

Chi-square Test for Independence

A random sample of 400 residents of large western city are polled to determine their attitudes concerning the affirmative action admissions policy of the local university. The residents are classified according to ethnicity (white, black, Asian) and whether or not they favor the affirmative action policy. The results are presented in the following table.

| | | Favor | Do Not Favor | Total |
|-----------|-------|-------|--------------|-------|
| Ethnicity | White | 130 | 120 | 250 |
| | Black | 75 | 35 | 110 |
| | Asian | 28 | 12 | 40 |
| | Total | 233 | 167 | 400 |

The χ^2 -test for independence can be summarized as follows.

| HYPOTHESES | CONDITIONS | TEST STATISTIC |
|---|---|--|
| <ul style="list-style-type: none"> Null hypothesis: H_0: The row and column variables are independent (or: they are not related). Alternative hypothesis: H_1: The row and column variables are not independent (or: they are related). | <ul style="list-style-type: none"> Observations are based on a random sample. The number of each expected count is at least 5. <p>(Some texts use the following condition: all expected counts are greater than 1, and at least 80% of the expected counts are greater than 5.)</p> | $X^2 = \sum \frac{(O - E)^2}{E}$ $df = (r-1)(c-1)$ |

example: A study of 150 cities was conducted to determine if crime rate is related to outdoor temperature. The results of the study are summarized in the following table:

Crime Rate

| | BELOW | NORMAL | ABOVE |
|--------------------|--------------|---------------|--------------|
| Temperature | | | |
| Below | 12 | 8 | 5 |
| Normal | 35 | 41 | 24 |
| Above | 4 | 7 | 14 |

Do these data provide evidence, at the 0.02 level of significance, that the crime rate is related to the temperature at the time of the crime?

example: A university dean suspects that there is a difference between how tenured and nontenured professors view a proposed salary increase. She randomly selects 20 nontenured instructors and 25 tenured staff to see if there is a difference. She gets the following results.

| | FAVOR PLAN | DO NOT FAVOR PLAN |
|------------|------------|-------------------|
| Tenured | 15 | 10 |
| Nontenured | 8 | 12 |

Do these data provide good statistical evidence that tenured and nontenured faculty differ in their attitudes toward the proposed salary increase?