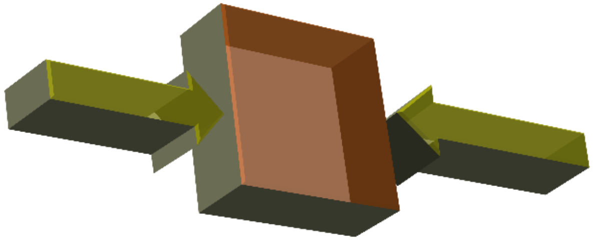
Calculating Truss Forces

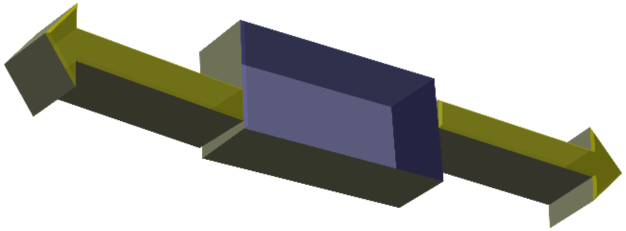
Forces

Compression



A body being squeezed

Tension



A body being stretched

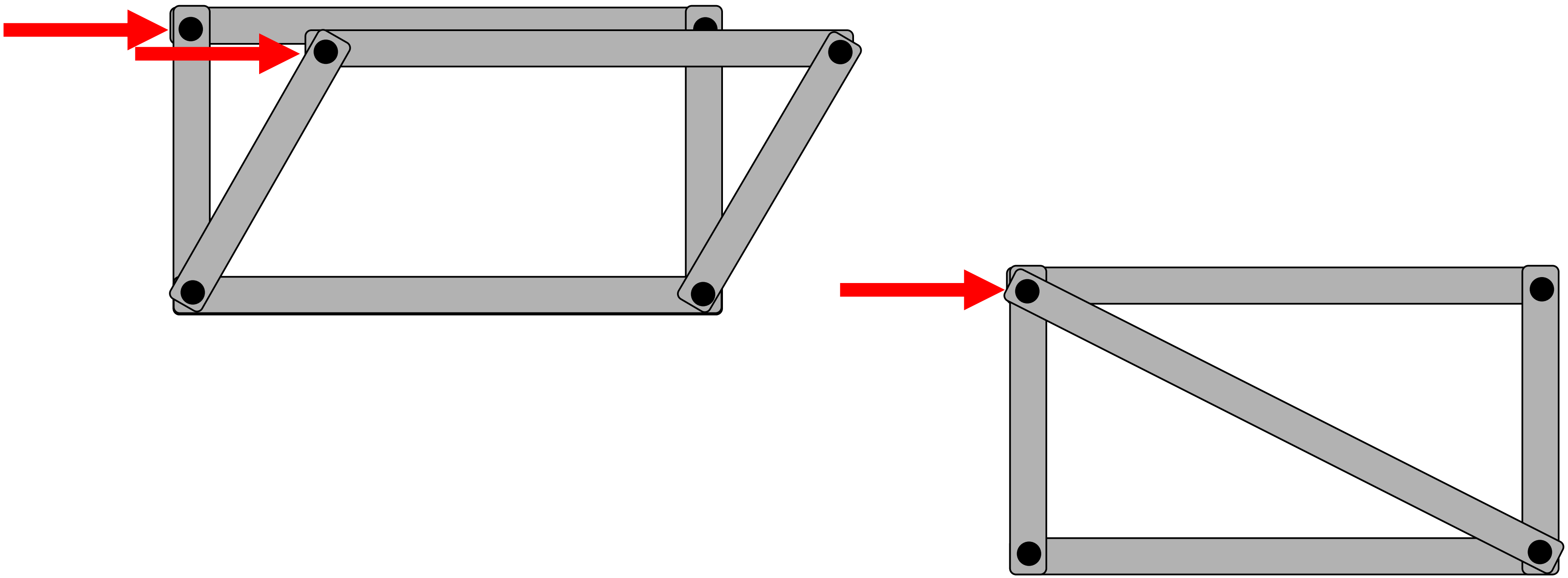
Truss

A truss is composed of slender members joined together at their end points.

- They are usually joined by **welds** or **gusset** plates.

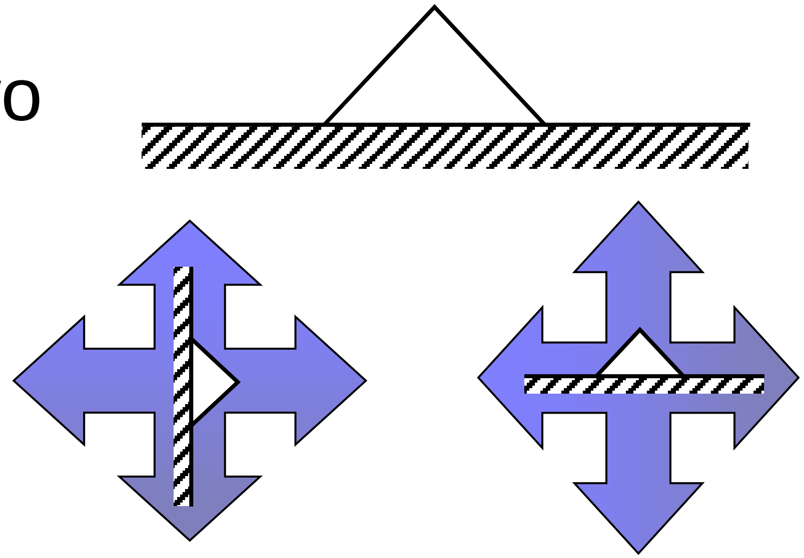
Simple Truss

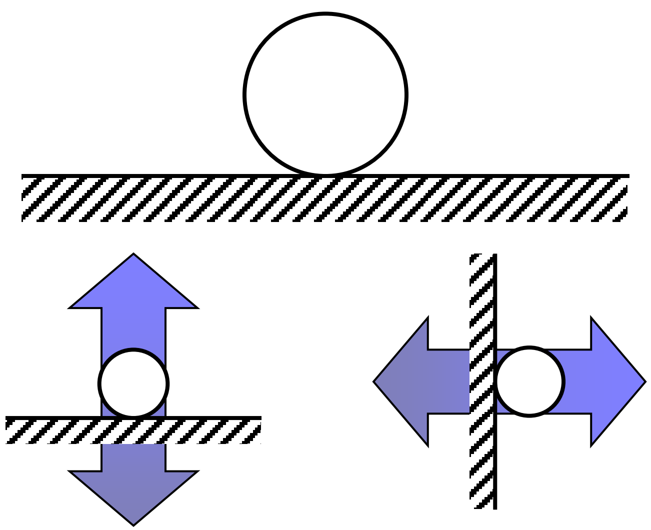
A simple truss is composed of **triangles**, which will retain their shape even when removed from supports.



Pinned and Roller Supports

A **pinned** support can support a structure in two dimensions.





A **roller** support can support a structure in only one dimension.

Solving Truss Forces

Assumptions:

All members are perfectly **straight**.

All loads are applied at the **joints**.

All joints are pinned and **frictionless**.

Each member has no **weight**.

Members can only experience **tension** or **compression** forces.

What risks might these assumptions pose if we were designing an actual bridge?

Static Determinacy

A statically **determinate** structure is one that can be mathematically solved.

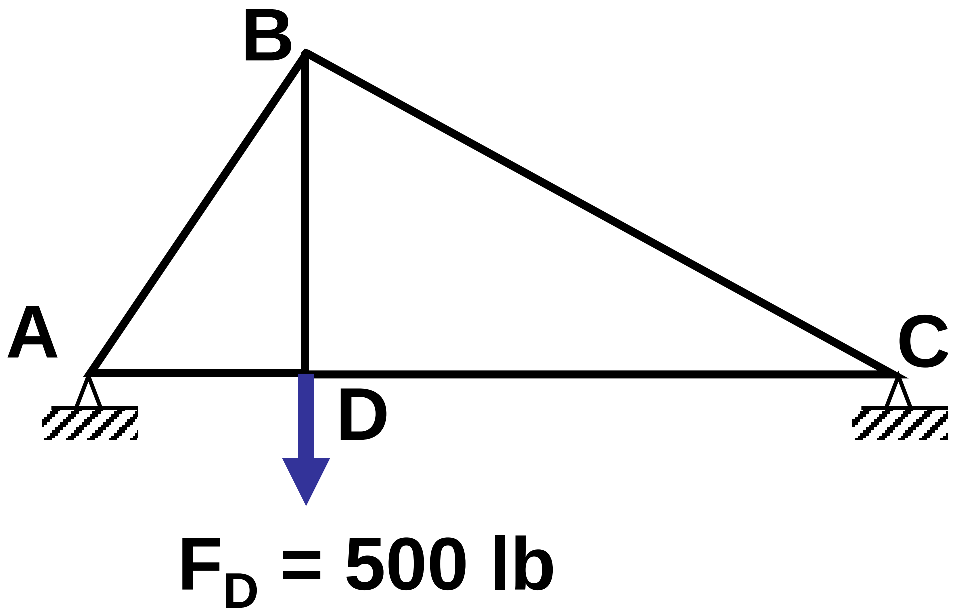
$$2J = M + R$$

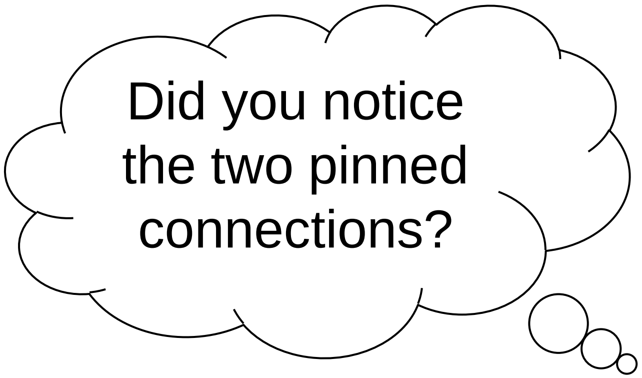
J = Number of **Joints**

M = Number of **Members**

R = Number of **Reactions**

Statically Indeterminate

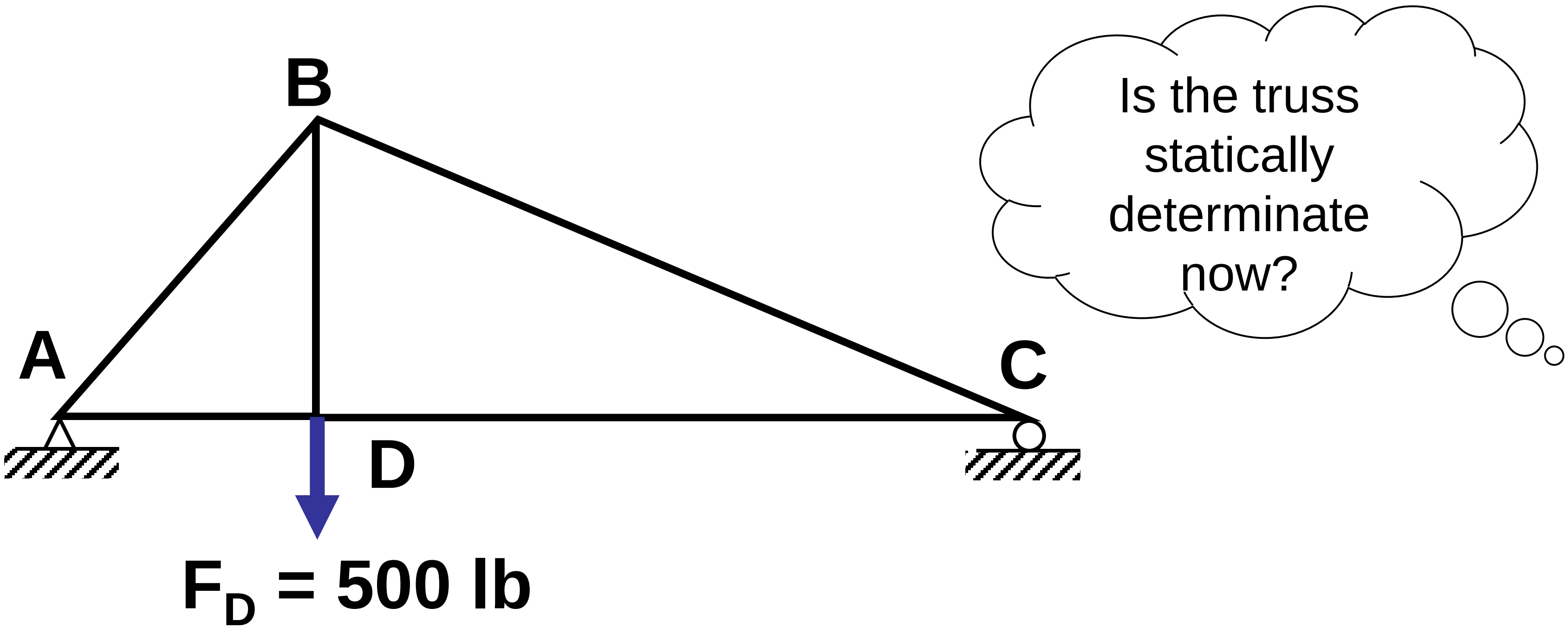




A truss is considered **statically <u>indeterminate</u>** when the static equilibrium equations are not sufficient to find the reactions on that structure. There are simply too many unknowns.

$$2J = M + R$$

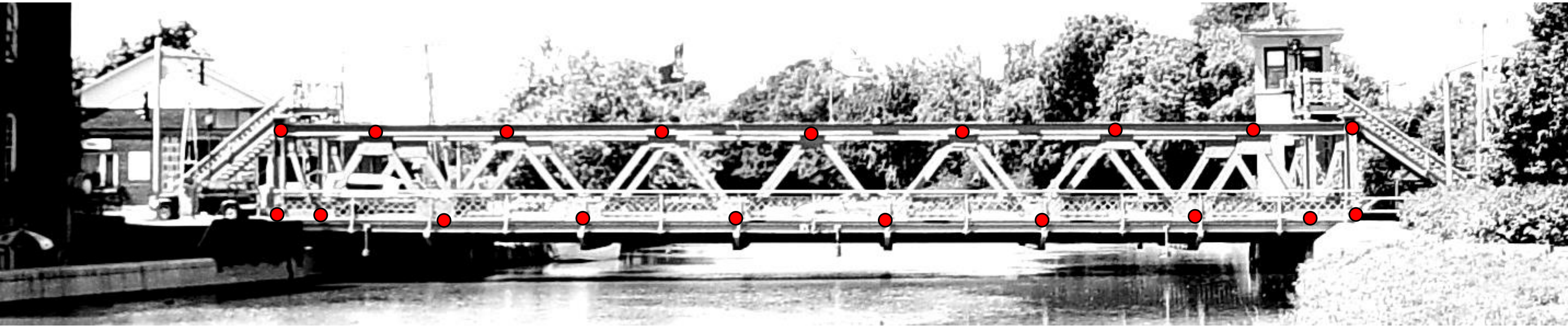
Statically Determinate



A truss is considered **statically <u>determinate</u>** when the static equilibrium equations can be used to find the reactions on that structure.

$$2J = M + R$$

Static Determinacy Example

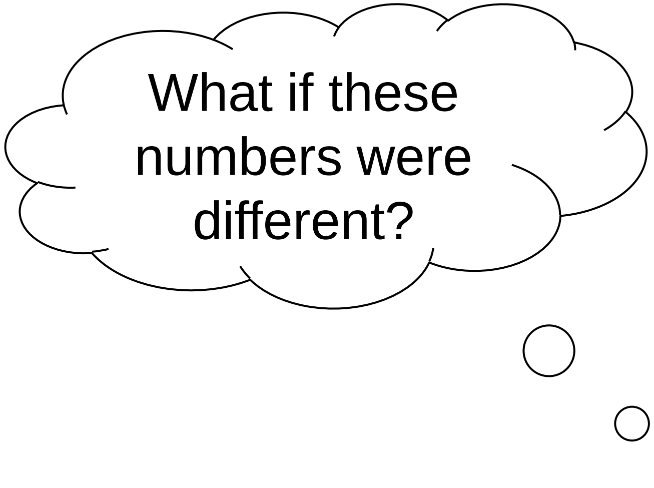


Each side of the main street bridge in Brockport, NY has 19 joints, 35 members, and three reaction forces (pin and roller), making it a statically determinate truss.

$$2J = M + R$$

$$2(19) = 35 + 3$$

$$38 = 38$$



Equilibrium Equations

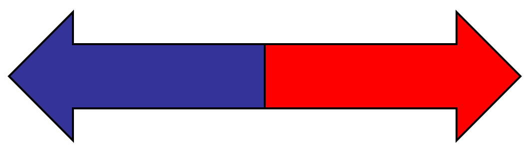
$$\Sigma M = 0$$

The sum of the moments about a given point is **zero**.

Equilibrium Equations

$$\Sigma F_x = 0$$

The sum of the forces in the x-direction is **zero**.

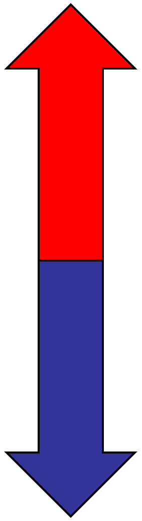


Do you remember the Cartesian coordinate system? A vector that acts to the right is **positive**, and a vector that acts to the left is **negative**.

Equilibrium Equations

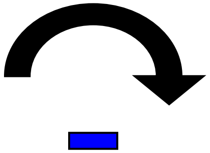
$$\Sigma F_y = 0$$

The sum of the forces in the y-direction is **zero**.



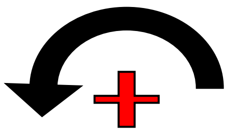
A vector that acts **up** is positive, and a vector that acts down is negative.

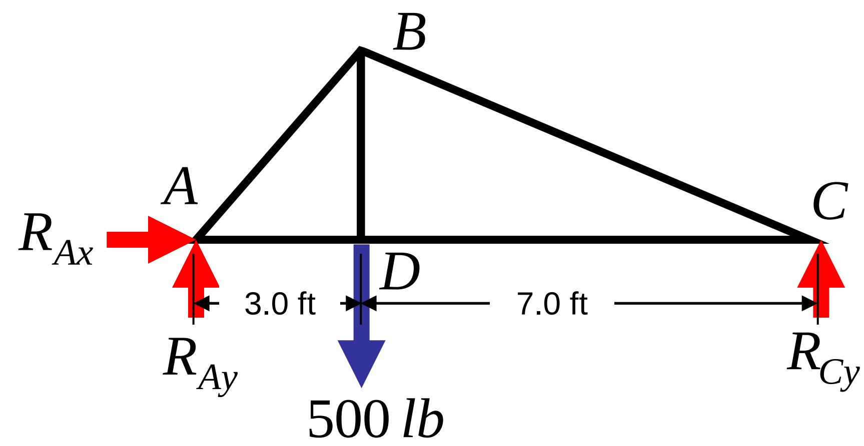
Using Moments to Find R_{CY}



A force that causes a **clockwise** moment is **negative.**

A force that causes a **counterclockwise** moment is **positive.**





F_D contributes a **negative** moment because it causes a **clockwise** moment.

R_{Cy} contributes a **positive** moment because it causes a **counterclockwise** moment.

$$\Sigma M_A = 0$$

$$-F_D(3.0\,ft) + R_{Cy}(10.0\,ft) = 0$$

$$-500lb \cdot (3.0\,ft) + R_{Cy}(10.0\,ft) = 0$$

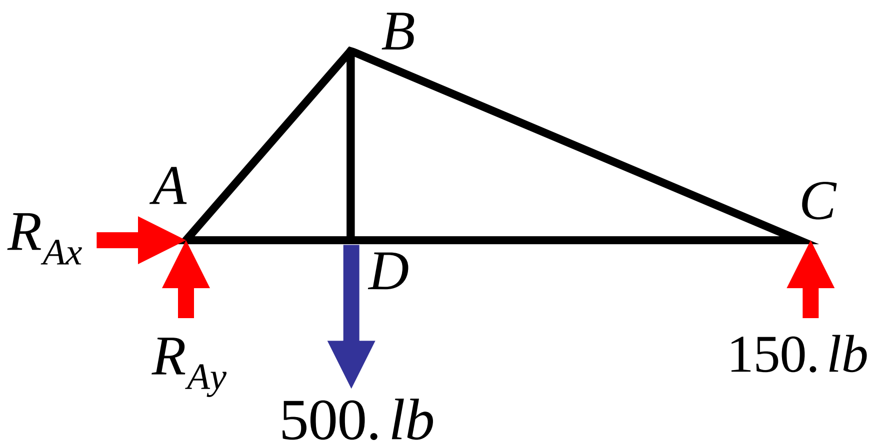
$$-1500lb \cdot ft + R_{Cy}(10.0\,ft) = 0$$

$$R_{Cy}(10.0\,\cancel{ft}) = 1500lb \cdot \cancel{ft}$$

$$R_{Cy} = 150lb$$

Sum the y Forces to Find R_{Ay}

We know two out of the three forces acting in the y-direction. By simply summing those forces together, we can find the unknown reaction at point A.



$$\Sigma F_y = 0$$

$$-F_D + R_{Cy} + R_{Ay} = 0$$

$$-500.\,lb + 150.\underline{00}\,lb + R_{Ay} = 0$$

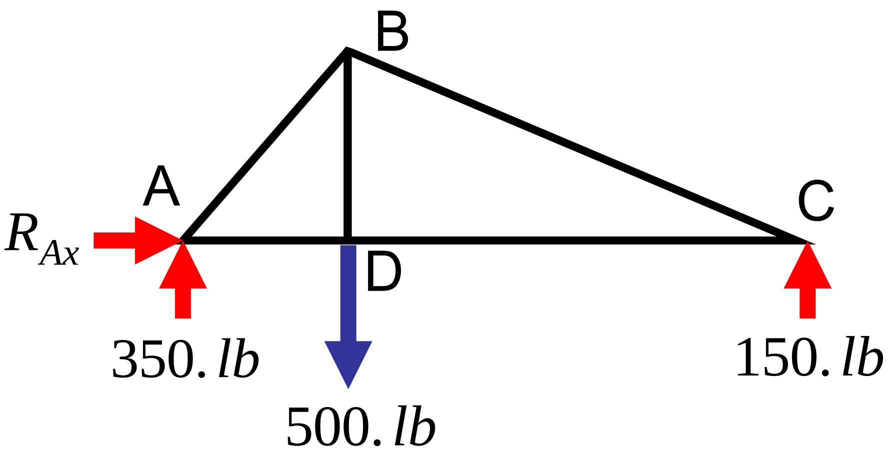
$$-350.\,lb + R_{Ay} = 0$$

$$R_{Ay} = 350.\,lb$$

Please note that F_D is shown as a **negative** because of its direction.

Sum the x Forces to Find A_x

Because joint **A** is **pinned**, it is capable of reacting to a force applied in the **x**-direction.



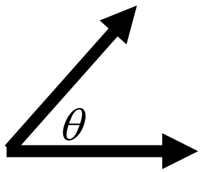
However, since the only load applied to this truss (**F_D**) has no x-component, **R_{Ax}** must be **zero**.

$$\Sigma F_x = 0$$

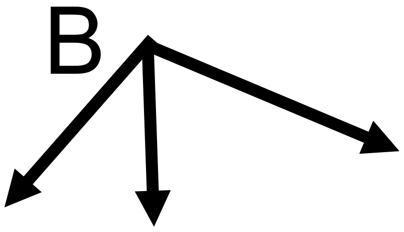
$$A_x = 0$$

Method of Joints

Use cosine and sine to determine x and y vector components.



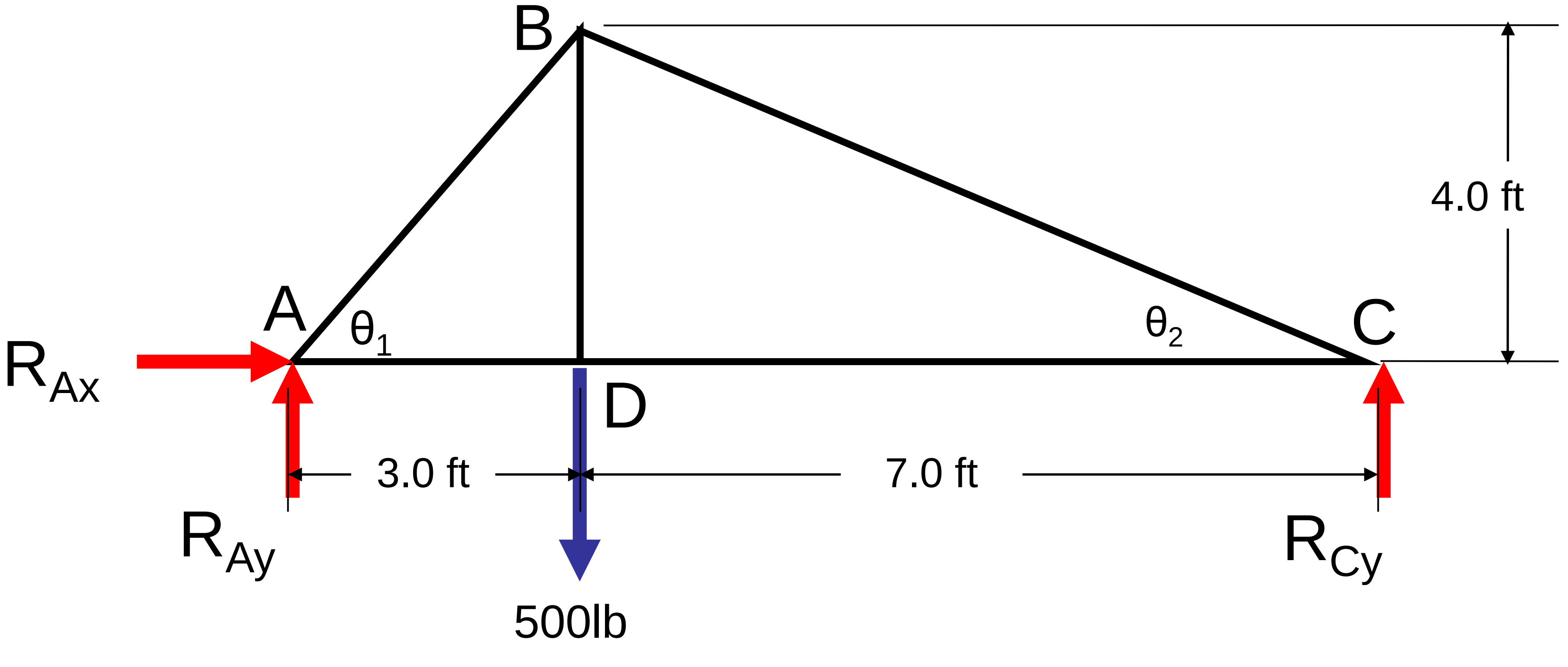
Assume all members to be in tension. A **positive** answer will mean the member is in **tension**, and a **negative** number will mean the member is in **compression**.



As forces are solved, update free body diagrams. Use correct magnitude and sense for subsequent joint free body diagrams.

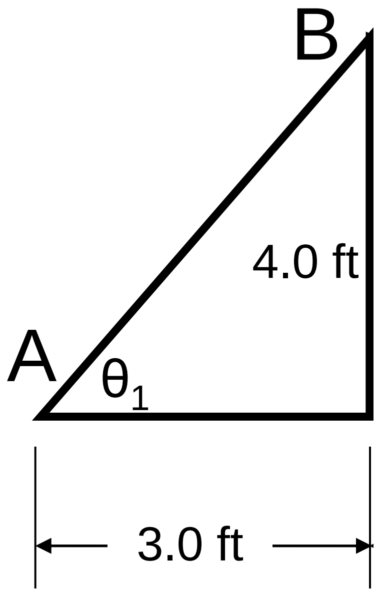
Method of Joints

Truss Dimensions



Method of Joints

Using Truss Dimensions to Find Angles



$$\tan\theta_1 = \frac{opp}{adj}$$

$$\tan\theta_1 = \frac{4.0\,\cancel{ft}}{3.0\,\cancel{ft}}$$

$$\theta_1 = \tan^{-1}\frac{4.0}{3.0}$$

$$\theta_1 = 53.130°$$

Method of Joints

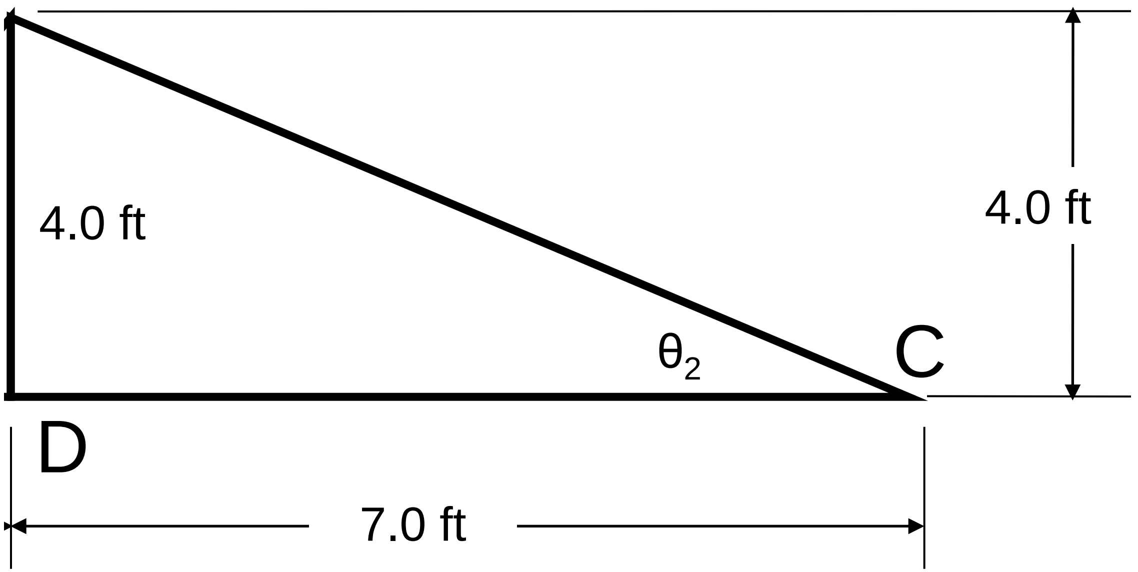
Using Truss Dimensions to Find Angles

$$\tan\theta_1 = \frac{opp}{adj}$$

$$\tan\theta_1 = \frac{4.0\ \cancel{ft}}{7.0\ \cancel{ft}}$$

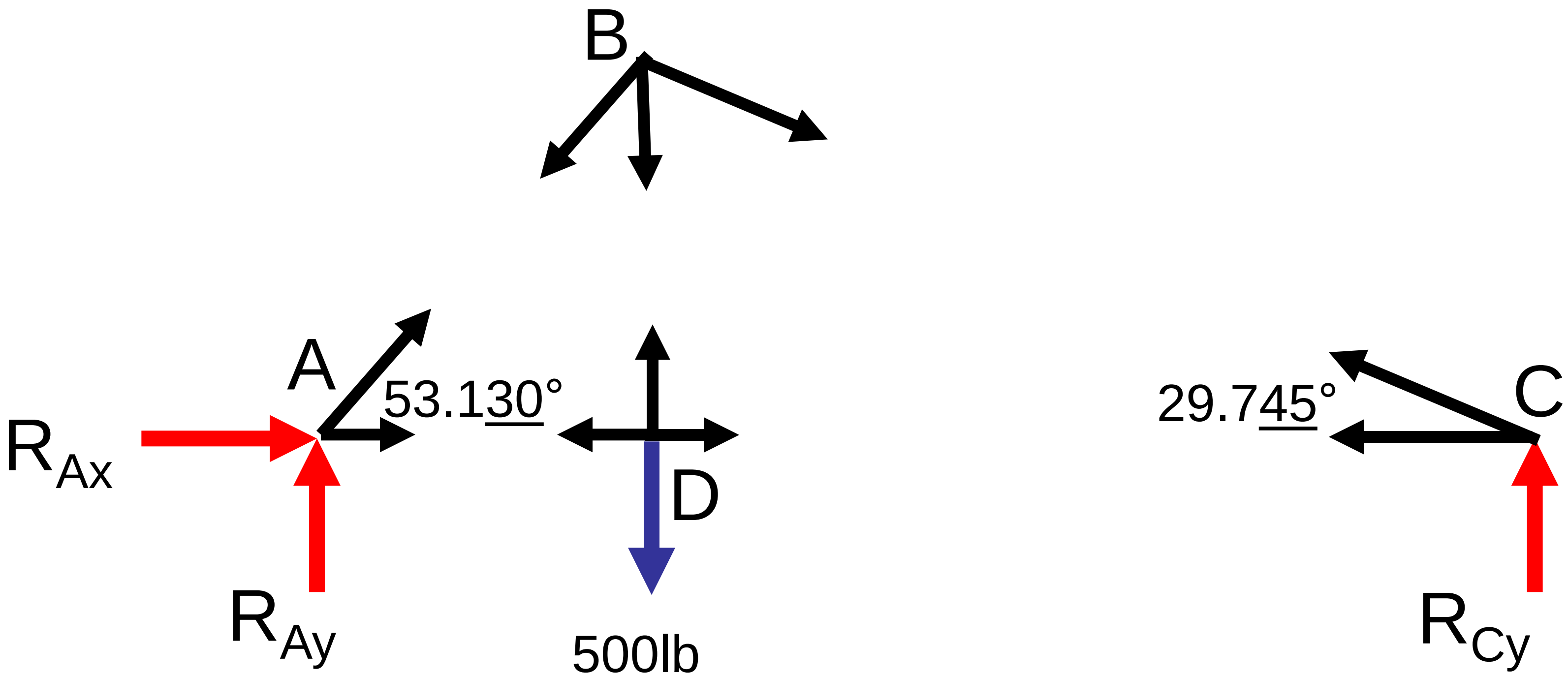
$$\theta_1 = \tan^{-1}\frac{4.0}{7.0}$$

$$\theta_1 = 29.745°$$



Method of Joints

Draw a free body diagram of each pin.



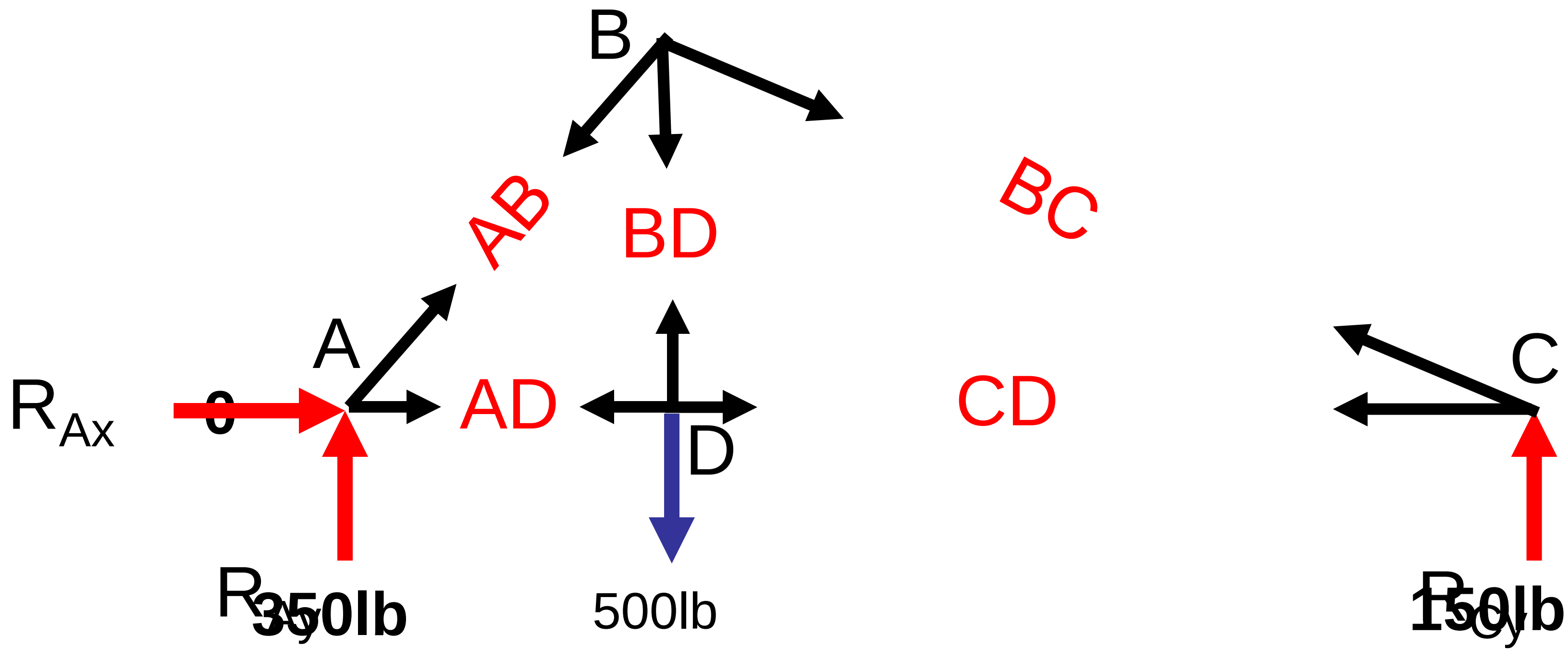
Every member is assumed to be in tension. A **positive** answer indicates the member is in **tension**, and a **negative** answer indicates the member is in **compression**.

Method of Joints

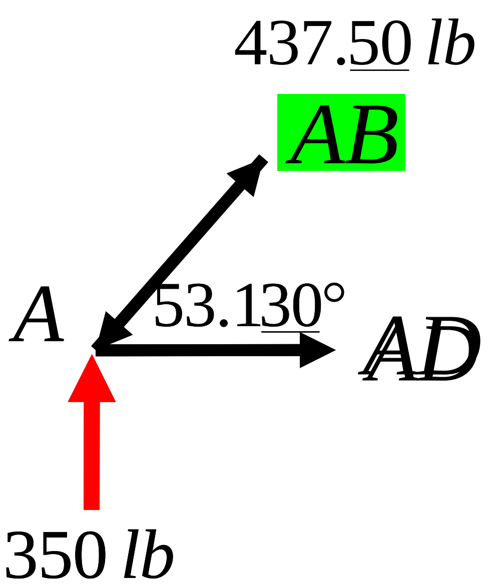
Where to Begin

Choose the joint that has the **least** number of **unknowns**.

Reaction forces at joints **A** and **C** are both good choices to begin our calculations.



Method of Joints



$$\Sigma F_Y = 0$$

$$R_{Ay} + AB_y = 0$$

$$350lb + AB\sin 53.1\underline{30}^\circ = 0$$

$$AB\sin 53.1\underline{30}^\circ = -350lb$$

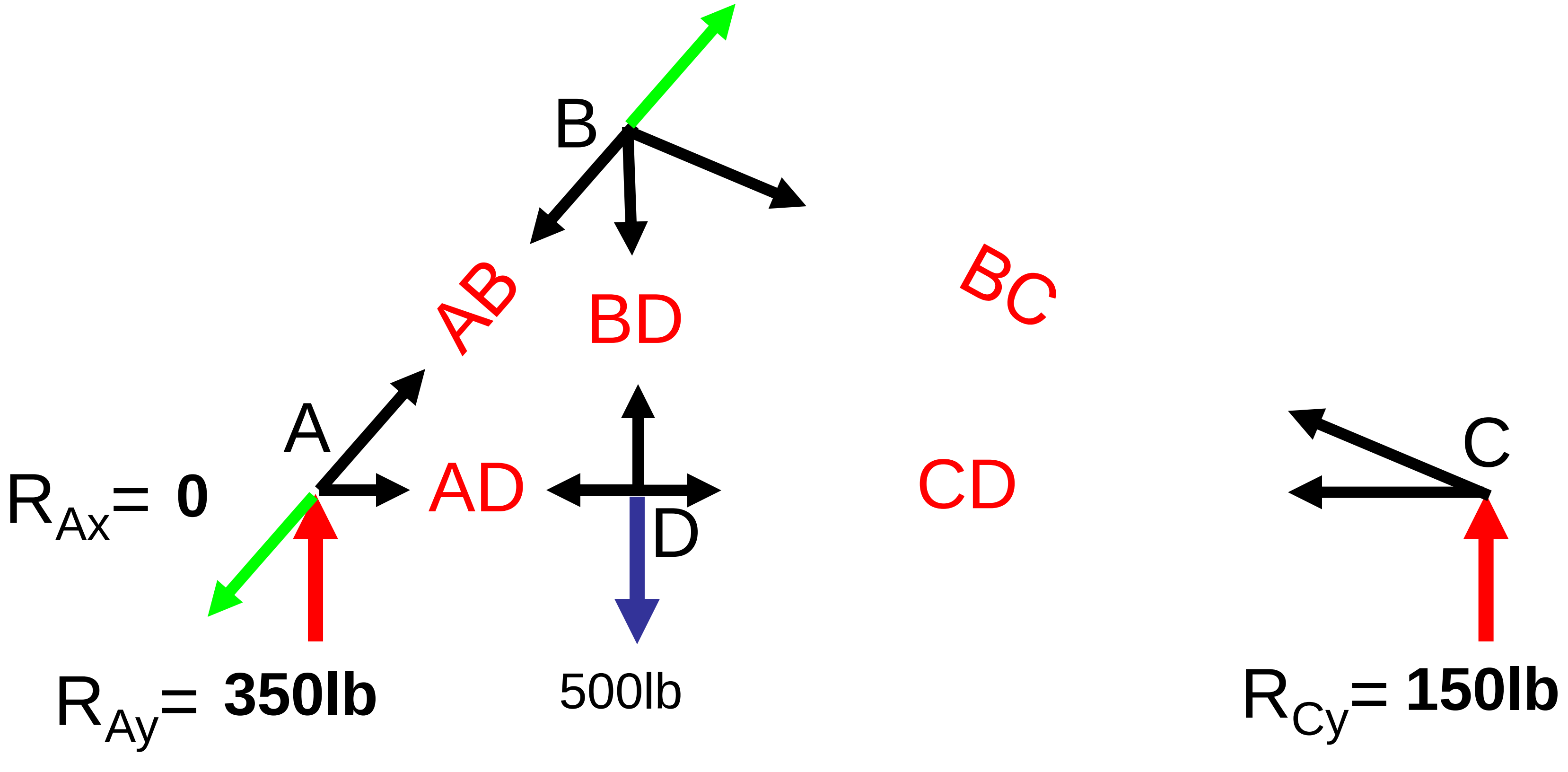
$$AB = \frac{-350lb}{\sin 53.1\underline{30}^\circ}$$

$$AB = -438\ lb$$

COMPRESSION

Method of Joints

Update the all force diagrams based on AB being under compression.

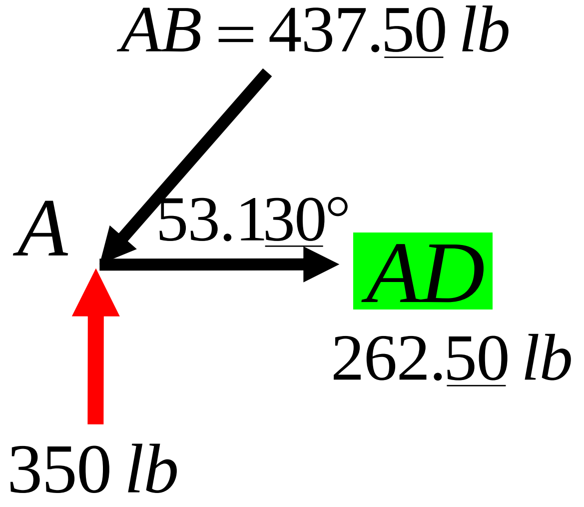


Method of Joints

$$\Sigma F_X = 0$$

$$-AB_x + AD = 0$$

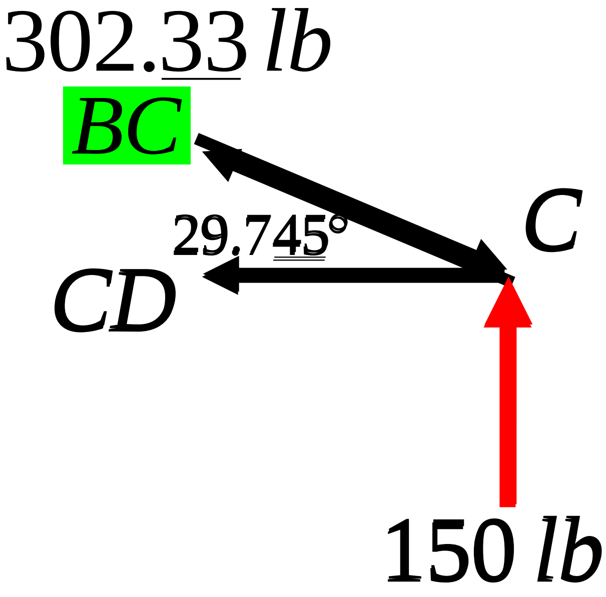
$$-437.\underline{50}\ lb \cos 53.1\underline{30}^\circ + AD = 0$$



$$AD = 437.\underline{50}\ lb \cos 53.1\underline{30}^\circ$$

$$AD = 262.50\ lb$$ **TENSION**

Method of Joints



$$\Sigma F_Y = 0$$

$$R_{Cy} + BC_y = 0$$

$$150\ lb + BC \sin 29.745^\circ = 0$$

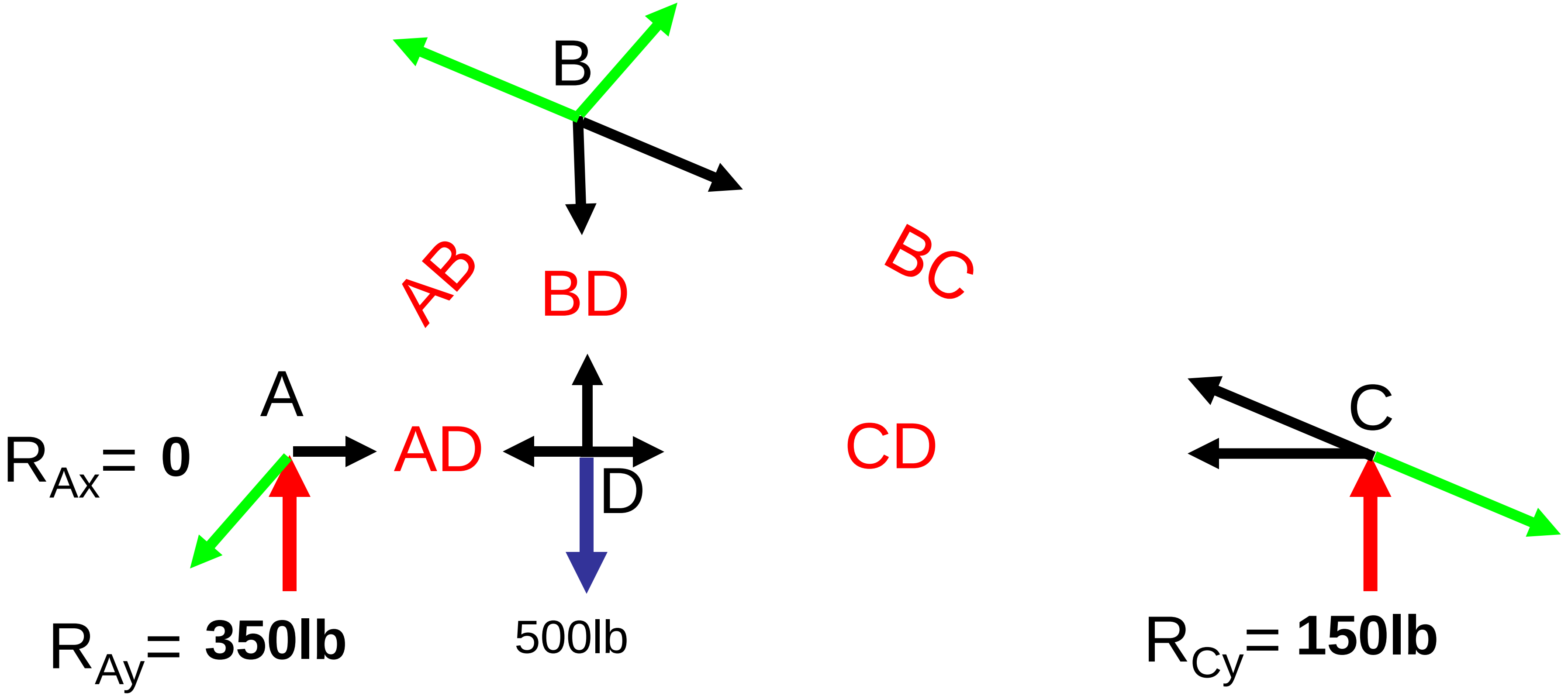
$$BC \sin 29.745^\circ = -150\ lb$$

$$BC = \frac{-150 lb}{\sin 29.745^\circ}$$

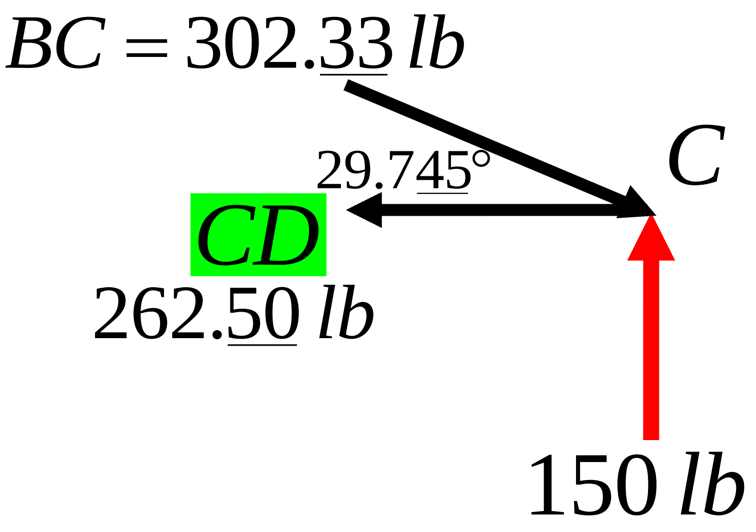
$$BC = -302\ lb$$ **COMPRESSION**

Method of Joints

Update the all force diagrams based on BC being under compression.



Method of Joints



$$\Sigma F_X = 0$$

$$BC_x - CD = 0$$

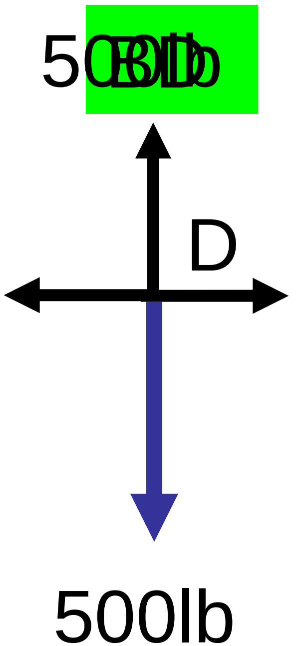
$$302.\underline{33}\,lb\cos 29.7\underline{45}^\circ - CD = 0$$

$$CD = 302.\underline{33}\,lb\cos 29.7\underline{45}^\circ$$

$$CD = 262.50\,lb$$

TENSION

Method of Joints



$$\Sigma F_Y = 0$$

$$BD - F_D = 0$$

$$BD - 500lb = 0$$

$$BD = 500lb$$ TENSION