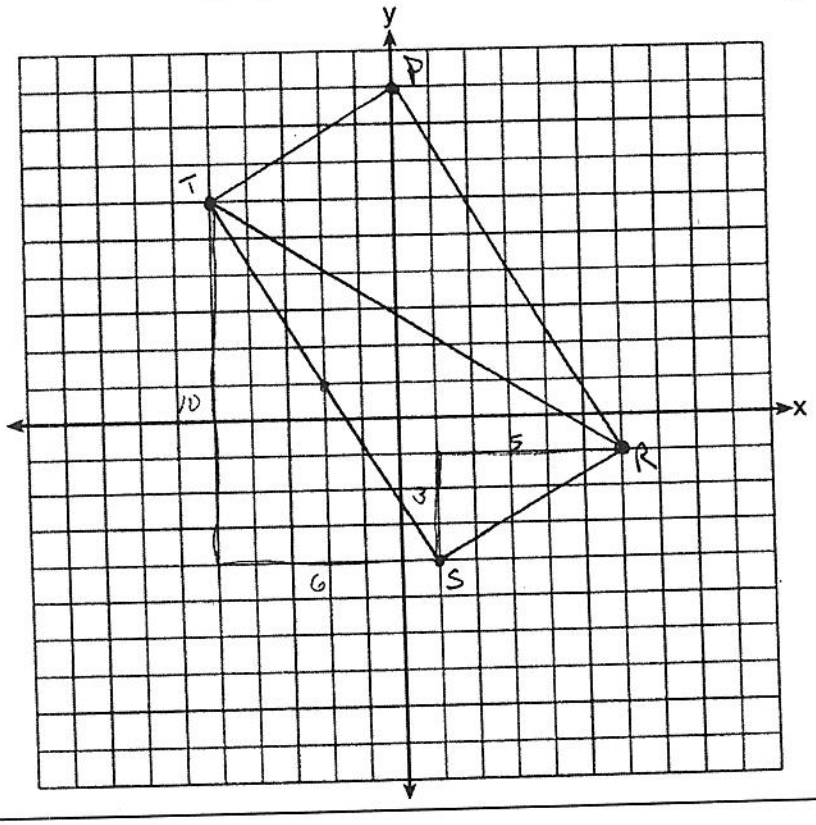
Question 36 continued

Prove that your quadrilateral *RSTP* is a rectangle.
[The use of the set of axes below is optional.]

$m_{\overline{RS}} = \frac{3}{5}$
$m_{\overline{PT}} = \frac{3}{5}$
$\therefore \overline{RS} \parallel \overline{PT}$

$m_{\overline{ST}} = \frac{-10}{6} = \frac{-5}{3}$
$m_{\overline{RP}} = \frac{-10}{6} = \frac{-5}{3}$
$\therefore \overline{ST} \parallel \overline{RP}$

Since RSTP is a quadrilateral with both pairs of opposite sides $\parallel$ and one rt $\angle$ at S, it must be a rectangle.



Score 6: The student has a complete and correct response.