Key

Intro to Confidence Intervals

Up until this point in AP Stats we have learned to:

1) Collect data
2) Organize data
3) Make calculations with the data
4) Find different probabilities of certain events
5) Design experiments and observational studies
6) Understand the distribution of sample means and sample proportions

Now with all of these skills we will make *inferences* about the population from the sample that we collect.

Statistical Inference: Providing methods for drawing conclusions about a population from the sample.

i.e. A tire manufacturer claims that the average tread life of a certain tire is 70,000 miles with a standard deviation of 1,500 miles. I conducted a study to test that claim of 70,000 miles. I took a simple random sample of 50 cars to have these tires. After a few years I found the average tread life of 60,000 miles. Did I prove the claim wrong?

60,000 does seem considerably smaller than 70,000 but maybe the sample I chose happened to be smaller. after all, samples vary. We will learn to be more confident in our answer.

Two types of statistical inference we will study:

1) **Confidence Intervals** – giving a range of values where the true population parameter should lie with a certain level of confidence.

   i.e. From my sample of 50 cars I am 95% confident the true mean tread life is between (59 576, 60 424).

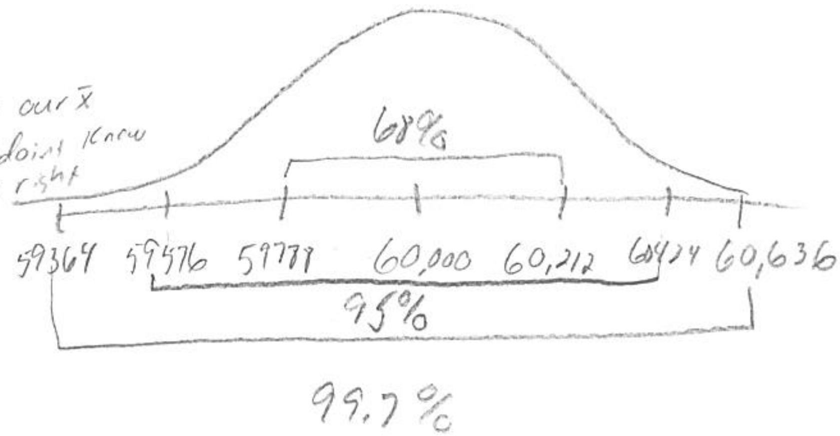
2) **Significance Testing** – Stating whether a claim is wrong based off of calculated probabilities.

   Since my sample average of 60,000 is 47 standard Deviations to the left of 70,000, I am very confident that 70,000 is an over estimate.

Examples:

1) A tire manufacturer claims that the average tread life of a certain tire is 70,000 miles with a standard deviation of 1,500 miles. I conducted a study to test that claim of 70,000 miles. I took a simple random sample of 50 cars to have these tires. After a few years I found the average tread life of 60,000 miles. Find certain levels of confidence to capture the true tread life of these tires using the empirical rule.

68 - 95 - 99.7



Sample means

we will use our $\bar{x}$ because we don't know if they are right

$\mu_{\bar{x}} = \mu \neq 60{,}000$

$\bar{x}$?

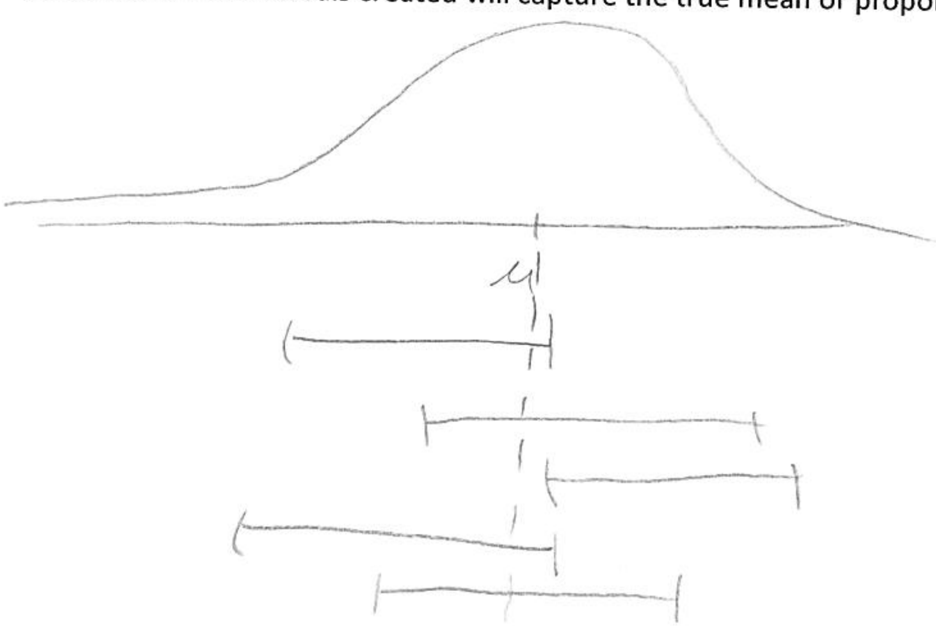
$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1500}{\sqrt{50}} = 212.$

I am 68% confident that the true mean tread life is between (59788, 60212)

" 95% " (59576, 60424)

" 99.7 " (59,364, 60,636)

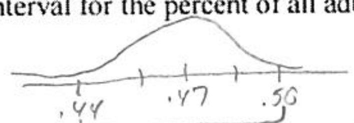
- **What is meant by "Confidence"?**
  If I am 95% confident, then it means that if I were to take infinite samples of size 50, then 95% of the intervals created will capture the true mean or proportion.



Practice Questions:

10.1 **POLLING WOMEN** A *New York Times* poll on women's issues interviewed 1025 women randomly selected from the US, excluding Alaska and Hawaii. The poll found that 47% of the women said they do not get enough time for themselves.

(a) The poll announced a margin of error of ± 3 percentage points for 95% confidence in its conclusions. What is the 95% confidence interval for the percent of all adult women who think they do not get enough time for themselves?



(b) Explain to someone who knows no statistics why we can't just say that 47% of all adult women do not get enough time for themselves.

Samples vary.

(c) Then explain clearly what "95% confidence" means.

10.2 **NAEP SCORES** Young people have a better chance of full-time employment and good wages if they are good with numbers. How strong are the quantitative skills of young Americans of working age? One source of data is the National Assessment of Educational Progress (NAEP) Young Adult Literacy Assessment Survey, which is based on a nationwide probability sample of households. The NAEP survey includes a short test of quantitative skills, covering mainly basic arithmetic and the ability to apply it to realistic problems. Scores on the test range from 0 to 500. For example, a person who scores 233 can add the amounts of two checks appearing on a bank deposit slip; someone scoring 325 can determine the price of a meal from a menu; a person scoring 375 can transform a price in cents per ounce into dollars per pound.

Suppose that you give the NAEP test to a SRS of 840 people from a large population in which the scores have mean 280 and standard deviation $\sigma = 60$. The mean $\bar{x}$ of the 840 scores will vary if you take repeated samples.

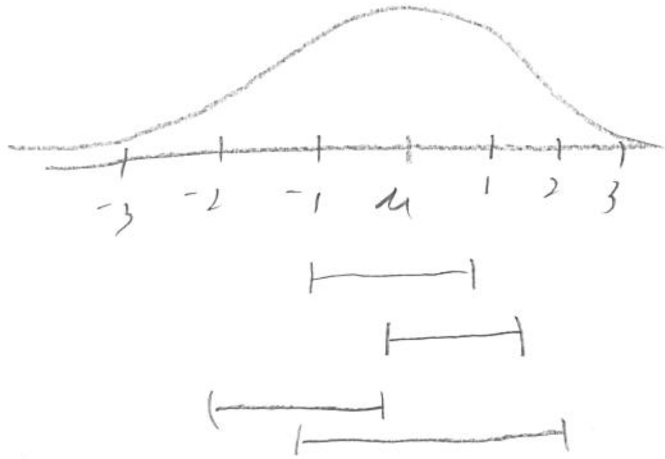
(a) Describe the shape, center, and spread of the sampling distribution of $\bar{x}$. What guarantees this?

1) CLT $n \geq 30$ $\bar{x}$ is approx Normal

2) $\mu_{\bar{x}} = 280$

3) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{60}{\sqrt{840}} \approx 2.1$

(b) Sketch the normal curve that describes how $\bar{x}$ varies in many samples from this population. Mark its mean and the values 1, 2, and 3 standard deviations on either side of the mean.



(c) According to the 68-95-99.7% rule, about 95% of all the values of $\bar{x}$ fall within $\pm 2\sigma = 4.2$ of the mean of the curve. What is the missing number? Call it m for "margin of error". Sketch the region from the mean minus m to the mean plus m on the axis of your sketch.

(d) Whenever $\bar{x}$ falls in the region you shaded, the true value of the population mean, $\mu = 280$, lies in the confidence interval between $\bar{x} - m$ and $\bar{x} + m$. Draw the confidence interval below your sketch for one value of $\bar{x}$ inside the shaded region and one value of $\bar{x}$ outside the shaded region.

(e) In what percent of all samples will the true mean $\mu = 280$ be covered the confidence interval $\bar{x} \pm m$?