Circuit Simplification: DeMorgan's Theorems

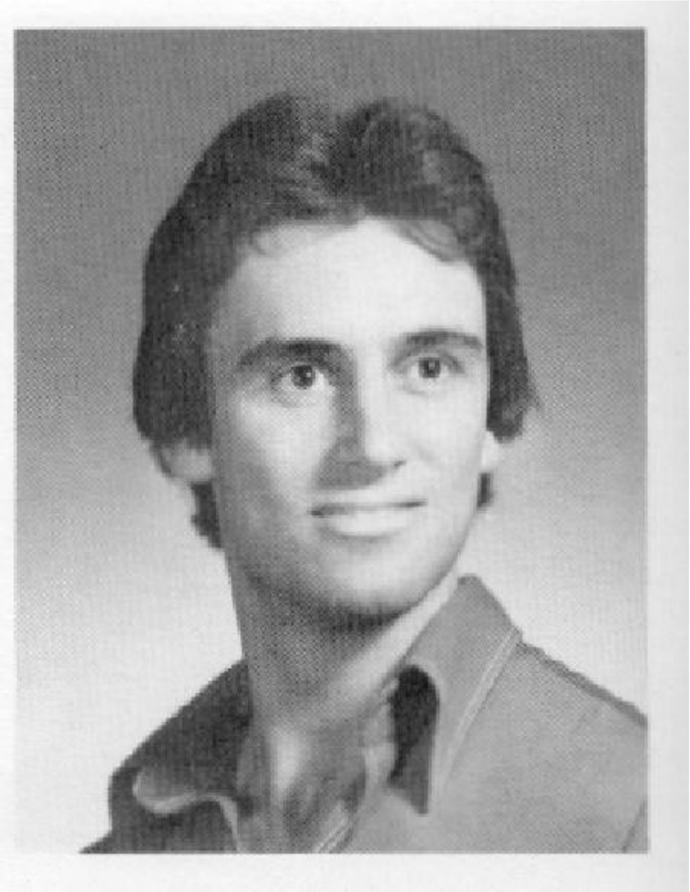
DeMorgan’s Theorems

DeMorgan’s Theorems are **two** additional **simplification** techniques that can be used to simplify Boolean expressions. Again, the simpler the Boolean expression, the **simpler** the resulting logic.

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

Augustus DeMorgan



Augustus DeMorgan, an Englishman, born in India in 1806. He was instrumental in the advancement of mathematics and is best known for the **logic theorems** that bear his name.

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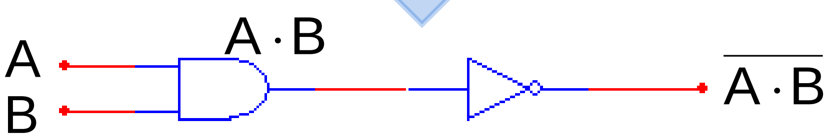
DeMorgan's Theorem #1: $\overline{A \cdot B} = \overline{A} + \overline{B}$

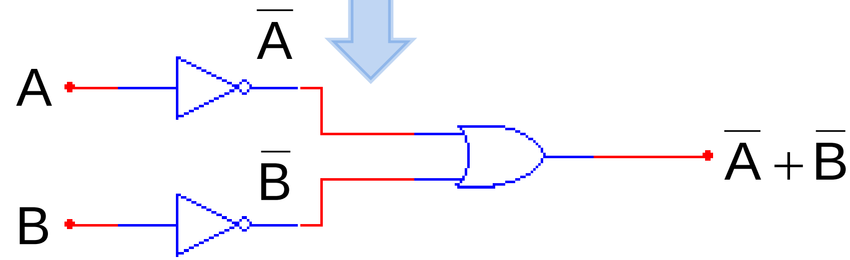
Proof

$$\overline{A \cdot B}$$



A	B	$A \cdot B$	$\overline{A \cdot B}$
0	0	0	
0	1	0	
1	0	0	
1	1	1	

$$\overline{A} + \overline{B}$$



A	B	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$
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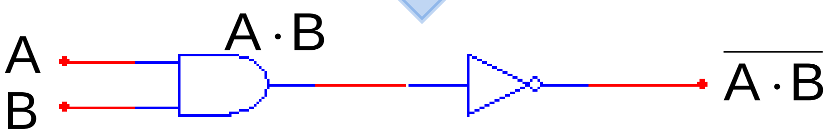

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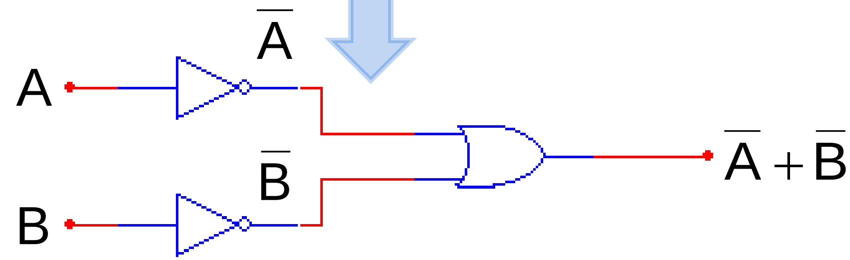

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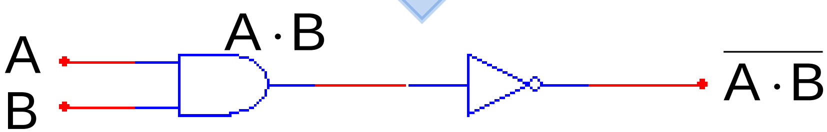

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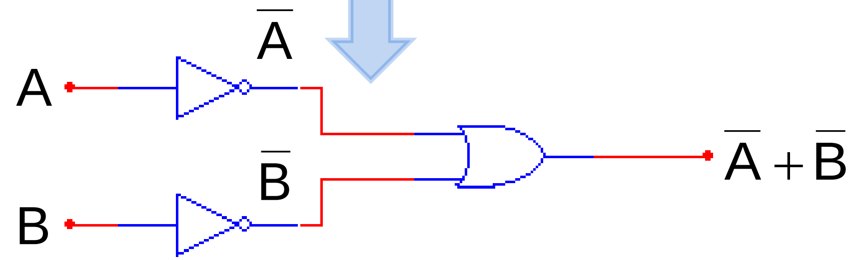

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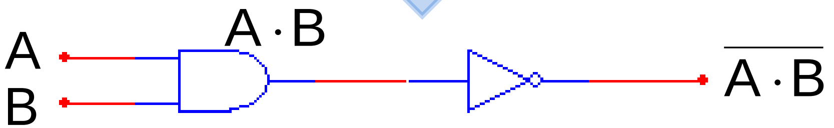

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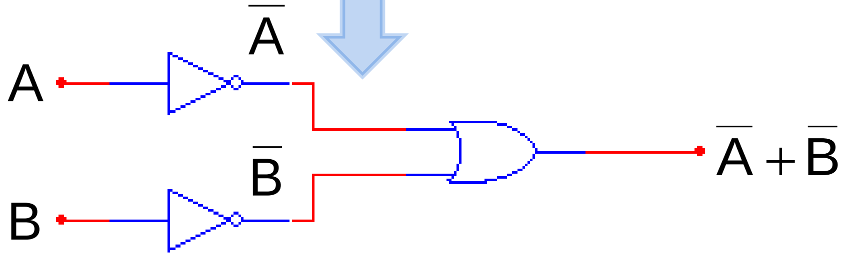

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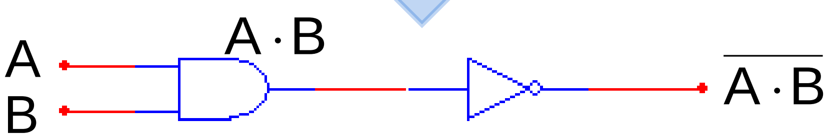

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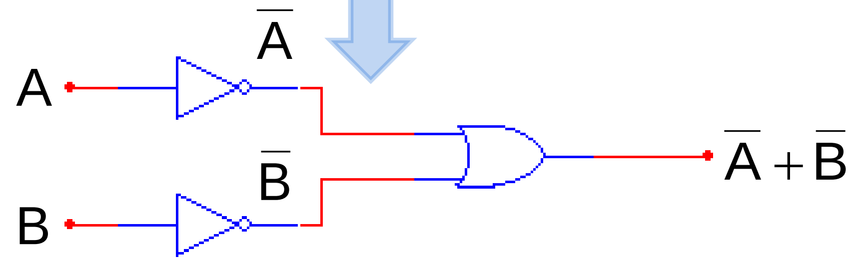

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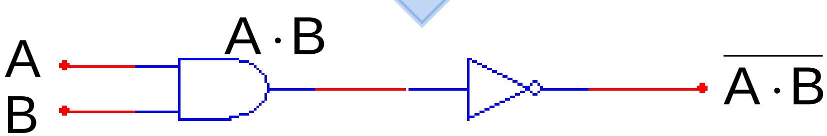

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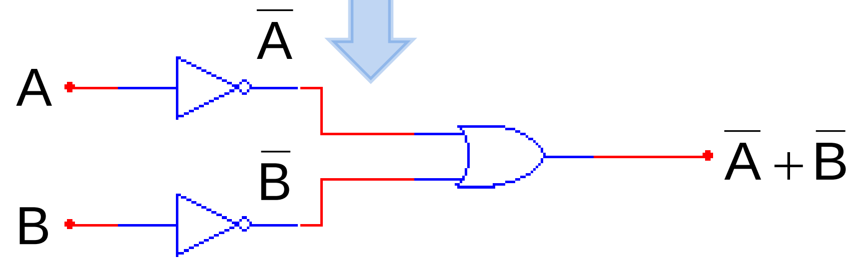

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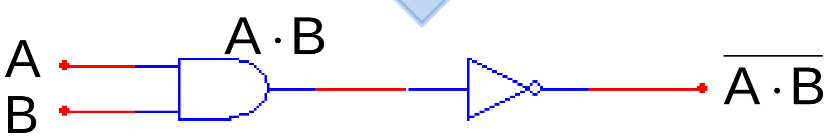

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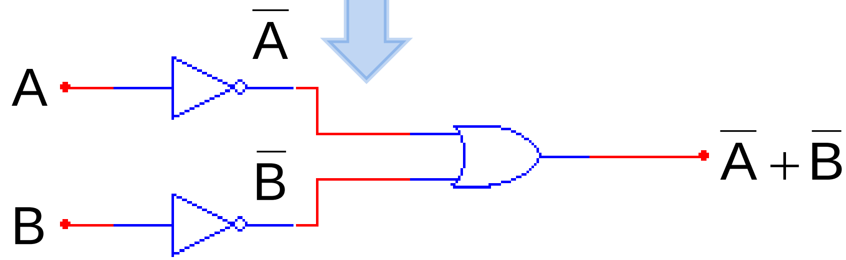

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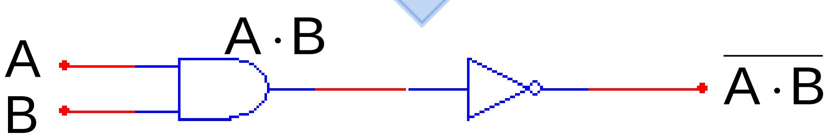

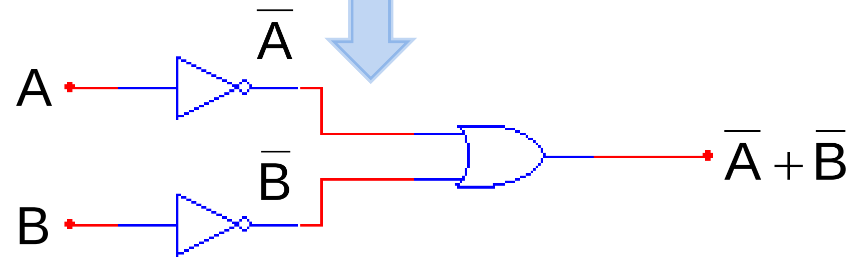

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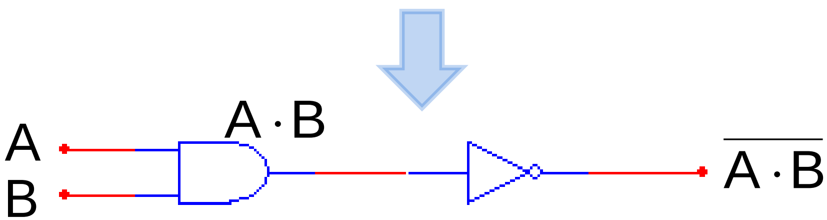

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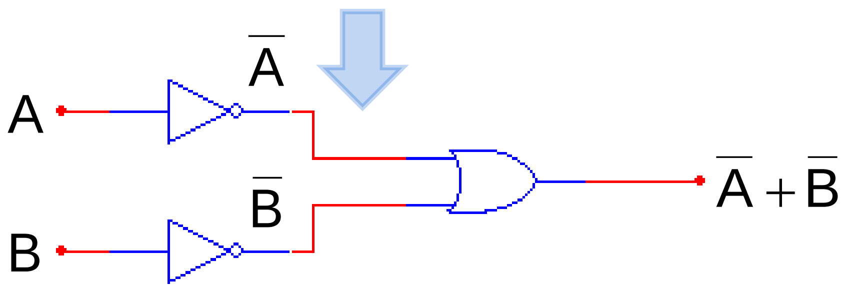

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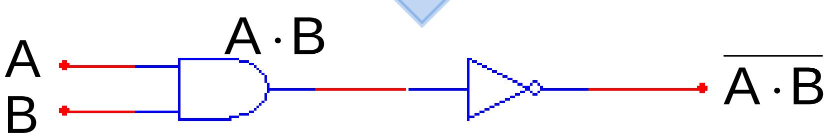

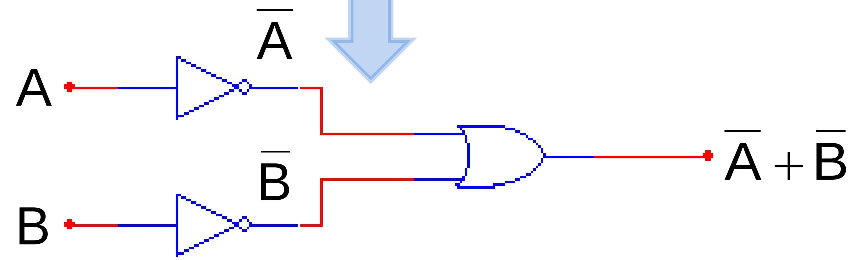

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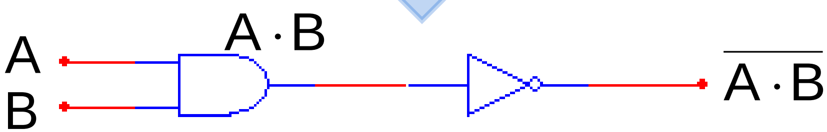

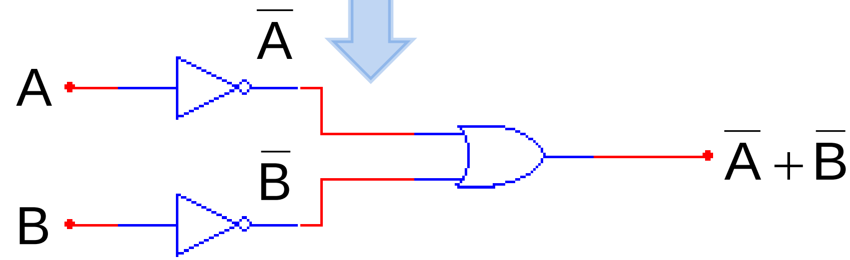

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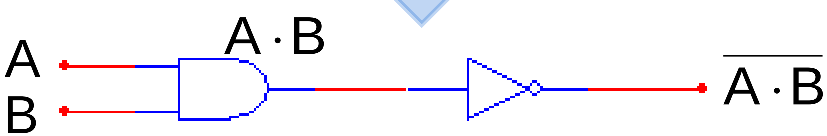

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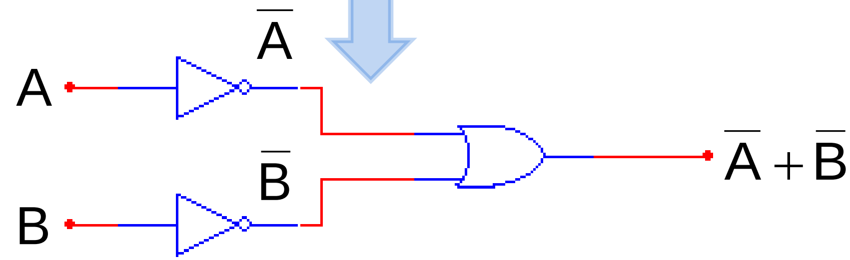

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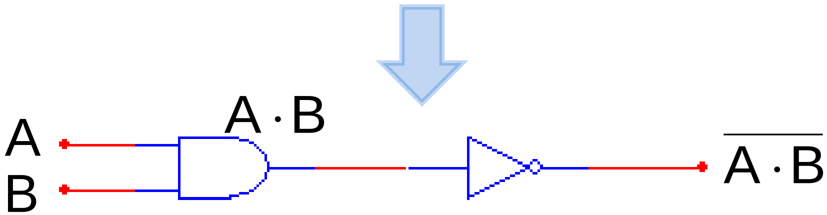

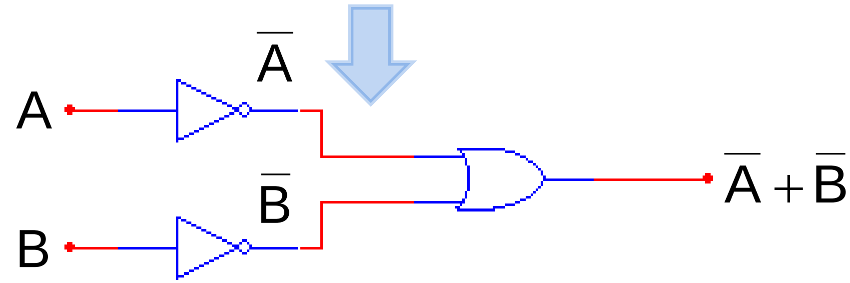

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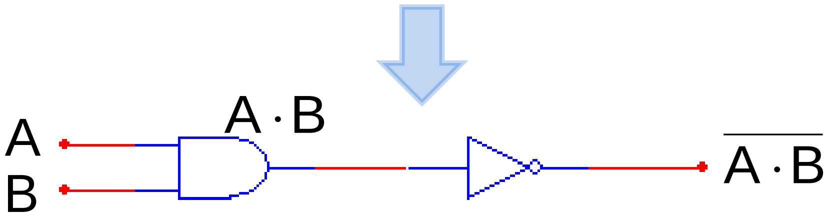

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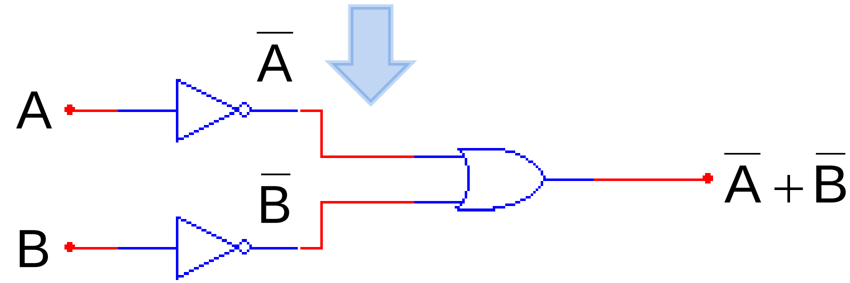

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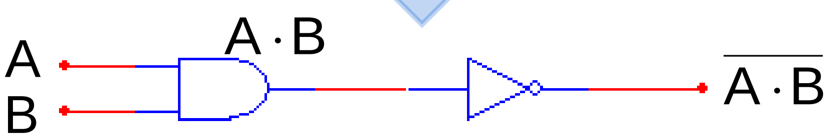

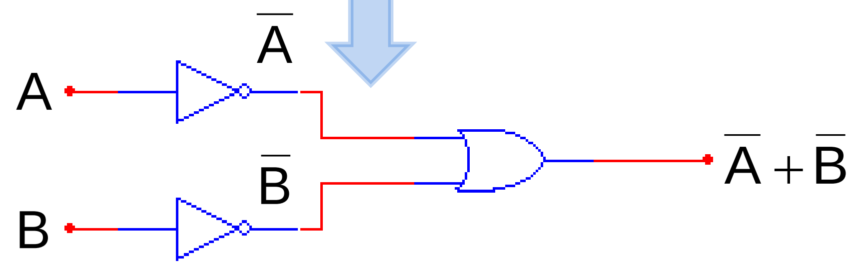

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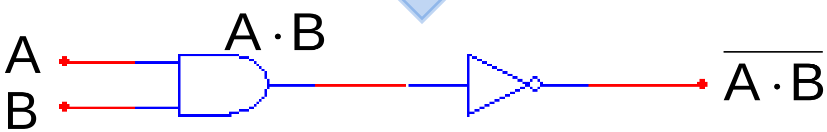

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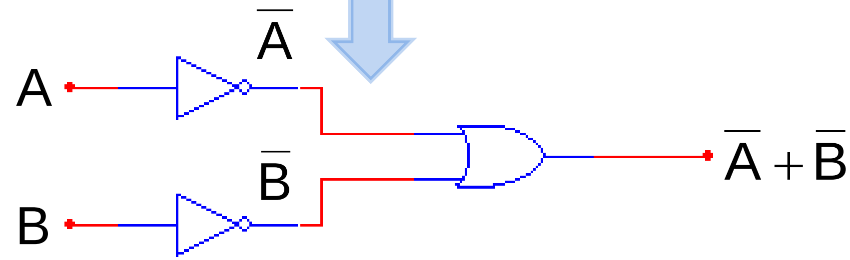

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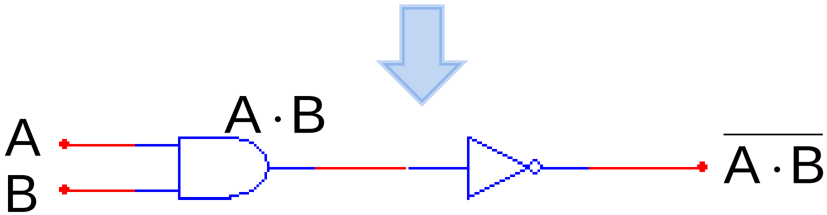

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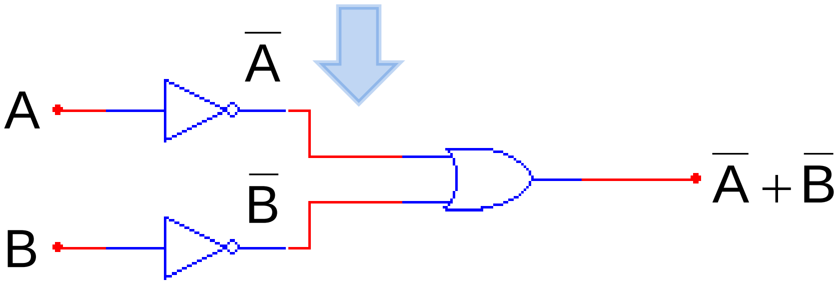

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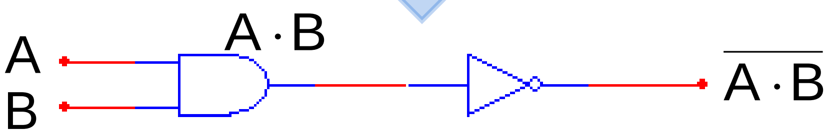

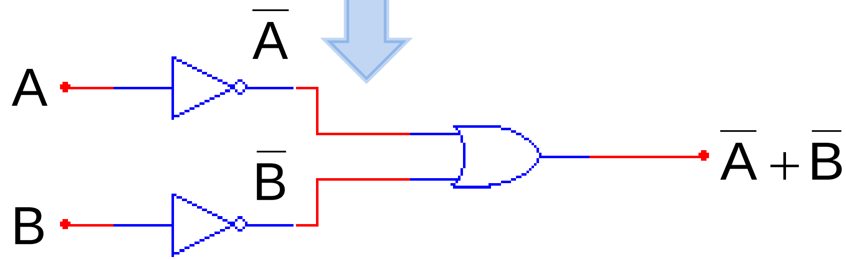

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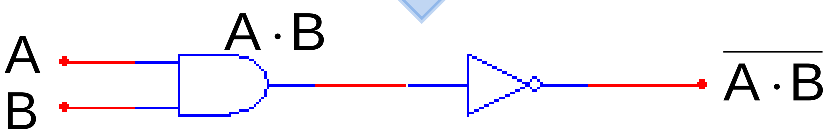
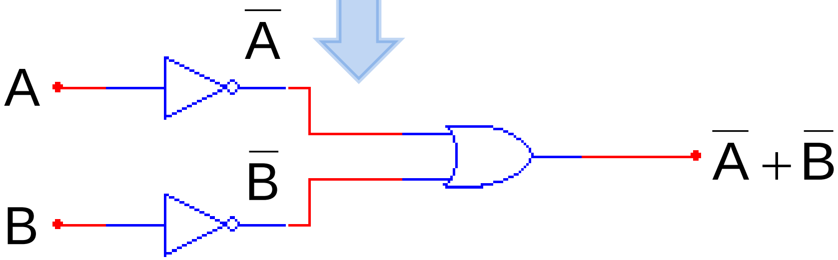


$\overline{A} + \overline{B}$



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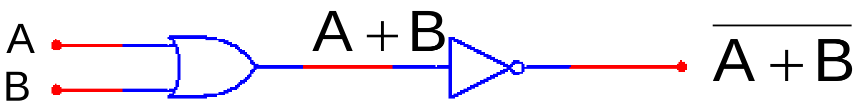
The truth-tables are **equal**; therefore, the Boolean equations must be **equal**!

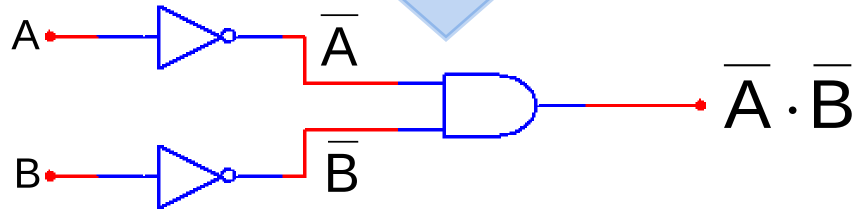
DeMorgan's Theorem #2 $\overline{A+B} = \overline{A} \cdot \overline{B}$

Proof

$\overline{A+B}$



$\overline{A} \cdot \overline{B}$



A	B	$A+B$	$\overline{A+B}$
0	0	0	
0	1	1	
1	0	1	
1	1	1	

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
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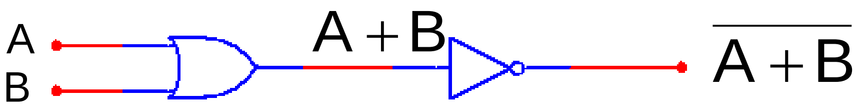
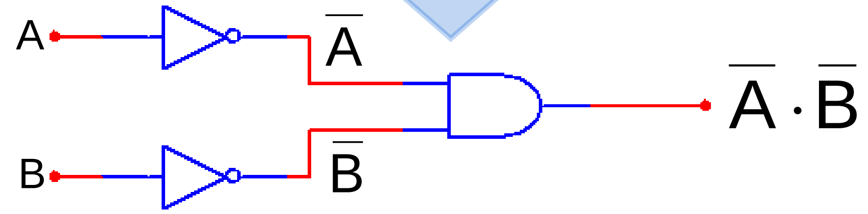
Proof

$\overline{A+B}$



A	B	$A+B$	$\overline{A+B}$
0	0	0	
0	1	1	
1	0	1	
1	1	1	

$\overline{A} \cdot \overline{B}$

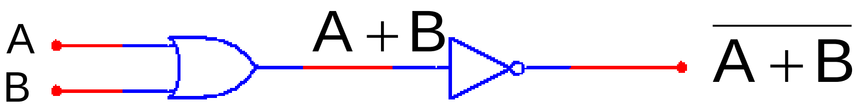


A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	
0	1	1	0	
1	0	0	1	
1	1	0	0	

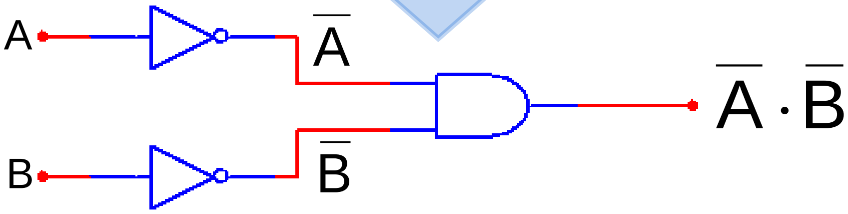
DeMorgan's Theorem #2 $\overline{A+B} = \overline{A}\cdot\overline{B}$

Proof

$\overline{A+B}$



$\overline{A}\cdot\overline{B}$



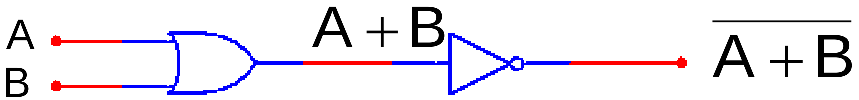
A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	
1	0	1	
1	1	1	

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$
0	0	1	1	
0	1	1	0	
1	0	0	1	
1	1	0	0	

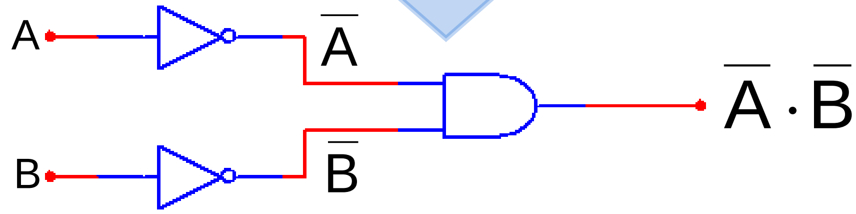
DeMorgan's Theorem #2 $\overline{A+B} = \overline{A}\cdot\overline{B}$

Proof

$\overline{A+B}$



$\overline{A}\cdot\overline{B}$



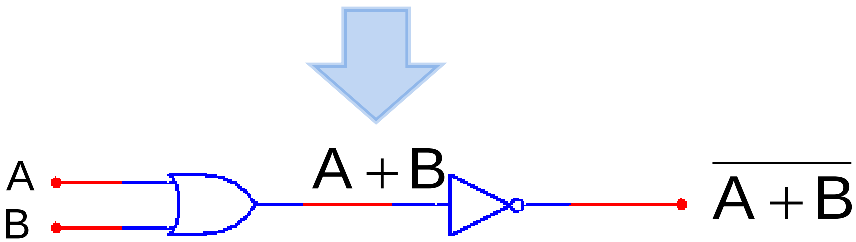
A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	
1	0	1	
1	1	1	

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$
0	0	1	1	
0	1	1	0	
1	0	0	1	
1	1	0	0	

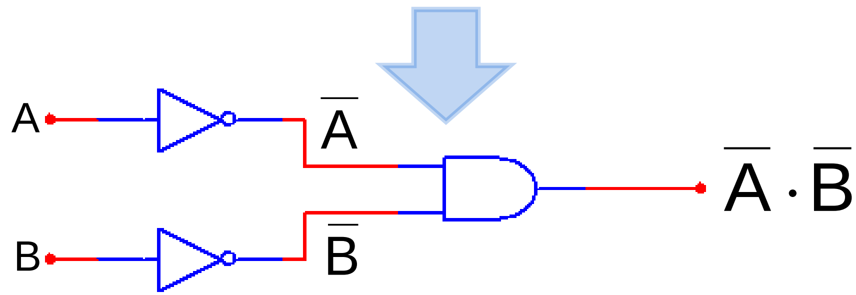
DeMorgan's Theorem #2 $\overline{A+B} = \overline{A} \cdot \overline{B}$

Proof

$\overline{A+B}$



$\overline{A} \cdot \overline{B}$



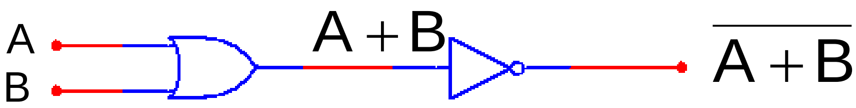
A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	
1	1	1	

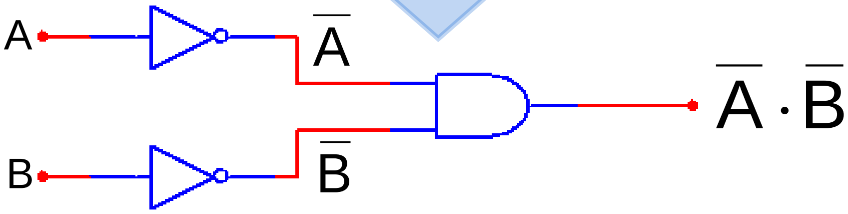
A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	
0	1	1	0	
1	0	0	1	
1	1	0	0	

DeMorgan's Theorem #2 $\overline{A+B} = \overline{A} \cdot \overline{B}$

Proof

$\overline{A+B}$ $\qquad\qquad$ $\overline{A} \cdot \overline{B}$





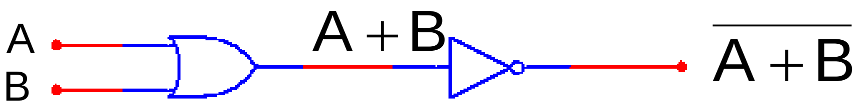
A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	
1	1	1	

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	
0	1	1	0	
1	0	0	1	
1	1	0	0	

DeMorgan's Theorem #2 $\overline{A+B} = \overline{A} \cdot \overline{B}$

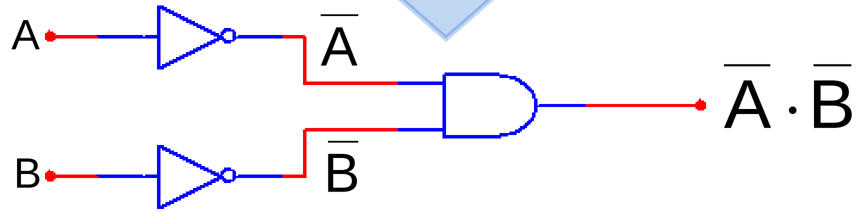
Proof

$\overline{A+B}$



A	B	A + B	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	

$\overline{A} \cdot \overline{B}$

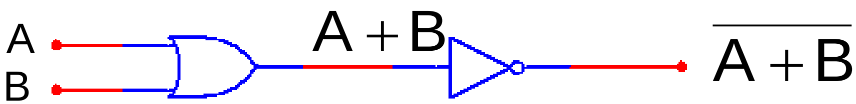


A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	
0	1	1	0	
1	0	0	1	
1	1	0	0	

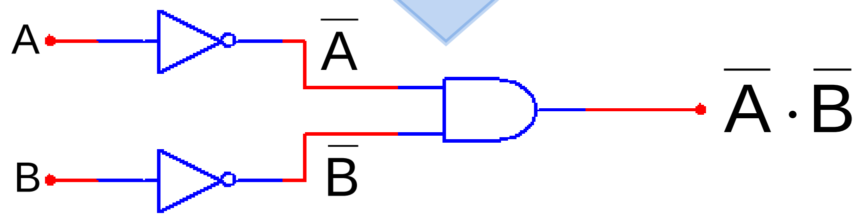
DeMorgan's Theorem #2 $\overline{A+B} = \overline{A}\cdot\overline{B}$

Proof

$\overline{A+B}$



$\overline{A}\cdot\overline{B}$



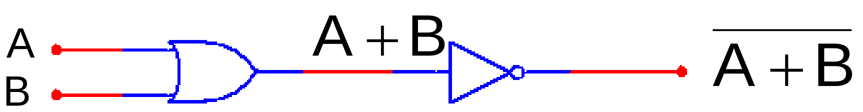
A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$
0	0	1	1	
0	1	1	0	
1	0	0	1	
1	1	0	0	

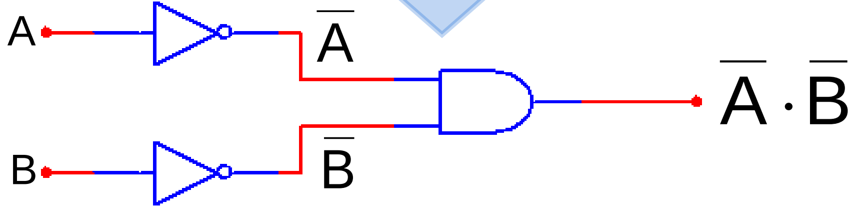
DeMorgan's Theorem #2 $\overline{A+B} = \overline{A} \cdot \overline{B}$

Proof

$\overline{A+B}$



$\overline{A} \cdot \overline{B}$



A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

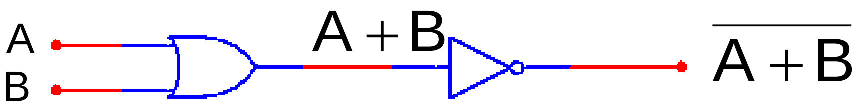
A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	
0	1	1	0	
1	0	0	1	
1	1	0	0	

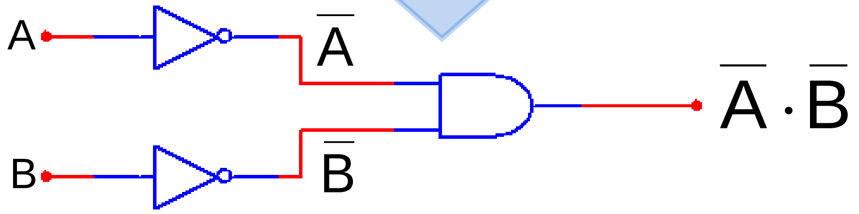
DeMorgan's Theorem #2 $\overline{A+B} = \overline{A} \cdot \overline{B}$

Proof

$\overline{A+B}$

$\overline{A} \cdot \overline{B}$





A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	
0	1	1	0	
1	0	0	1	
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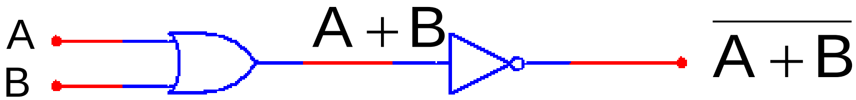
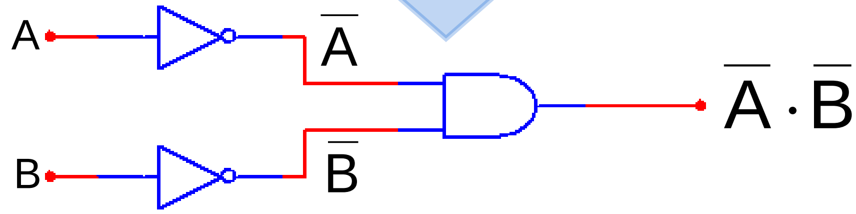
DeMorgan's Theorem #2 $\overline{A+B} = \overline{A} \cdot \overline{B}$

Proof

$\overline{A+B}$



$\overline{A} \cdot \overline{B}$



A	B	$A+B$	$\overline{A+B}$
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1	0	1	0
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A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
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DeMorgan's Theorem #2 $\overline{A+B} = \overline{A} \cdot \overline{B}$

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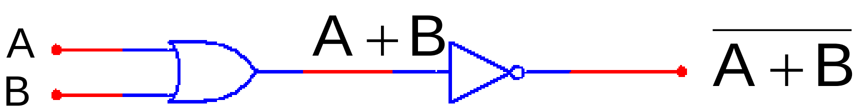
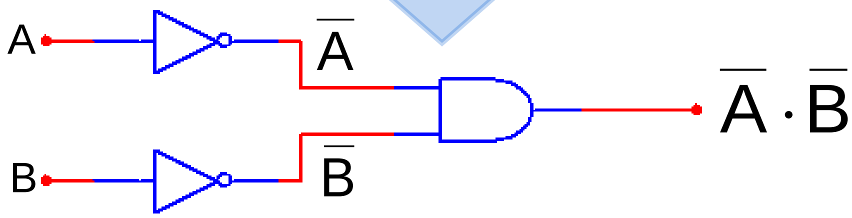


$\overline{A} \cdot \overline{B}$



A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
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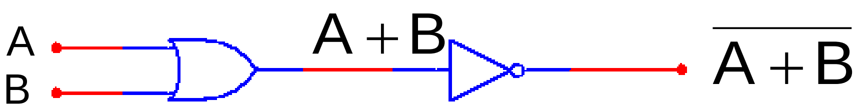

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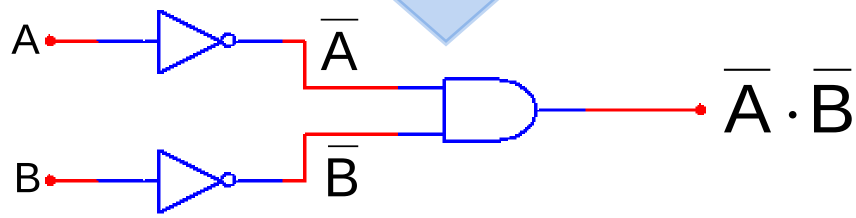

A	B	$A+B$	$\overline{A+B}$
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1	0	1	0
1	1	1	0

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
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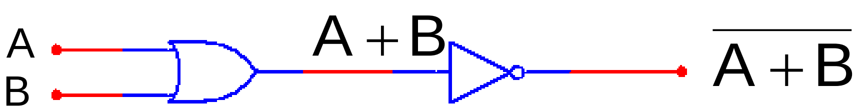

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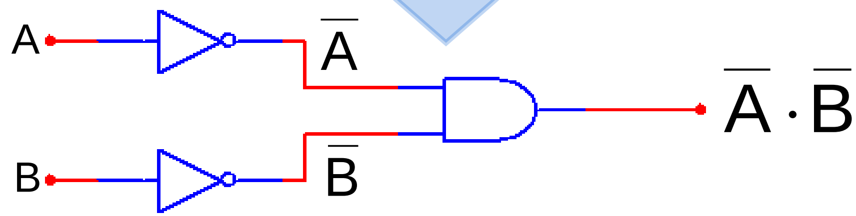

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0	1	1	0
1	0	1	0
1	1	1	0

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
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0	1	1	0	0
1	0	0	1	
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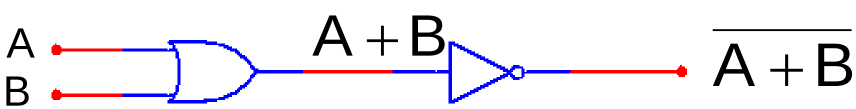
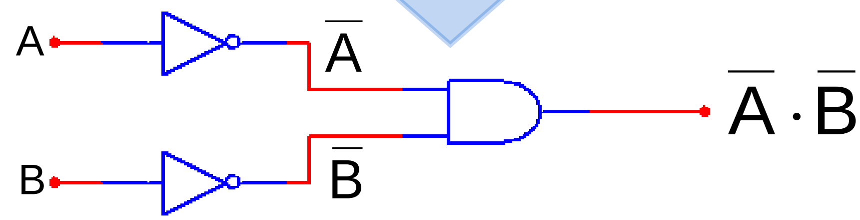
DeMorgan's Theorem #2 $\overline{A+B} = \overline{A} \cdot \overline{B}$

Proof

$\overline{A+B}$



$\overline{A} \cdot \overline{B}$



A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
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DeMorgan's Theorem #2 $\overline{A+B} = \overline{A}\cdot\overline{B}$

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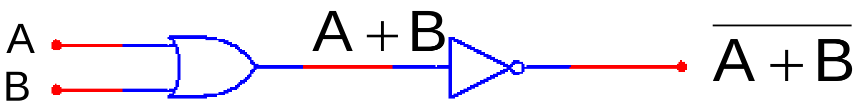

A	B	$A+B$	$\overline{A+B}$
0	0	0	1
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1	0	1	0
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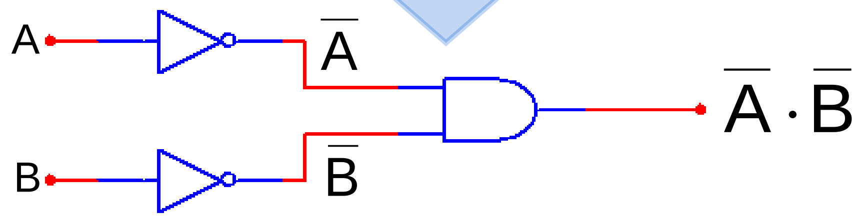
$\overline{A}\cdot\overline{B}$

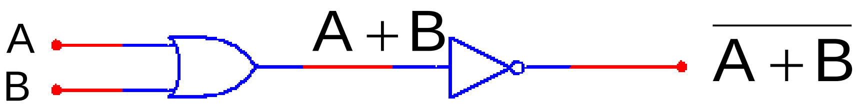


A	B	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	

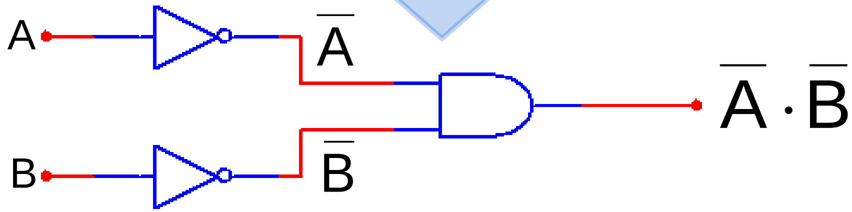
DeMorgan's Theorem #2 $\overline{A+B}=\overline{A}\cdot\overline{B}$

Proof

$\overline{A+B}$



$\overline{A}\cdot\overline{B}$



A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$
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0	1	1	0	0
1	0	0	1	0
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DeMorgan's Theorem #2 $\overline{A+B} = \overline{A} \cdot \overline{B}$

Proof

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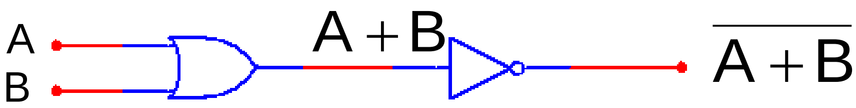
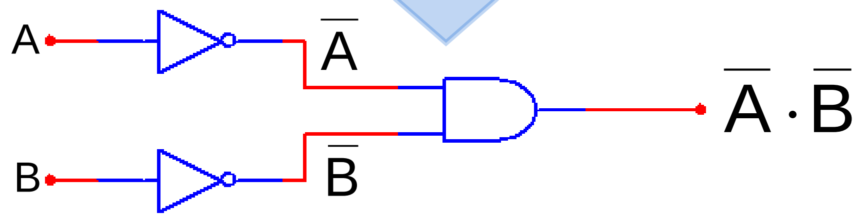


$\overline{A} \cdot \overline{B}$



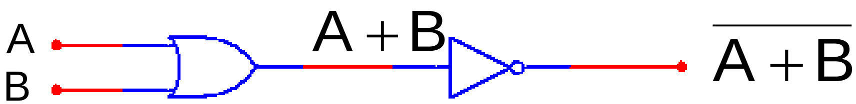
A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

DeMorgan's Theorem #2 $\overline{A+B} = \overline{A} \cdot \overline{B}$

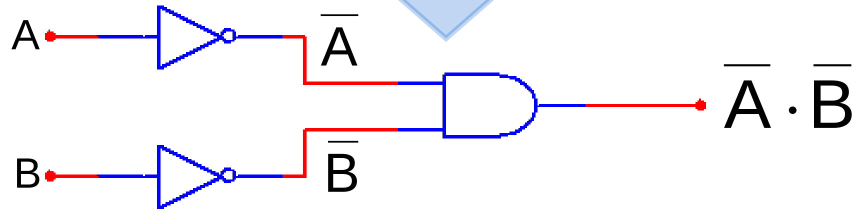
Proof

$\overline{A+B}$



A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

$\overline{A} \cdot \overline{B}$

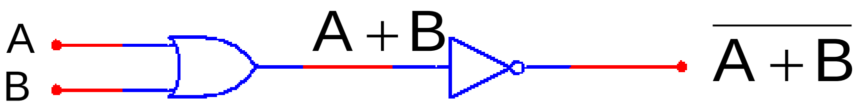


A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

DeMorgan's Theorem #2 $\overline{A+B} = \overline{A} \cdot \overline{B}$

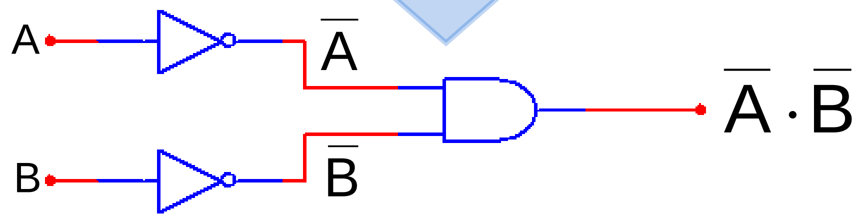
Proof

$\overline{A+B}$



A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

$\overline{A} \cdot \overline{B}$



A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

The truth-tables are **equal**; therefore, the Boolean equations must be **equal**!

Summary

Boolean & DeMorgan's Theorems

1) $X \cdot 0 = 0$
2) $X \cdot 1 = X$
3) $X \cdot X = X$
4) $X \cdot \overline{X} = 0$
5) $X + 0 = X$
6) $X + 1 = 1$
7) $X + X = X$
8) $X + \overline{X} = 1$
9) $\overline{\overline{X}} = X$

10A) $X \cdot Y = Y \cdot X$
10B) $X + Y = Y + X$
} Commutative Law

11A) $X(YZ) = (XY)Z$
11B) $X + (Y + Z) = (X + Y) + Z$
} Associative Law

12A) $X(Y + Z) = XY + XZ$
12B) $(X + Y)(W + Z) = XW + XZ + YW + YZ$
} Distributive Law

13A) $X + \overline{X}Y = X + Y$
13B) $\overline{X} + XY = \overline{X} + Y$
13C) $X + \overline{X}\,\overline{Y} = X + \overline{Y}$
13D) $\overline{X} + X\overline{Y} = \overline{X} + \overline{Y}$
} Consensus Theorem

14A) $\overline{X\,Y} = \overline{X} + \overline{Y}$
14B) $\overline{X + Y} = \overline{X}\ \overline{Y}$
} DeMorgan's

DeMorgan Shortcut

BREAK THE **LINE**, CHANGE THE **SIGN**!

Break the LINE over the two variables,
and change the SIGN directly under the line.

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

For Theorem #**14A**, break the line, and change the **AND** function to an **OR** function. *Be sure to keep the lines over the variables.*

DeMorgan Shortcut

BREAK THE **<u>LINE</u>**, CHANGE THE **<u>SIGN</u>**!

Break the LINE over the two variables,
and change the SIGN directly under the line.

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

For Theorem #**<u>14B</u>**, break the line, and change the **<u>OR</u>** function to an **<u>AND</u>** function. Be sure to keep the lines over the variables.

DeMorgan’s: Example #1

Simplify the following Boolean expression and note the Boolean or DeMorgan’s theorem used at each step. Put the answer in SOP form.

$$F_1 = \overline{\overline{(X \cdot \overline{Y})} \cdot (\overline{Y} + Z)}$$

DeMorgan’s: Example #1

$$F_1 = \overline{(\overline{X \cdot \overline{Y}}) \cdot (\overline{Y} + Z)}$$

Theorem #

DeMorgan's: Example #1

$$F_1 = \overline{(\overline{X \cdot \overline{Y}}) \cdot (\overline{Y} + Z)}$$

Theorem #14A

DeMorgan’s: Example #1

$$F_1 = \overline{(\overline{X \cdot \overline{Y}}) \cdot (\overline{Y} + Z)}$$

$$F_1 = (\overline{\overline{X \cdot \overline{Y}}}) + (\overline{\overline{Y} + Z}) \qquad \text{Theorem \#14A}$$

DeMorgan's: Example #1

$$F_1 = \overline{(\overline{X \cdot \overline{Y}}) \cdot (\overline{Y} + Z)}$$

$$F_1 = (\overline{\overline{X \cdot \overline{Y}}}) + (\overline{\overline{Y} + Z})$$

Theorem #14A

Theorem #   &   #

DeMorgan's: Example #1

$$F_1 = \overline{(\overline{X \cdot \overline{Y}}) \cdot (\overline{Y} + Z)}$$

$$F_1 = (\overline{\overline{X \cdot \overline{Y}}}) + (\overline{\overline{Y} + Z})$$

Theorem #14A

Theorem #9 & #14B

DeMorgan's: Example #1

$$F_1 = \overline{(\overline{X \cdot \overline{Y}}) \cdot (\overline{Y} + Z)}$$

$$F_1 = (\overline{\overline{X \cdot \overline{Y}}}) + (\overline{\overline{Y} + Z})$$ Theorem #14A

$$F_1 = (X \cdot \overline{Y}) + (\overline{\overline{Y}} \cdot \overline{Z})$$ Theorem #9 & #14B

DeMorgan's: Example #1

$$F_1 = \overline{(\overline{X \cdot \overline{Y}}) \cdot (\overline{Y} + Z)}$$

$$F_1 = (\overline{\overline{X \cdot \overline{Y}}}) + (\overline{\overline{Y} + Z})$$ Theorem #14A

$$F_1 = (X \cdot \overline{Y}) + (\overline{\overline{Y}} \cdot \overline{Z})$$ Theorem #9 & #14B

Theorem #

DeMorgan's: Example #1

$$F_1 = \overline{\overline{(X \cdot \overline{\overline{Y}})} \cdot (\overline{Y} + Z)}$$

$$F_1 = \overline{\overline{(X \cdot \overline{\overline{Y}})}} + \overline{(\overline{Y} + Z)}$$ Theorem #14A

$$F_1 = (X \cdot \overline{Y}) + (\overline{\overline{Y}} \cdot \overline{Z})$$ Theorem #9 & #14B

Theorem #9

DeMorgan's: Example #1

$$F_1 = \overline{(\overline{X \cdot \overline{Y}}) \cdot (\overline{Y} + Z)}$$

$$F_1 = (\overline{\overline{X \cdot \overline{Y}}}) + (\overline{\overline{Y} + Z})$$ Theorem #14A

$$F_1 = (X \cdot \overline{Y}) + (\overline{\overline{Y}} \cdot \overline{Z})$$ Theorem #9 & #14B

$$F_1 = (X \cdot \overline{Y}) + (Y \cdot \overline{Z})$$ Theorem #9

<u>DeMorgan's: Example #1</u>

$$F_1 = \overline{\overline{(X \cdot \overline{Y})} \cdot (\overline{Y} + Z)}$$

$$F_1 = \overline{\overline{(X \cdot \overline{Y})}} + \overline{(\overline{Y} + Z)}$$ Theorem #14A

$$F_1 = (X \cdot \overline{Y}) + (\overline{\overline{Y}} \cdot \overline{Z})$$ Theorem #9 & #14B

$$F_1 = (X \cdot \overline{Y}) + (Y \cdot \overline{Z})$$ Theorem #9

Rewritten without AND symbols and parentheses

DeMorgan’s: Example #1

$$F_1 = \overline{(\overline{X \cdot \overline{Y}}) \cdot (\overline{Y} + Z)}$$

$$F_1 = (\overline{\overline{X \cdot \overline{Y}}}) + (\overline{\overline{Y} + Z})$$ Theorem #14A

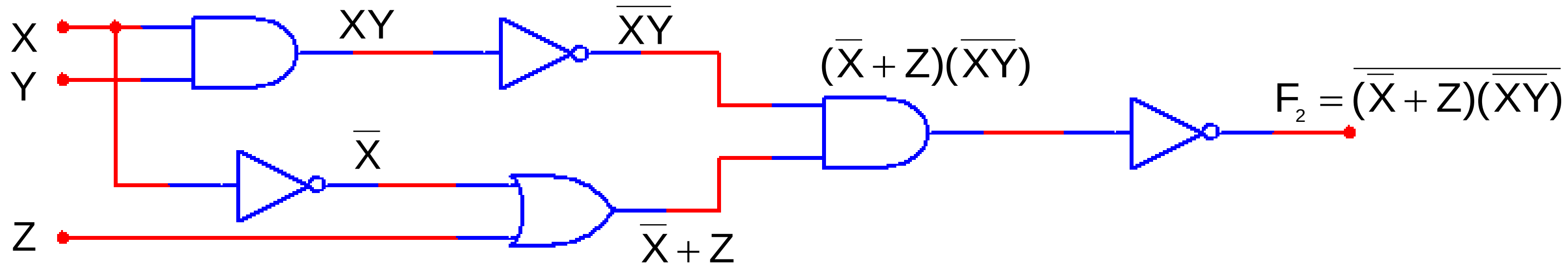
$$F_1 = (X \cdot \overline{Y}) + (\overline{\overline{Y}} \cdot \overline{Z})$$ Theorem #9 & #14B

$$F_1 = (X \cdot \overline{Y}) + (Y \cdot \overline{Z})$$ Theorem #9

$$F_1 = X\overline{Y} + Y\overline{Z}$$ Rewritten without AND symbols and parentheses

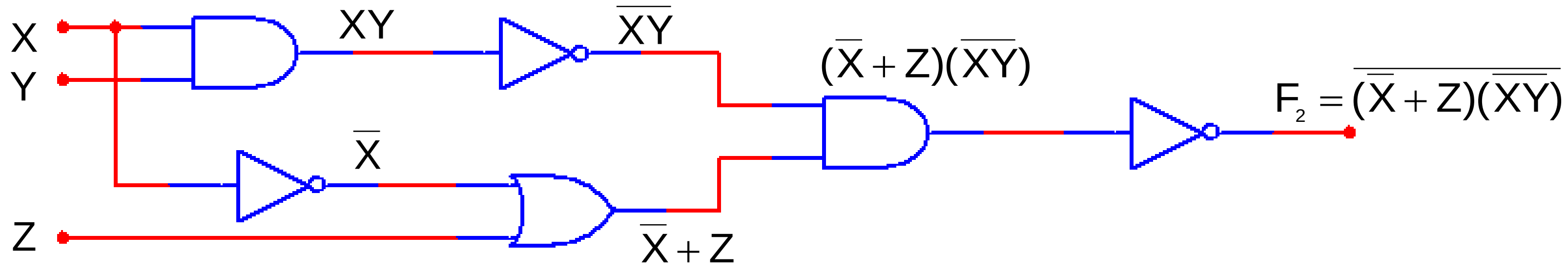
DeMorgan's: Example #2

Take a look at the VERY poorly designed logic circuit shown below. If you were to analyze this circuit to determine the output function F_2, you would obtain the results shown.



DeMorgan's: Example #2

Simplify the output function F_2. Be sure to note the Boolean or DeMorgan's theorem used at each step. Put the answer in SOP form.



DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

DeMorgan’s: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

Theorem #

DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

Theorem #14A

DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = (\overline{\overline{X} + Z}) + (\overline{\overline{XY}})$$ Theorem #14A

DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = \overline{(\overline{X} + Z)} + (\overline{\overline{XY}})$$ Theorem #14A

Theorem #

DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = (\overline{\overline{X} + Z}) + (\overline{\overline{XY}})$$ Theorem #14A

Theorem #9

DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = (\overline{\overline{X} + Z}) + (\overline{\overline{XY}})$$ Theorem #14A

$$F_2 = (\overline{\overline{X} + Z}) + (XY)$$ Theorem #9

DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = (\overline{\overline{X} + Z}) + (\overline{\overline{XY}})$$ Theorem #14A

$$F_2 = (\overline{\overline{X} + Z}) + (XY)$$ Theorem #9

Theorem #

DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = (\overline{\overline{X} + Z}) + (\overline{\overline{XY}})$$ Theorem #14A

$$F_2 = (\overline{\overline{X} + Z}) + (XY)$$ Theorem #9

Theorem #14B

DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = (\overline{\overline{X} + Z}) + (\overline{\overline{XY}})$$ Theorem #14A

$$F_2 = (\overline{\overline{X} + Z}) + (XY)$$ Theorem #9

$$F_2 = (\overline{\overline{X}}\,\overline{Z}) + (XY)$$ Theorem #14B

DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = (\overline{\overline{X} + Z}) + (\overline{\overline{XY}})$$ Theorem #14A

$$F_2 = (\overline{\overline{X} + Z}) + (XY)$$ Theorem #9

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Theorem #

DeMorgan’s: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = \overline{(\overline{X} + Z)} + \overline{\overline{(XY)}}$$ Theorem #14A

$$F_2 = \overline{(\overline{X} + Z)} + (XY)$$ Theorem #9

$$F_2 = (\overline{\overline{X}}\,\overline{Z}) + (XY)$$ Theorem #14B

Theorem #9

DeMorgan’s: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = \overline{(\overline{X} + Z)} + (\overline{\overline{XY}})$$ Theorem #14A

$$F_2 = \overline{(\overline{X} + Z)} + (XY)$$ Theorem #9

$$F_2 = (\overline{\overline{X}}\,\overline{Z}) + (XY)$$ Theorem #14B

$$F_2 = (X\,\overline{Z}) + (XY)$$ Theorem #9

DeMorgan’s: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = \overline{(\overline{X} + Z)} + \overline{\overline{(XY)}}$$ Theorem #14A

$$F_2 = (\overline{\overline{X} + Z}) + (XY)$$ Theorem #9

$$F_2 = (\overline{\overline{X}}\,\overline{Z}) + (XY)$$ Theorem #14B

$$F_2 = (X\,\overline{Z}) + (XY)$$ Theorem #9

Rewritten without parentheses

DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = \overline{(\overline{X} + Z)} + \overline{\overline{(XY)}}$$ Theorem #14A

$$F_2 = \overline{(\overline{X} + Z)} + (XY)$$ Theorem #9

$$F_2 = (\overline{\overline{X}}\,\overline{Z}) + (XY)$$ Theorem #14B

$$F_2 = (X\,\overline{Z}) + (XY)$$ Theorem #9

$$F_2 = X\,\overline{Z} + X\,Y$$ Rewritten without parentheses

DeMorgan's: Example #2

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

can be simplified to…

$$F_2 = X\,\overline{Z} + X\,Y$$