More Probability Rules

- **Complement:** The probability of the complement of an event A is given by

$$P(A') = 1 - P(A)$$

- **Union** (addition rule): The probability of the union of two events A and B is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If the events A and B are disjoint, then $P(A \cap B) = 0$, and

$$P(A \cup B) = P(A) + P(B)$$

- **Intersection** (multiplication rule): For events A and B defined in a sample space S

$$P(A \cap B) = P(A) \circ P(B \mid A) = P(B) \circ P(A \mid B)$$

- **Conditional probabilities:** The probability of A given B is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

- **Independence:** Two events A and B are independent if and only if

$$P(A \mid B) = P(A) \text{ and } P(B \mid A) = P(B)$$

In other words, two events A and B are independent if and only if

$$P(A \cap B) = P(A) \circ P(B)$$

Example 3: Imagine that you shuffle a standard deck of 52 cards and draw a card at random. Let

D = diamond	C = club		
H = heart	S = spade		
J = jack	Q = queen	K = king	1 = ace

Then

$$S = \{D1, \ldots, D10, DJ, DQ, DK, C1, \ldots, C10, CJ, CQ, CK, H1, \ldots, H10, HJ, HQ, HK, S1, \ldots, S10, SJ, SQ, SK\}$$

Suppose we define the following events:

A = Getting an ace = $\{D1, C1, H1, S1\}$

B = Getting a diamond = $\{D1, \ldots, D10, DJ, DQ, DK\}$

C = Getting a club = $\{C1, C2, \ldots, CJ, CQ, CK\}$

Then

- $P(A) =$ ___ $P(B) =$ ___ $P(C) =$ ___
- A' = Getting a non-ace card

  $P(A') =$ ______
- B' = Getting a non-diamond card

  $P(B') =$ ______
- $(A \cap B)$ = Getting an ace of diamonds = $\{D1\}$

  $P(A \cap B) =$ ___

  Events A and B are not disjoint, because an ace of diamonds ($D1$) is a common outcome for both the events.
- $(B \cap C)$ = Getting a card that is a club and a diamond = $\{\ \}$

  $P(B \cap C) =$ ___

  The events B and C are disjoint, because no outcome is common to them. Each card in a deck belongs to only one suit.

- $(A \cup B)$ = Getting an ace or a diamond

$$\{D1,\ldots, D10, DJ, DQ, DK, C1, H1, S1\}$$

$$P(A \cup B) =$$

Alternatively,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$=$$

$$=$$

- $P(B \cup C)$ = Getting a diamond or a club

$$= \{D1,\ldots, D10, DJ, DQ, DK, C1,\ldots, C10, CJ, CQ, CK\}$$

$$P(B \cup C) =$$

Alternatively, because B and C are disjoint, $P(B \cap C) = 0$. Therefore,

$$P(B \cup C) = P(B) + P(C)$$

$$=$$

$$=$$

- $(A \mid B)$ = Getting an ace given that a diamond has been drawn = $\{D1\}$

$$P(A \mid B) =$$

Alternatively,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} =$$

Note that $P(A \mid B) = P(A)$. Therefore, events A and B are independent.

Example 4: Seventy-five percent of people who purchase hair dryers are female. Of these female purchasers of hair dryers, 30 percent are over 50 years old. What is the probability that a randomly selected hair dryer purchaser is a female over 50 years old?

Solution: Let us define the events as follows:

W = The purchaser of a hair dryer is a female.
F = The purchaser of a hair dryer is over 50 years old.

It is known that $P(W) =$ ___ and $P(F \mid W) =$ ___.

Thus,

$$P(W \cap F) = P(F \mid W) \circ P(W) = \underline{\hspace{3cm}}$$

There is a ___ percent chance that a randomly selected hair dryer purchaser is a female over 50 years old.

Example 5: Company I has 24 total employees that are classified as associates, partners, managers or entrepreneurs. There are 10 associates, six partners, five managers, and three entrepreneurs. There are three levels of experience within each of these classifications: entry, junior, or senior (i.e., an employee can be a junior associate, a senior manager, etc.). The percentage of Company I employees that are entry, junior, and senior level is 50.0 percent, 37.5 percent, and 12.5 percent, respectively. An employee from Company II needs to arrange a conference meeting with four employees from Company I.

(a) Find the probability that two of the people she meets with are senior managers.

(b) Find the probability that one is a junior partner, and one is an entry entrepreneur.

(c) Find the probability that one is a senior associate, and two are entry-level associates.

Example 6: An insurance agent knows that 70 percent of her customers carry adequate collision coverage. She also knows that of those who carry adequate coverage, 5 percent have been involved in accidents, and of those who do not carry adequate coverage, 12 percent have been involved in accidents. If one of her clients is involved in an auto accident, then what is the probability that the client does not have adequate insurance coverage?

Example 7: The local Chamber of Commerce conducted a survey of 1,000 randomly selected shoppers at a mall. For all shoppers, gender of shopper and items shopping for was recorded. The data collected is summarized in the following table:

	Shopping For			
Gender	**Clothing**	**Shoes**	**Other**	**Total**
Male	75	25	150	250
Female	350	230	170	750
Total	425	255	320	1,000

If a shopper is selected at random from this mall,

(a) What is the probability that the shopper is a female?

(b) What is the probability that the shopper is shopping for shoes?

(c) What is the probability that the shopper is a female shopping for shoes?

(d) What is the probability that the shopper is shopping for shoes given that the shopper is a female?

(e) Are the events "female" and "shopping for shoes" disjoint?

(f) Are the events "female" and "shopping for shoes" independent?