Basic Probability Rules and Terms

There are two rules that all the probabilities must satisfy:

- **Rule 1:** For any event A, the probability of A is always greater than or equal to 0 and less than or equal to 1.

$$0 \leq P(A) \leq 1$$

- **Rule 2:** The sum of the probabilities for all possible outcomes in a sample space is always 1.

As a result, we can say the following:

- If an event can never occur, its probability is 0. Such an event is known as an **impossible event.**
- If an event must occur every time, its probability is 1. Such an event is known as a **sure event.**

The **odds in favor of an event** is a ratio of the probability of the occurrence of an event to the probability of the nonoccurrence of that event.

$$\text{Odds in favor of an event} = \frac{P(\text{Event } A \text{ occurs})}{P(\text{Event } A \text{ does not occur})}$$

or

$$P(\text{Event } A \text{ occurs}) : P(\text{Event } A \text{ does not occur})$$

Example 1: When tossing a die, what are the odds in favor of getting the number 2?

Solution: When tossing a die,

$P(\text{Getting the number 2}) = \frac{1}{6}$ and

$P(\text{Not getting the number 2}) = P(\text{Getting the numbers 1, 3, 4, 5, or 6}) = \frac{5}{6}$.

Thus, the odds in favor of getting the number 2 are $\frac{1}{6} : \frac{5}{6}$ or 1 to 5 (or 1:5).

More Terms

The Venn diagrams shown in Figures 3–6 illustrate some of the following terms. The rectangular box indicates the A' sample space. Circles indicate different events.

The **complement** of an event is the set of all possible outcomes in a sample space that does not lead to the event. The complement of an event A is denoted by A' (or A^C). See Figure 3.

Disjoint or **mutually exclusive events** are events that have no outcome in common. In other words, they cannot occur together. See Figure 4.

The **union** of events A and B is the set of all possible outcomes that lead to at least one of two events A and B. The union of events A and B is denoted by $(A \cup B)$ or (A or B). See Figure 5.

The **intersection** of events A and B is the set of all possible outcomes that lead to *both* events A and B. The intersection of events A and B is denoted by $(A \cap B)$ or (A and B). See Figure 6.

A **conditional event:** A given B is a set of outcomes for event A that occurs if B has occurred. It is indicated by $(A \mid B)$ and reads "A given B."

Two events A and B are considered **independent** if the occurrence of one event does not depend on the occurrence of the other.

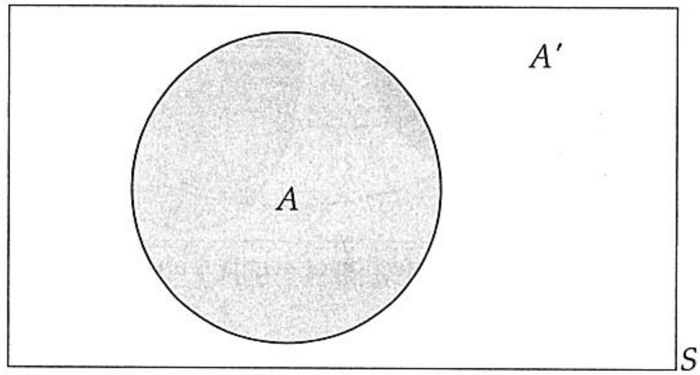

Figure 3: Event A and its complement

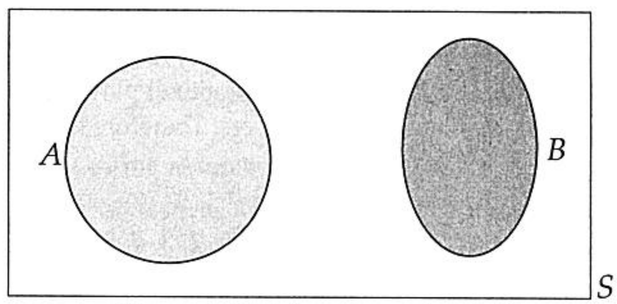

Figure 4: Disjoint events A and B

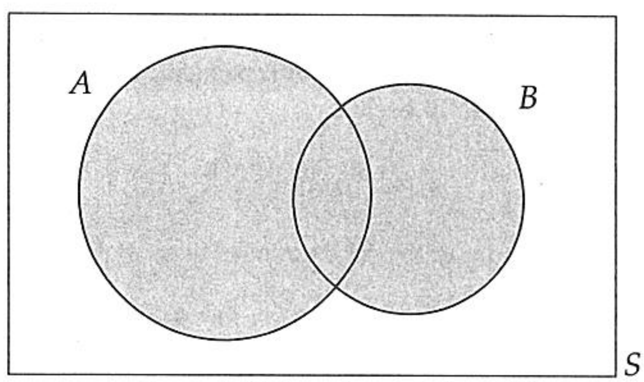

Figure 5: Union of events A and B

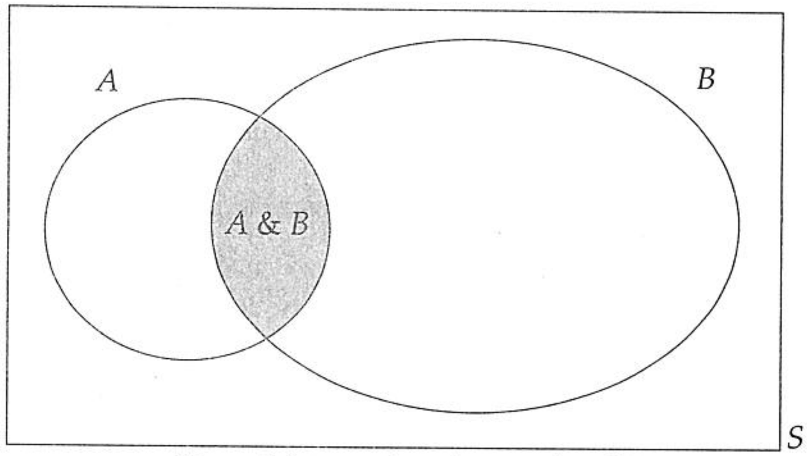

Figure 6: Intersection of events A and B

Independence versus Dependence

Imagine that you shuffle a standard deck of cards and then draw a card at random. The chance of your getting an ace is the same across all four suits (hearts, clubs, diamonds, and spades). In other words, the likelihood of your getting an ace does not depend on the suit of the card. So we can say that the events "getting an ace" and "getting a particular suit" are *independent*.

Now consider a doctor examining patients in an emergency room. The likelihood of a patient being diagnosed for a knee injury is higher if that patient is a football player, because football players are more likely to suffer knee injuries than nonfootball players. Therefore the event "knee injury" *depends* on the event "football player."

Example 2: The sample space for throwing a die is $S = \{1, 2, 3, 4, 5, 6\}$. Suppose events A, B, and C are defined as follows:

A = Getting an even number = $\{2, 4, 6\}$
B = Getting at least 5 = $\{5, 6\}$
C = Getting at most 3 = $\{1, 2, 3\}$

Find the probability of each of these events and its complement. Then, find the union, intersection and conditional probability of each pair of events.