Key

Finding Different Levels of Confidence for Sample Means

Opening Exercise:

1) How many standard deviations away from the mean would produce:

a) 68% confidence level

$\pm 1\sigma$

b) 95% confidence level

$\pm 2\sigma$

c) 99.7% confidence level

$\pm 3\sigma$

t-table

d) 98% confidence level

$\pm 2.326\,\sigma$

e) 99% confidence level

$\pm 2.576\,\sigma$

How many standard deviations away from the mean to produce a certain level of confidence is called the **Critical Value** or **z*.**

Now we will construct different confidence intervals:

4 Steps for conducting a legitimate confidence interval study:

1) Name the population and parameter of interest in the study.
2) Name the confidence level you are finding and the conditions needed for the calculations. i.e. CLT, np >10, n(1-p) > 10, population is greater than 10n. SRS
3) Show the calculation
4) Interpret the results.

The calculation for confidence levels are as follows:

Sample means

$$\text{CI} = \bar{x} \pm z^* \cdot \underbrace{\frac{\sigma}{\sqrt{n}}}_{\sigma_{\bar{x}}}$$

Sample proportions

$$\text{CI} = \hat{p} \pm z^* \cdot \underbrace{\sqrt{\frac{p(1-p)}{n}}}_{\sigma_{\hat{p}}}$$

We will just focus on Sample means for this unit.

Examples:

10.5 **ANALYZING PHARMACEUTICALS** A manufacturer of pharmaceutical products analyzes a specimen from each batch of a product to verify the concentration of active ingredient. The chemical analysis is not perfectly precise. Repeated measurements on the same specimen give slightly different results. The results of repeated measurements follow a normal distribution quite closely. The analysis procedure has no bias, so the mean μ of all measurements is the true concentration in the specimen. The standard deviation of this distribution is known to be $\sigma = 0.0068$ grams per liter. The laboratory analyzes each specimen three times and reports the mean result. Three analyses of one specimen give concentrations {0.8403, 0.8363, 0.8447}.
Construct a 99% confidence interval for the true concentration μ

Steps

1) The population of the study are specimen from pharm comp. the parameter of the study is the average concentration of active ingredient.

2) We are going to produce a 99% C.I. to find true Average concentration of active ingredient. assuming the population is greater than 30 specimen we can use $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, and since the population is normal we can use a Normal Approx.

3) $C.I. = \bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} = .8404 \pm 2.576\left(\frac{.0068}{\sqrt{3}}\right) = (.8303, .8505)$

4) I am 99% confident the true mean concentration of active ing. is between (.8303, .8505)

10.6 **SURVEYING HOTEL MANAGERS** A study of the career paths of hotel general managers sent questionnaires to a SRS of 160 hotels belonging to major U.S. hotel chains. There were 114 responses. The average time these 114 general managers had spent with their current company was 11.78 years. Find a 99% confidence interval for the mean number of years general managers of major-hotel chains have spent with their current company. (Take it as known that the standard deviation of time with the company for all general managers is 3.2 years.)

1) population of the study are hotel general man. parameter of the study is the average time spent with current company.

2) I am going to do a 99% C.I to find the true average time spent at the current company by hotel managers. assuming there are more than 1140 hotel managers in the U.S. we can use $\sigma_{\bar{x}}$, Since $n > 30$ we can use a Normal Approx. for the Dist. of $\bar{x}$ by the CLT. SRS Not used, proceed with Caution.

3) $C.I = \bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} = 11.78 \pm 2.576 \left(\frac{3.2}{\sqrt{114}}\right)$

$(11.008, 12.552)$

4) I am 99% Confident that the true average time spent at current company for US hotel managers is between (11.008, 12.552)

10.7 **ENGINE CRANKSHAFTS** Here are measurements (in millimeters) of a critical dimension on a sample of auto engine crankshafts:

224.120	224.001	224.017	223.982	223.989	223.961	223.960	224.089
223.987	223.902	223.980	224.098	224.057	223.913	223.999	223.976

The data come from a production process that is known to have standard deviation $\sigma = 0.060$mm. The process mean is supposed to be $\mu = 224$ mm but can drift away from this target during production.

Produce a 95 % Confidence interval for the mean Crank Shaft measurement.

1) Pop= Engine Crank shafts

Parameter = Average dimension of Crank shafts.

2) 95% C.I

SRS - Assuming

Pop ~ N - Assuming

Normal -

3) $C.I = \bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$

(223.91, 223.97)

4) I am 95% Conf.