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AP CALCULUS AB AND BC

Course Content

Based on the Understanding by Design® (Wiggins and McTighe) model, this course framework provides a clear and detailed description of the course requirements necessary for student success. The framework specifies what students must know, be able to do, and understand, with a focus on big ideas that encompass core principles, theories, and processes of the discipline. The framework also encourages instruction that prepares students for advanced coursework in mathematics or other fields engaged in modeling change (e.g., pure sciences, engineering, or economics) and for creating useful, reasonable solutions to problems encountered in an ever-changing world.

Big Ideas

The big ideas serve as the foundation of the course and allow students to create meaningful connections among concepts. They are often abstract concepts or themes that become threads that run throughout the course. Revisiting the big ideas and applying them in a variety of contexts allows students to develop deeper conceptual understanding. Below are the big ideas of the course and a brief description of each.

BIG IDEA 1: CHANGE (CHA)

Using derivatives to describe rates of change of one variable with respect to another or using definite integrals to describe the net change in one variable over an interval of another allows students to understand change in a variety of contexts. It is critical that students grasp the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus—a central idea in AP Calculus.

BIG IDEA 2: LIMITS (LIM)

Beginning with a discrete model and then considering the consequences of a limiting case allows us to model real-world behavior and to discover and understand important ideas, definitions, formulas, and theorems in calculus: for example, continuity, differentiation, integration, and series **BC ONLY**.

BIG IDEA 3: ANALYSIS OF FUNCTIONS (FUN)

Calculus allows us to analyze the behaviors of functions by relating limits to differentiation, integration, and infinite series and relating each of these concepts to the others.

UNITS

The course content is organized into commonly taught units. The units have been arranged in a logical sequence frequently found in many college courses and textbooks.

The eight units in AP Calculus AB and ten units in AP Calculus BC, and their weighting on the multiple-choice section of the AP Exam, are listed on the following page.

Pacing recommendations at the unit level and on the Course at a Glance provide suggestions for how teachers can teach the required course content and administer the Personal Progress Checks.

The suggested class periods are based on a schedule in which the class meets five days a week for 45 minutes each day. While these recommendations have been made to aid planning, teachers are of course free to adjust the pacing based on the needs of their students, alternate schedules (e.g., block scheduling), or their school's academic calendar.

TOPICS

Each unit is broken down into teachable segments called topics. The topic pages (starting on p. 35) contain the required content for each topic. Although most topics can be taught in one or two class periods, teachers should pace the course to suit the needs of their students and school.

Exam Weighting for the Multiple-Choice Section of the AP Exam

Units	Exam Weighting (AB)	Exam Weighting (BC)
Unit 1: Limits and Continuity	**10–12%**	**4–7%**
Unit 2: Differentiation: Definition and Fundamental Properties	**10–12%**	**4–7%**
Unit 3: Differentiation: Composite, Implicit, and Inverse Functions	**9–13%**	**4–7%**
Unit 4: Contextual Applications of Differentiation	**10–15%**	**6–9%**
Unit 5: Analytical Applications of Differentiation	**15–18%**	**8–11%**
Unit 6: Integration and Accumulation of Change	**17–20%**	**17–20%**
Unit 7: Differential Equations	**6–12%**	**6–9%**
Unit 8: Applications of Integration	**10–15%**	**6–9%**
Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions **BC ONLY**		**11–12%**
Unit 10: Infinite Sequences and Series **BC ONLY**		**17–18%**

Spiraling the Big Ideas

The following table shows how the big ideas spiral across units.

Big Ideas	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
	Limits and Continuity	Differentiation: Definition and Fundamental Properties	Differentiation: Composite, Implicit, and Inverse Functions	Contextual Applications of Differentiation	Analytical Applications of Differentiation
Change CHA	✓	✓		✓	
Limits LIM	✓	✓		✓	
Analysis of Functions FUN	✓	✓	✓		✓

Spiraling the Big Ideas *(cont'd)*

Big Ideas	Unit 6 Integration and Accumulation of Change	Unit 7 Differential Equations	Unit 8 Applications of Integration	Unit 9 Parametric Equations, Polar Coordinates, and Vector-Valued Functions BC ONLY	Unit 10 Infinite Sequences and Series BC ONLY
Change CHA	✓		✓	✓	
Limits LIM	✓				✓
Analysis of Functions FUN	✓	✓		✓	

Course at a Glance

Plan

The Course at a Glance provides a useful visual organization of the AP Calculus AB and AP Calculus BC curricular components, including:

- Sequence of units, along with approximate weighting and suggested pacing. Please note, pacing is based on 45-minute class periods, meeting five days each week for a full academic year.
- Progression of topics within each unit.
- Spiraling of the big ideas and mathematical practices across units.

Teach

MATHEMATICAL PRACTICES

Mathematical practices spiral throughout the course.

- 1 Implementing Mathematical Processes
- 2 Connecting Representations
- 3 Justification
- 4 Communication and Notation

BIG IDEAS

Big ideas spiral across topics and units.

- CHA Change
- LIM Limits
- FUN Analysis of Functions

BC ONLY

The purple shading represents BC only content.

Assess

Assign the Personal Progress Checks—either as homework or in class—for each unit. Each Personal Progress Check contains formative multiple-choice and free-response questions. The feedback from the Personal Progress Checks shows students the areas where they need to focus.

UNIT 1 Limits and Continuity

AP EXAM WEIGHTING	10–12% AB 4–7% BC
CLASS PERIODS	~22–23 AB ~13–14 BC

CHA 2	1.1	Introducing Calculus: Can Change Occur at an Instant?
LIM 2	1.2	Defining Limits and Using Limit Notation
LIM 2	1.3	Estimating Limit Values from Graphs
LIM 2	1.4	Estimating Limit Values from Tables
LIM 1	1.5	Determining Limits Using Algebraic Properties of Limits
LIM 1	1.6	Determining Limits Using Algebraic Manipulation
LIM 1	1.7	Selecting Procedures for Determining Limits
LIM 3	1.8	Determining Limits Using the Squeeze Theorem
LIM 2	1.9	Connecting Multiple Representations of Limits
LIM 3	1.10	Exploring Types of Discontinuities
LIM 3	1.11	Defining Continuity at a Point
LIM 1	1.12	Confirming Continuity over an Interval
LIM 1	1.13	Removing Discontinuities
LIM 3	1.14	Connecting Infinite Limits and Vertical Asymptotes
LIM 2	1.15	Connecting Limits at Infinity and Horizontal Asymptotes
FUN 3	1.16	Working with the Intermediate Value Theorem (IVT)

Personal Progress Check 1

Multiple-choice: ~45 questions
Free-response: 3 questions (partial)

UNIT 2 Differentiation: Definition and Basic Derivative Rules

AP EXAM WEIGHTING	10–12% AB 4–7% BC
CLASS PERIODS	~13–14 AB ~9–10 BC

CHA 2	2.1	Defining Average and Instantaneous Rates of Change at a Point
CHA 1 4	2.2	Defining the Derivative of a Function and Using Derivative Notation
CHA 1	2.3	Estimating Derivatives of a Function at a Point
FUN 3	2.4	Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist
FUN 1	2.5	Applying the Power Rule
FUN 1	2.6	Derivative Rules: Constant, Sum, Difference, and Constant Multiple
FUN LIM 1	2.7	Derivatives of cos x, sin x, e^x, and ln x
FUN 1	2.8	The Product Rule
FUN 1	2.9	The Quotient Rule
FUN 1	2.10	Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions

Personal Progress Check 2

Multiple-choice: ~30 questions
Free-response: 3 questions (partial)

NOTE: Partial versions of the free-response questions are provided to prepare students for more complex, full questions that they will encounter on the AP Exam.

UNIT 3 Differentiation: Composite, Implicit, and Inverse Functions

AP EXAM WEIGHTING	9–13% AB 4–7% BC
CLASS PERIODS	~10–11 AB ~8–9 BC

- FUN 1 — 3.1 The Chain Rule
- FUN 1 — 3.2 Implicit Differentiation
- FUN 3 — 3.3 Differentiating Inverse Functions
- FUN 1 — 3.4 Differentiating Inverse Trigonometric Functions
- FUN 1 — 3.5 Selecting Procedures for Calculating Derivatives
- FUN 1 — 3.6 Calculating Higher-Order Derivatives

Personal Progress Check 3

Multiple-choice: ~15 questions
Free-response: 3 questions
(partial/full)

UNIT 4 Contextual Applications of Differentiation

AP EXAM WEIGHTING	10–15% AB 6–9% BC
CLASS PERIODS	~10–11 AB ~6–7 BC

- CHA 1 — 4.1 Interpreting the Meaning of the Derivative in Context
- CHA 1 — 4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration
- CHA 2 — 4.3 Rates of Change in Applied Contexts Other Than Motion
- CHA 1 — 4.4 Introduction to Related Rates
- CHA 3 — 4.5 Solving Related Rates Problems
- CHA 1 — 4.6 Approximating Values of a Function Using Local Linearity and Linearization
- LIM 3 — 4.7 Using L'Hospital's Rule for Determining Limits of Indeterminate Forms

Personal Progress Check 4

Multiple-choice: ~15 questions
Free-response: 3 questions

UNIT 5 Analytical Applications of Differentiation

AP EXAM WEIGHTING	15–18% AB 8–11% BC
CLASS PERIODS	~15–16 AB ~10–11 BC

- FUN 3 — 5.1 Using the Mean Value Theorem
- FUN 3 — 5.2 Extreme Value Theorem, Global Versus Local Extrema, and Critical Points
- FUN 2 — 5.3 Determining Intervals on Which a Function Is Increasing or Decreasing
- FUN 3 — 5.4 Using the First Derivative Test to Determine Relative (Local) Extrema
- FUN 1 — 5.5 Using the Candidates Test to Determine Absolute (Global) Extrema
- FUN 2 — 5.6 Determining Concavity of Functions over Their Domains
- FUN 3 — 5.7 Using the Second Derivative Test to Determine Extrema
- FUN 2 — 5.8 Sketching Graphs of Functions and Their Derivatives
- FUN 2 — 5.9 Connecting a Function, Its First Derivative, and Its Second Derivative
- FUN 2 — 5.10 Introduction to Optimization Problems
- FUN 3 — 5.11 Solving Optimization Problems
- FUN 1 3 — 5.12 Exploring Behaviors of Implicit Relations

Personal Progress Check 5

Multiple-choice: ~35 questions
Free-response: 3 questions

UNIT 6 Integration and Accumulation of Change

AP EXAM WEIGHTING	17–20% AB 17–20% BC
CLASS PERIODS	~18–20 AB ~15–16 BC

- CHA 4 — 6.1 Exploring Accumulations of Change
- LIM 1 — 6.2 Approximating Areas with Riemann Sums
- LIM 2 — 6.3 Riemann Sums, Summation Notation, and Definite Integral Notation
- FUN 1 — 6.4 The Fundamental Theorem of Calculus and Accumulation Functions
- FUN 2 — 6.5 Interpreting the Behavior of Accumulation Functions Involving Area
- FUN 3 — 6.6 Applying Properties of Definite Integrals
- FUN 3 — 6.7 The Fundamental Theorem of Calculus and Definite Integrals
- FUN 4 — 6.8 Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation
- FUN 1 — 6.9 Integrating Using Substitution
- FUN 1 — 6.10 Integrating Functions Using Long Division and Completing the Square
- FUN 1 — 6.11 Integrating Using Integration by Parts BC ONLY
- FUN 1 — 6.12 Using Linear Partial Fractions BC ONLY
- LIM 1 — 6.13 Evaluating Improper Integrals BC ONLY
- FUN 1 — 6.14 Selecting Techniques for Antidifferentiation

Personal Progress Check 6

Multiple-choice:
- ~25 questions (AB)
- ~35 questions (BC)

Free-response: 3 questions

UNIT 7 Differential Equations

AP EXAM WEIGHTING	6–12% AB 6–9% BC
CLASS PERIODS	~8–9 AB ~9–10 BC

- FUN 2 — 7.1 Modeling Situations with Differential Equations
- FUN 3 — 7.2 Verifying Solutions for Differential Equations
- FUN 2 — 7.3 Sketching Slope Fields
- FUN 4 — 7.4 Reasoning Using Slope Fields
- FUN 1 — 7.5 Approximating Solutions Using Euler's Method BC ONLY
- FUN 1 — 7.6 Finding General Solutions Using Separation of Variables
- FUN 1 — 7.7 Finding Particular Solutions Using Initial Conditions and Separation of Variables
- FUN 3 — 7.8 Exponential Models with Differential Equations
- FUN 3 — 7.9 Logistic Models with Differential Equations BC ONLY

Personal Progress Check 7

Multiple-choice:
- ~15 questions (AB)
- ~20 questions (BC)

Free-response: 3 questions

UNIT 8 Applications of Integration

AP EXAM WEIGHTING	10–15% AB 6–9% BC
CLASS PERIODS	~19–20 AB ~13–14 BC

- CHA 1 — 8.1 Finding the Average Value of a Function on an Interval
- CHA 1 — 8.2 Connecting Position, Velocity, and Acceleration of Functions Using Integrals
- CHA 3 — 8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts
- CHA 4 — 8.4 Finding the Area Between Curves Expressed as Functions of x
- CHA 1 — 8.5 Finding the Area Between Curves Expressed as Functions of y
- CHA 2 — 8.6 Finding the Area Between Curves That Intersect at More Than Two Points
- CHA 3 — 8.7 Volumes with Cross Sections: Squares and Rectangles
- CHA 3 — 8.8 Volumes with Cross Sections: Triangles and Semicircles
- CHA 3 — 8.9 Volume with Disc Method: Revolving Around the x- or y-Axis
- CHA 2 — 8.10 Volume with Disc Method: Revolving Around Other Axes
- CHA 4 — 8.11 Volume with Washer Method: Revolving Around the x- or y-Axis
- CHA 2 — 8.12 Volume with Washer Method: Revolving Around Other Axes
- CHA 3 — 8.13 The Arc Length of a Smooth, Planar Curve and Distance Traveled BC ONLY

Personal Progress Check 8

Multiple-choice: ~30 questions
Free-response: 3 questions

UNIT 9 Parametric Equations, Polar Coordinates, and Vector-Valued Functions BC ONLY

AP EXAM WEIGHTING	N/A AB	11–12% BC
CLASS PERIODS	N/A AB	~10–11 BC

CHA 2	9.1	Defining and Differentiating Parametric Equations
CHA 1	9.2	Second Derivatives of Parametric Equations
CHA 1	9.3	Finding Arc Lengths of Curves Given by Parametric Equations
CHA 1	9.4	Defining and Differentiating Vector-Valued Functions
FUN 1	9.5	Integrating Vector-Valued Functions
FUN 1	9.6	Solving Motion Problems Using Parametric and Vector-Valued Functions
FUN 2	9.7	Defining Polar Coordinates and Differentiating in Polar Form
CHA 3	9.8	Find the Area of a Polar Region or the Area Bounded by a Single Polar Curve
CHA 3	9.9	Finding the Area of the Region Bounded by Two Polar Curves

Personal Progress Check 9

Multiple-choice: ~25 questions
Free-response: 3 questions

UNIT 10 Infinite Sequences and Series BC ONLY

AP EXAM WEIGHTING	N/A AB	17–18% BC
CLASS PERIODS	N/A AB	~17–18 BC

LIM 3	10.1	Defining Convergent and Divergent Infinite Series
LIM 3	10.2	Working with Geometric Series
LIM 3	10.3	The nth Term Test for Divergence
LIM 3	10.4	Integral Test for Convergence
LIM 3	10.5	Harmonic Series and p-Series
LIM 3	10.6	Comparison Tests for Convergence
LIM 3	10.7	Alternating Series Test for Convergence
LIM 3	10.8	Ratio Test for Convergence
LIM 3	10.9	Determining Absolute or Conditional Convergence
LIM 1	10.10	Alternating Series Error Bound
LIM 3 2	10.11	Finding Taylor Polynomial Approximations of Functions
LIM 1	10.12	Lagrange Error Bound
LIM 2	10.13	Radius and Interval of Convergence of Power Series
LIM 2	10.14	Finding Taylor or Maclaurin Series for a Function
LIM 3	10.15	Representing Functions as Power Series

Personal Progress Check 10

Multiple-choice: ~45 questions
Free-response: 3 questions