Name Key

AP Statistics - Mr. Bannon

Date ____________

Period ____________

Unit 8 Review

Part I. Multiple Choice

1) Approximately 3 in 10,000 people will develop a certain skin problem each year. In a group of 1000 people, what is the probability that at least 2 people will develop the skin problem next year?

A. 0.033
B. 0.037
C. 0.259
D. 0.741
E. 0.963

$p = \frac{3}{10,000}$ $n = 1000$ $r = 2, 3, 4 \ldots$

$1 -$ Binomcdf$(1000, \frac{3}{10000}, 1)$

2) A 12-sided die has faces numbered from 1–12. Assuming the die is fair (that is, each face is equally likely to appear each time), which of the following would give the exact probability of getting at least 10 3s out of 50 rolls?

$n = 50$

$p = \frac{1}{12}$ $r = 10, 11, 12$

Binomial Formula

$\binom{n}{r} p^r (1-p)^{n-r}$

(a) $\binom{50}{0}(0.083)^0(0.917)^{50} + \binom{50}{1}(0.083)^1(0.917)^{49} + \ldots + \binom{50}{9}(0.083)^9(0.917)^{41}$.

(b) $\binom{50}{11}(0.083)^{11}(0.917)^{39} + \binom{50}{12}(0.083)^{12}(0.917)^{38} + \ldots + \binom{50}{50}(0.083)^{50}(0.917)^0$.

(c) $1-\left[\binom{50}{0}(0.083)^0(0.917)^{50} + \binom{50}{1}(0.083)^1(0.917)^{49} + \ldots + \binom{50}{10}(0.083)^{10}(0.917)^{40}\right]$.

(d) $1-\left[\binom{50}{0}(0.083)^0(0.917)^{50} + \binom{50}{1}(0.083)^1(0.917)^{49} + \ldots + \binom{50}{9}(0.083)^9(0.917)^{41}\right]$.

(e) $\binom{50}{0}(0.083)^0(0.917)^{50} + \binom{50}{1}(0.083)^1(0.917)^{49} + \ldots + \binom{50}{10}(0.083)^{10}(0.917)^{40}$.

3) Twenty percent of all trucks undergoing a certain inspection will fail the inspection. Assume that trucks are independently undergoing this inspection, one at a time. The expected number of trucks inspected before a truck fails inspection is

(a) 2
(b) 4
(c) 5
(d) 20
(e) The answer cannot be computed from the information given.

Geometric Average

$\mu_{Geo} = \frac{1}{p}$ $\frac{1}{.2}$

$= 5$

Part II - Free Response

4) Which of the following is not a common characteristic of binomial and geometric experiments?

(a) There are exactly two possible outcomes: success or failure.
(b) There is a random variable X that counts the number of successes. Bin
(c) Each trial is independent (knowledge about what has happened on previous trials gives you no information about the current trial).
(d) The probability of success stays the same from trial to trial.
(e) P(success) + P(failure) = 1.

1) A brake inspection station reports that 15% of all cars tested have brakes in need of replacement pads. For a sample of 20 cars that come to the inspection station,

(a) what is the probability that exactly 3 have defective breaks?
(b) what is the mean and standard deviation of the number of cars that need replacement pads?

a) binompdf (20, .15, 3) = $\boxed{.2428}$

b) $\mu_{bin} = n \cdot p = 20 \cdot .15 = \boxed{3}$

$\sigma_{bin} = \sqrt{n \cdot p \cdot q} = \boxed{1.5969}$

2) The probability of winning a bet on red in roulette is 0.474. The binomial probability of winning money if you play 10 games is 0.31 and drops to 0.27 if you play 100 games. Use a normal approximation to the binomial to estimate your probability of coming out ahead (that is, winning more than $^1/_2$ of your bets) if you play 1000 times. Justify being able to use a normal approximation for this situation.

$n \cdot p \geq 10$ $1000(.474) = 474$

$n \cdot q \geq 10$ $1000(.526) = 526$

✓

$p = .474$ $n = 1000$ ~~binompdf(10~~ $P(x > 500)$

$\mu_{bin} = 1000(.474) = 474$ $\sigma_{bin} = \sqrt{1000(.474)(.526)}$

$= 15.8$

$z_{500} = \frac{500 - 474}{15.8} = \boxed{1.65}$ $1 - (\ \ ) = .05$

3) The coin of problem #1 is flipped 50 times. Let X be the number of heads. What is

(a) the probability of *exactly* 20 heads?
(b) the probability of *at least* 20 heads?

a) binompdf(50, .5, 20) = $\boxed{.04}$

b) 1-binomcdf(50, .5, 19) = $\boxed{.94}$

4) The probability that a person recovers from a particular type of cancer operation is 0.7. Suppose 8 people have the operation. What is the probability that

(a) exactly 5 recover?
(b) they all recover?
(c) at least one of them recovers?

a) binompdf(8, .7, 5) = .25

b) binompdf(8, .7, 8) = .0576

c) 1-binomcdf(8, .7, 0) = 1 - .00006 .99994

5) At a school better known for football than academics (a school its football team can be proud of), it is known that only 20% of the scholarship athletes graduate within 5 years. The school is able to give 55 scholarships for football. What are the expected mean and standard deviation of the number of graduates for a group of 55 scholarship athletes?

$n = 55$ $p = .20$ binomial

$\mu_{bin} = 55(.20) = \boxed{11}$

$\sigma_{bin} = \sqrt{55(.2)(.8)} = \boxed{2.966}$

$P = .95$

6) An airline, believing that 5% of passengers fail to show up for flights, overbooks (sells more tickets than there are seats). Suppose a plane will hold 265 passengers, and the airline sells 275 seats. What's the probability the airline will not have enough seats so someone gets bumped?

.195

$\mu = n \cdot p$ $\quad \sigma = \sqrt{n \cdot p \cdot q}$

$\mu = 275(.95$

$\mu = 261.25$ $\quad \sigma = \sqrt{\quad} = 13.0619$

3.614

$$Z = \frac{265 - 261.25}{3.614} = 1.037$$

$P(X > 265) = 14.92\%$

7) An orchard owner knows that he'll have to use about 6% of the apples he harvests for cider because they will have bruises or blemishes. He expects a tree to produce about 300 apples. n

a. Is this problem binomial or geometric? Why?

binomial, 1) n 2) 2 outcomes 3) Ind. 4) prob =

a. Find the probability there will be no more than a dozen cider apples?

0,1,...12 $\quad$ binomcdf(300, .06, 12) = .0849686059

b. Is it likely there will be more than 50 cider apples? Explain.

1 - binomcdf(300, .06, 50) = really small

8) **POLLING** *Many local polls of public opinion use samples of size 400 to 800. Consider a poll of 400 adults in Richmond that asks the question "Do you approve of President George W. Bush's response to the World Trade Center terrorists attacks in September 2001?"* Suppose we know that President Bush's approval rating on this issue nationally is 92% a week after the incident.

(a) What is the random variable X? Is X binomial? Explain.
X = the number of people in the sample of 400 adult Richmonders who approve of the President's reaction.

1) Fixed n
2) 2 outcomes
3) Ind.
4) Prob stays

(b) Calculate the binomial probability that at most 358 of the 400 adults in the Richmond poll answer "Yes" to this question.

binomcdf(400, .92, 358) = $\boxed{.044102}$

c) J

(c) Find the expected number of people in the sample who indicate approval. Find the standard deviation of X.

$\mu = n \cdot p$
$= 368$

$\sigma = \sqrt{n \cdot p \cdot (1-p)}$
$= 5.425864977$

(d) Perform a normal approximation to answer the question in (b), and compare the results of the binomial calculation and the normal approximation. Is the normal approximation satisfactory?

$n \cdot p > 10$ $n(1-p) > 10$

$368 > 10$ ✓ $32 > 10$ ✓

We can use normal Distr.

$z = \frac{358 - 368}{5.425}$

$= -1.84$

$P(X < 358) = \boxed{.0329}$