

Activity 2.1.5 Circuit Simplification: DeMorgan's Theorems

Introduction

Despite all of the work done by George Boole, there was still more work to be done. Expanding on Boole's studies, Augustus DeMorgan (1806-1871) developed two additional theorems that now bear his name. Without DeMorgan's Theorems, the complete simplification of logic expression would not be possible.

$$\text{Theorem \#1: } \overline{X \bullet Y} = \overline{X} + \overline{Y}$$

$$\text{Theorem \#2: } \overline{\overline{X} + \overline{Y}} = \overline{\overline{X}} \bullet \overline{\overline{Y}}$$

As we will see in later activities, DeMorgan's Theorems are the foundation for the NAND and NOR logic gates.

In this activity you will learn how to simplify logic expressions and digital logic circuits using DeMorgan's two theorems along with the other laws of Boolean algebra.

Procedure

Using DeMorgan's theorems and the other theorems and laws of Boolean algebra, simplify the following logic expressions. Note the theorem/law used at each simplification step. Be sure to put your answer in Sum-Of-Products (SOP) form.

$$1. F_1 = \overline{\overline{X} + Y}$$

$$2. F_2 = \overline{\overline{(R + \overline{S})}(\overline{T} + U)}$$

$$3. F_3 = \overline{\overline{MN}}$$

$$4. F_4 = \overline{\overline{W} X (\overline{Y} + Z)}$$

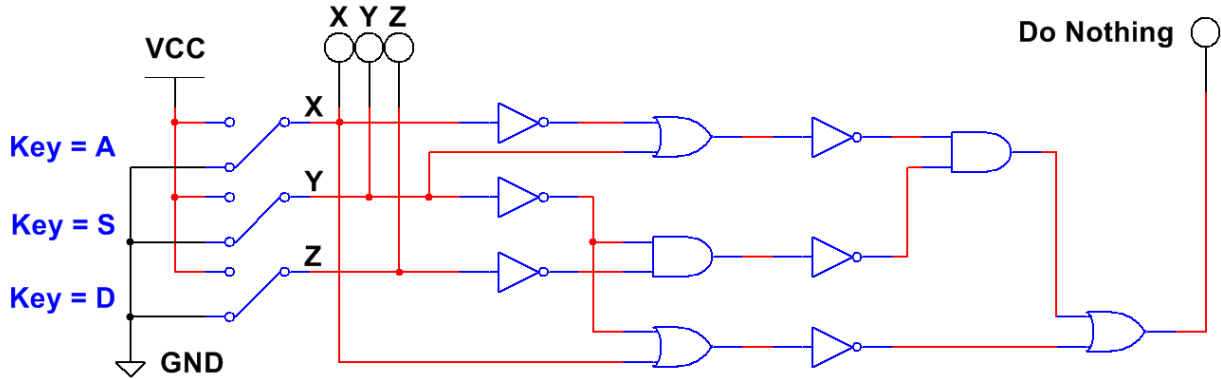
$$5. F_5 = \overline{\overline{PQR} + \overline{\overline{PQR}}}$$

$$6. F_6 = \overline{\overline{\overline{W} X Y Z}}$$

Let's see how we would utilize DeMorgan's theorems to simplify a digital logic circuit.

- Write the UN-SIMPLIFIED logic expression for the output **Do-Nothing** in the logic circuit shown below.

The output is call **Do-Nothing** because that is exactly what the circuit does, nothing; yet it's a good example for learning about DeMorgan's theorems.



8. Using DeMorgan's theorems and the other theorems and laws of Boolean algebra, simplify the logic expression **Do Nothing**. Be sure to put your answer in Sum-Of-Products (SOP) form.

9. In the space provided, draw an AOI circuit that implements the simplified logic expression **Do Nothing**. For this implementation you may assume that AND & OR gates are available with any number of inputs.

Do Nothing – I

10. Re-implement the circuit assuming that only 2-input AND gates (74LS08), 2-input OR gates (74LS32), and inverters (74LS04) are available. Draw this circuit in the space provided.

Do Nothing – II

Conclusion

1. Draw the gate equivalent for DeMorgan's two theorems.

$$\text{Theorem \#1: } \overline{X \bullet Y} = \overline{X} + \overline{Y}$$

$$\text{Theorem \#2: } \overline{X + Y} = \overline{X} \bullet \overline{Y}$$

2. How would you prove that the original **Do Nothing** circuit and the simplified version are equivalent?
3. If each GATE cost 5¢ and you made 100,000 of the unsimplified units, how much of the company's money did you waste on the unsimplified Do-Nothing project?