Roanoke County Public Schools

Geometry

Curriculum Guide

2019

Mathematics Curriculum Guide

Revised 2019. Available at www.rcps.us.

Acknowledgements

The following people have made tremendous contributions to the completion of this curriculum guide and all are appreciated.

Mike Bass
William Byrd High School

Terri Preston
Hidden Valley High School

Jennifer Sprouse
Cave Spring Middle School

Roanoke County Public Schools Administration

Dr. Ken Nicely
Superintendent

James Soltis
Director of Secondary Instruction

Dr. Rebecca Eastwood
Director of Elementary Instruction

Terry Hartley
Mathematics Supervisor

Preface

This guide will assist the mathematics teacher in preparing students for the challenges of the twenty-first century. As established by the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics*, educational goals for students are changing. A comprehensive and coherent set of mathematics standards for each and every student from prekindergarten through grade 12, *Principles and Standards* is the first set of rigorous, college and career readiness standards for the 21st century. Students should have many and varied experiences in their mathematical training to help them learn to value mathematics, become confident in their ability to do mathematics, become problem solvers, and learn to communicate and reason mathematically. This guide, along with the available division resources, VDOE resources, professional literature, alternative assessment methods, and in-service activities will assist the mathematics teacher in continuing to integrate these student goals into the curriculum.

Table of Contents

Introduction/General Comments

This curriculum guide follows the 2016 Virginia Geometry SOLs as adopted by the Virginia Department of Education. It is extremely important and required that the Sequence of Instruction and Pacing be followed as presented in this guide. Topic references from the textbook, *Virginia Geometry,* (2019 edition) by Houghton-Mifflin-Harcourt are listed in this curriculum guide in the Resources sections.

Students will take three formative assessments during the year (dates to be determined annually). Each teacher-designed test will assess skill levels of the SOLs as presented in the Sequence of Instruction and Pacing and the formative assessment blueprint. The data collected from the formative assessments will help teachers determine students' strengths and weaknesses, and inform instructional decisions.

The Mapping for Instruction is based on specified SOLs which are to be taught in the predetermined order. Note, some SOLs (or their parts) may be taught over multiple 9-week periods.

Refer to the Mathematics 2016 Standards of Learning Geometry Curriculum Framework during every lesson. It is located at the back of this guide. This will provide valuable information for the teacher (Understanding the Standard) and desired goals for instruction (Essential Knowledge and Skills). Examples of teaching techniques and strategies, definitions, and recommended manipulatives are included in the Curriculum Framework and on the VDOE website under Mathematics Instructional Plans (MIPs) http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/mip/index.shtml.

Resources Overview

Resources for all SOLs
IXL
PowerSchool
BrainPop
Pearsonsuccessnet.com
VDOE
Promethean Planet
Number Talks
Performance Tasks
RCPS Common Assessments
get2math

Sequence of Instruction and Pacing

First Nine Weeks			Second Nine Weeks			Third Nine Weeks			Fourth Nine Weeks		
SOL	**Instructional Focus**	**BLKS**	**SOL**	**Instructional Focus**	**BLKS**	**SOL**	**Instructional Focus**	**BLKS**	**SOL**	**Instructional Focus**	**BLKS**
	Definitions & notation Segment Relationships Angle Relationships	3	**G.6**	Triangle Congruence: - Corresponding parts - Direct proof - Coordinate methods	5	**G.8a**	Pythagorean Theorem: - determine if a right triangle exists - Solve practical problems	1	**G.11a**	Circles: - Central angle, inscribed angle - Measures of angles of inscribed quadrilaterals	1
G.4 aef	Constructions: - Segment congruent to a segment - Bisector of an angle - Angle congruent to an angle *Tested on FA 3.*	1	**G.6**	Overlapping Triangles - Direct proof	1	**G.8b**	Special Right Triangles: 45°-45°-90° 30°-60°-90° - Solve for missing lengths - Rationalize denominators	2	**G.11 ab**	Circles: - Tangents and circumscribed angles - find angle measures and segment lengths	1
G.3a	Midpoint, distance, and slope	1	**G.6**	Isosceles and equilateral triangles: - Theorems/properties - Solve for sides and angles	1	**G.8c**	Trigonometry: - Sine, cosine, and tangent ratios - Solve for missing side lengths and angle measures	2	**G.11b**	Segment relationships in circles: - chords, secants and tangents	1.5
G.3d	Translations, reflections, rotations in the coordinate plane	1	**G.4h**	Construction: - Equilateral triangle inscribed in a circle	.5	**G.8c**	Angles of elevation and depression application problems	1	**G.11a**	Angle relationships in circles: - chords, secants, and tangents	2
G.1a	Logic: - Conditionals - Converse, inverse, and contrapositive - Biconditionals	3	**SOL prep**	Special segments in triangles: - Perpendicular bisector - Angle bisector - Median and altitude - Midsegment	1	**G.10abc**	Convex polygons: - Sum of interior and exterior angles - Find measures of interior and exterior angles and number of sides of a regular polygon - Find angle measures of a regular polygon in a tessellation	2	**G.11 cd**	Length of Arcs Areas of Sectors	1
G.1c	Deductive reasoning: - Law of Detachment - Law of Syllogism - Law of Contrapositive - Counterexample	2	**G.5 abcd**	Triangle Inequalities: - Order the sides/angles - Determine if a triangle exists - Determine the range of the third side	2.5	**G.9**	Quadrilaterals: - Apply properties - Parallelograms, rhombi, squares, rectangles, trapezoids and isosceles trapezoids	4	**G.12**	Equations of Circles: - find the center of a circle - find length of radius and diameter - identify coordinates of points on the circle - determine equation of a circle	2

G.1b	Translate arguments into symbols Intro to proofs Properties of equality	1.5	**G.3cd**	Dilations and symmetry - scale factor - point and line symmetry	1	**G.9**	Quadrilateral Proofs: - Direct proof - Coordinate and algebraic methods (use distance, slope, and midpoint formulas)	2	**G.13**	Surface Area: prisms, cylinders, pyramids, and cones Lateral Area: cylinders, prisms, and regular pyramids Composite figures and determine missing measure	1
G.2b	Solve problems using angle relationships formed by parallel lines and transversals	2	**G.7 G.14 ad**	Similar Figures: - Proportional sides - Congruent angles	1	**G.4h**	Constructions: - Square in a circle - Regular hexagon in a circle	1	**G.13**	Volume: prisms, cylinders, pyramids and cones Composite figures and determine missing measure	1
G.2a	Prove lines parallel - given numerical or algebraic angle measures - use deductive proofs	1.5	**G.7**	Triangle Similarity: - Use angle and side relationships to prove similarity - Direct proof - Coordinate methods (distance and slope formulas)	3	**G.14**	Perimeter and area of figures: - Use formulas to find perimeters and areas	.5	**G.13**	Surface area and volume: spheres and hemispheres	1
G.4bcdg	Constructions: - Perpendicular bisector - Perpendicular to a line from point not on a line - Perpendicular to a line at a point on the line - A line parallel to a line through a point not on the line *Tested on FA 3.*	1	**G.7**	Similarity in Right Triangles - Geometric mean	1	**G.14abcd**	Similar figures (perimeter and area): - perimeter ratio - area ratio - describe how changes in one or more dimensions or measures effects other measures of figure	2	**G.14 abcd**	Similar solids (surface area and volume): - area ratio - volume ratio - describe how changes in one or more dimensions or measures effects other measures of figure/solid	1
G.3b	Use slopes to determine if lines are parallel, perpendicular, or neither	1.5	**G.7**	Proportions in Triangles: -Triangle proportionality theorem	.5						
	Remediation, Review, Assessment	4		Remediation, Review, Assessment	5		Remediation, Review, Assessment	5		Remediation, Review, Assessment	10
	Total Blocks	**22.5**		**Total Blocks**	**22.5**		**Total Blocks**	**22.5**		**Total Blocks**	**22.5**

DESMOS CALCULATOR

The Desmos Virginia Graphing Calculator will be used for instruction and assessment.

Mapping for Instruction - First Nine Weeks

SOLs

G.1 The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
- **a) identifying the converse, inverse, and contrapositive of a conditional statement;**
- **b) translating a short verbal argument into symbolic form; and**
- **c) determining the validity of a logical argument.**

G.2 The student will use the relationships between angles formed by two lines intersected by a transversal to
- **a) prove two or more lines are parallel; and**
- **b) solve problems, including practical problems, involving angles formed when parallel lines are intersected by a transversal.**

G.3 The student will solve problems involving symmetry and transformation. This will include
- **a) investigating and using formulas for determining distance, midpoint, and slope;**
- **b) applying slope to verify and determine whether lines are parallel or perpendicular;**
- **d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.**

G.4 The student will construct and justify the constructions of
- **a) a line segment congruent to a given line segment;**
- **b) the perpendicular bisector of a line segment;**
- **c) a perpendicular to a given line from a point not on the line;**
- **d) a perpendicular to a given line at a given point on the line;**
- **e) the bisector of a given angle,**
- **f) an angle congruent to a given angle;**
- **g) a line parallel to a given line through a point not on the line; and**

SOL	Instructional Focus	Vocabulary	Comments	Blocks
SOL prep	**Definitions and notation** **Segment relationships** **Angle relationships**	Point, line, plane, collinear points, coplanar, space, segment, ray, opposite rays, postulate, theorem, axiom, intersection, conjecture, coordinate, distance, segment addition, congruent, midpoint, segment bisector, angle, sides, vertex, interior, exterior, measure, protractor, acute, right, obtuse, straight, angle addition, adjacent angles, vertical angles, linear pair, complementary angles, supplementary angles, perpendicular, angle bisector	***May not need 3 days for this material.*	3
G.4aef	**Construct and justify:** **- Segment congruent to a segment** **- Bisector of an angle** **- Angle congruent to an angle**	Construct, compass, straight edge	*Constructions will be tested on FA 3.*	1
G.3a	**Midpoint, distance, and slope:** **- Determine coordinates of midpoint or endpoint** **- Determine slope of a line** **- Determine length of a line segment**	Midpoint, distance, and slope formulas, Pythagorean Theorem (distance formula is an application on this theorem)	***Find the endpoint as well as the midpoint of a segment using the midpoint formula.*	1
G.3d	**Translations, reflections, and rotations in the coordinate plane:** **Translations in the coordinate plane (slide)** **Reflection over any horizontal or vertical line or**	Translation, reflection, rotation, line of reflection, center of rotation, angle of rotation, isometry, image, preimage, origin, slide	***Transformations may be a combination of transformations.*	1

	the lines $y = x$ or $y = -x$ Clockwise or counterclockwise rotation of 90°, 180°, 270°, or 360° (center of rotation is origin)			
G.1a	**Logic: - Conditional statements - Identify converse, inverse, and contrapositive of a conditional statement - Biconditional statement**	Inductive reasoning, conjecture, pattern, counterexample, conditional statement, if → then, hypothesis, conclusion, negation, converse, inverse, contrapositive, truth value, equivalent statements, biconditional statement, "if and only if", "iff",		3
G.1c	**Deductive reasoning: - Law of Detachment - Law of Syllogism - Law of Contrapositive - Counterexample**	Deductive reasoning, law of detachment, law of syllogism, law of contrapositive, counterexample	***Emphasize using a counterexample to prove an argument is false.*	2
G.1b	**Translate logic arguments to symbolic form Introduction to proofs Properties of Equality**	Addition property, subtraction property, multiplication property, division property, reflexive property, symmetric property, transitive property, substitution property, distributive property, proof, two-column proof, theorem Symbols: →, ↔, ~, ∴, ∧, and ∨		1.5
G.2b	**Parallel lines and transversals: - Angle relationships - Solve problems using these relationships**	Parallel lines, transversals, parallel planes, skew lines, same-side (consecutive) interior angles, alternate interior angles, same-side (consecutive) exterior angles, alternate exterior angles, corresponding angles	***Emphasize same-side (consecutive) exterior angles relationship.*	2
G.2a	**Proving lines parallel: - Prove two or more lines are parallel given angle measurements numerically or algebraically - Prove two lines are parallel given angle relationships**	(see above)		1.5
G.4 bcdg	**Construct and justify: - Perpendicular bisector of a line segment - Perpendicular to a line from a point not on a line - Perpendicular to a line at a given point on a line - A line parallel to a line through a point not on a line**	(see above)	*Constructions will be tested on FA 3.*	1
G.3b	**Compare slopes to verify and determine if lines are parallel, perpendicular, or neither**	Slope, slope-intercept form, parallel lines, perpendicular lines, reciprocal Symbols: Parallel – ∥; perpendicular – ⊥		1.5
			Remediation, Review, Assessment	4
			Total blocks	**22.5**

Resources – First Nine Weeks			
SOL	**Textbook**	**Links**	**Supplemental Materials**
prep	HMH 5-7; 41; 146-148		
G.4 a-f	HMH 5-16, 17-26, 169-178, 237-244 Virginia SOL Success 244A-244B		See MIPs: G.4a-h - Constructions (Word) / PDF Version
G.3a	HMH 5-16, 179-184, 425-434, 435-444, 445-456, 457-464, 465-476		See MIPs: G.3ab - Slope with Desmos (Word) / PDF Version G.3a - Distance, Midpoint, and Slope (Word) / PDF Version
G.3d	HMH 27-38, 55-64, 65-76, 77-88, 113-122, 201-210, 211220, 221-230, 489-498		See MIPs: G.3d - Transformations (Word) / PDF Version
G.1a	HMH 161-168 Virginia SOL Success 48A-48H		See MIPs: G.1ab - Logic and Conditional Statements (Word) / PDF Version
G.1c	HMH 39-48, 159, 380, 391, 455 Virginia SOL Success 48A-48H, 48I-48L		See MIPs: G.1c - Deductive Reasoning (Word) / PDF Version G.1c - Inductive and Deductive Reasoning (Word) / PDF Version
G.1b	Virginia SOL Success 48A-48H, 48I-48L		See MIPs: G.1ab - Logic and Conditional Statements (Word) / PDF Version
G.2ab	HMH 153-160, 161-168, 464 Virginia SOL Success 160A-160C		See MIPs: G.2ab - Parallel Lines and Angle Relationships (Word) /PDF Version
G.4 b-d, g	HMH 169-178, 237-244 Virginia SOL Success 244A-244B		See MIPs: G.4a-h - Constructions (Word) / PDF Version

G.3b	HMH 425-434, 435-444, 445-456, 457-464, 465-476		See MIPs: G.3ab - Slope with Desmos (Word) /PDF Version

Mapping for Instruction - Second Nine Weeks

SOLs

G.3 The student will solve problems involving symmetry and transformation. This will include
- **c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and**
- **d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.**

G.4 The student will construct and justify the constructions of
- **h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle.**

G.5 The student, given information concerning the lengths of sides and/or measures of angles in triangles, will solve problems, including practical problems. This will include
- **a) ordering the sides by length, given angle measures;**
- **b) ordering the angles by degree measure, given side lengths;**
- **c) determining whether a triangle exists; and**
- **d) determining the range in which the length of the third side must lie.**

G.6 The student, given information in the form of a figure or statement, will prove two triangles are congruent.

G.7 The student, given information in the form of a figure or statement, will prove two triangles are similar.

G.14 The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include
- **a) comparing ratios between lengths, perimeters, areas, and volumes of similar figures;**
- **d) solving problems, including practical problems, about similar geometric figures.**

SOL	Instructional Focus	Vocabulary	Comments	Blocks
G.6	**Identify corresponding parts of polygons and triangles** **Prove Triangles Congruent:** **- SAS** **- SSS** **- AAS** **- ASA** **- HL** **- Use coordinate methods (slope and distance formula)** **- Use direct proof** **- CPCTC**	Congruent polygons, corresponding parts, acute triangle, right triangle, obtuse triangle, equilateral triangle, isosceles triangle, scalene triangle, similar figures Properties of Congruence: - Reflexive - Symmetric - Transitive Properties of Equality: (see 1st nine weeks)		5
G.6	**Overlapping Triangles** **- Use direct proof to prove overlapping triangles congruent**	Overlapping, common side, common angle		1
G.6	**Isosceles and equilateral triangles:** **- Theorems and properties** **- Solve for angles and sides**	Opposite sides, base angles, vertex angle, legs, base, equiangular triangle, equilateral triangle		1
G.4h	**Construct and justify:** **- Equilateral triangle inscribed in a circle**	inscribed		.5
SOL prep	**Special segments in triangles:** **- Perpendicular bisector**	Midsegment, perpendicular bisector, angle bisector, equidistant, median, altitude	***For Pre-AP Geometry add circumcenter, incenter,*	1

	- Angle bisector **- Median and altitude** **- Midsegment**		*centroid, and orthocenter.*	
G.5abcd	**Triangle Inequalities:** **- Order the sides when given angle measures** **- Order the angles when given the side lengths** **- Determine if a triangle exists given three segment lengths** **- Determine the range of the third side of a triangle given the lengths of two sides**	Triangle Inequality Theorem, opposite, included angle, range		2.5
G.3cd	**Dilations and symmetry** **- Apply scale factor** **- Point and line symmetry**	Dilation, scale factor, ratio, point symmetry, line symmetry, rotational symmetry		1
G.7 **G.14ad**	**Similar figures:** **- Corresponding parts** **- Proportional sides** **- Congruent angles**	Similar, ratio, extended ratio, proportion, extremes, means, cross product property, similar figures, scale factor		1
G.7	**Triangle Similarity:** **- SAS~** **- SSS~** **- AA~** **- Prove two triangles are similar using relationships among angles and sides, direct proof, and coordinate methods (distance and slope formulas)**	Angle-Angle Similarity (AA~), Side-Angle-Side Similarity (SAS~), Side-Side-Side Similarity (SSS~)		3
G.7	**Similarity in Right Triangles**	Geometric mean		1
G.7	**Triangle Proportionality Theorem**	"Side-Splitter Theorem"		.5
			Remediation, Review, Assessment	5
			Total blocks	**22.5**

Resources – Second Nine Weeks			
SOL	**Textbook**	**Links**	**Supplemental Materials**
G.6	HMH 91-98, 191-200, 201-210, 211-220, 221-230, 245254, 255-262, 445-456		See MIPs: G.6 - Congruent Triangles (Word) /PDF Version
G.5a-d	HMH 293-302		See MIPs: G.5a-d - How Many Triangles? (Word) /PDF Version
G.3c	HMH 89-96 Virginia SOL Success 96A-96		See MIPs: G.3c - Symmetry (Word) / PDF Version
G.3d	HMH 27-38, 55-64, 65-76, 77-88, 113-122, 201-210, 211-220, 221-230, 489-498		See MIPs: G.3d - Transformations (Word) /PDF Version
G.14a,d	HMH 511-518, 519-528, 537-546, 555-562, 867-874		See MIPs: G.14 - Similar Solids and Proportional Reasoning (Word) / PDF Version
G.7	HMH 499-510, 519-528 Virginia SOL Success 528A-528C		See MIPs: G.7 - Similar Triangles (Word) / PDF Version

This page intentionally left blank.

Mapping for Instruction - Third Nine Weeks

SOLs

G.4 The student will construct and justify the constructions of
- **h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle.**

G.8 The student will solve problems, including practical problems, involving right triangles. This will include applying
- **a) the Pythagorean Theorem and its converse;**
- **b) properties of special right triangles; and**
- **c) trigonometric ratios.**

G.9 The student will verify and use properties of quadrilaterals to solve problems, including practical problems.

G.10 The student will solve problems, including practical problems, involving angles of convex polygons. This will include determining the
- **a) sum of the interior and/or exterior angles;**
- **b) measure of an interior and/or exterior angle; and**
- **c) number of sides of a regular polygon.**

G.14 The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include
- **a) comparing ratios between lengths, perimeters, areas, and volumes of similar figures;**
- **b) determining how changes in one or more dimensions of a figure affect area and/or volume of the figure;**
- **c) determining how changes in area and/or volume of a figure affect one or more dimensions of the figure; and**
- **d) solving problems, including practical problems, about similar geometric figures.**

SOL	Instructional Focus	Vocabulary	Comments	Blocks
G.8a	**Pythagorean Theorem:** **- Given three side lengths, determine if a right triangle exists** **- Solve practical problems**	Legs, hypotenuse, $a^2 + b^2 = c^2$		1
G.8b	**Special Right Triangles** **45°-45°-90°** **30°-60°-90°** **- Solve for missing lengths** **- Rationalize denominators**	Rationalize the denominator, legs, hypotenuse, radicals	***Curriculum Framework specifically states that rationalizing denominators may now be necessary.*	2
G.8c	**Trigonometry:** **- Solve for missing side lengths and angle measures using trigonometric ratios (sine, cosine, tangent)**	Opposite, adjacent, sine, cosine, tangent, ratios		2
G.8c	**Angles of elevation and depression** **- Application problems**	Elevation, depression, line of sight, horizontal	***Teacher may have to supplement here.*	1
G.10abc	**Convex polygons:** **- Determine the sum of the measures of interior and exterior angles of a convex polygon** **- Determine the measure of each interior and exterior angle of a regular polygon** **- Determine the angle measures of a regular polygon in a tessellation**	Polygon interior angle sum, equilateral polygon, equiangular polygon, regular polygon, polygon exterior angle sum, tessellation, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon, decagon, dodecagon, n-gon	***The Curriculum Framework now explicitly states that students must determine angle measures of a regular polygon in a tessellation.*	2

	- Determine the number of sides of a regular polygon given the measure of an interior or exterior angle			
G.9	**Quadrilaterals:** **- Verify and apply properties of quadrilaterals to solve problems** **- Parallelogram, rectangle, rhombus, square, trapezoid, isosceles trapezoid**	Quadrilateral, parallelogram, opposite angles, opposite sides, diagonal, rhombus, rectangle, square, trapezoid, isosceles trapezoid, base angles, leg, base, midsegment of a trapezoid, symmetry, consecutive sides, perpendicular	***The kite is not a required figure, but may be introduced to students.*	4
G.9	**Quadrilateral Proofs** **- Prove quadrilaterals have specific properties using algebraic and coordinate methods (distance, slope, and midpoint formulas)** **- Use direct proofs to prove properties of quadrilaterals**	(see above)		2
G.4h	**Construct and Verify:** **- Square inscribed in a circle** **- Regular hexagon inscribed in a circle**			1
G.14	**Perimeter and area of figures** **- Use formulas to find perimeter and area of figures**	Area, base of a parallelogram, altitude (height) of a parallelogram, base of a triangle, altitude (height) of a triangle, altitude (height) of a trapezoid, radius of a regular polygon, apothem	**Pre-AP Geometry course must add area of regular polygons with apothem.*	.5
G.14abcd	**Similar figures (perimeter and area):** **- Determine perimeter ratio** **- Determine area ratio** **- Compare ratios between side lengths, perimeters, and areas of similar figures** **- Describe how changes in one or more dimensions or measures affect other measures of a figure** **- Solve real world problems**	Perimeter, perimeter ratio, area ratio		2
			Remediation, Review, Assessment	5
			Total blocks	**22.5**

Resources – Third Nine Weeks			
SOL	**Textbook**	**Links**	**Supplemental Materials**
G.8a	HMH 5-16, 169-178, 220, 255-262, 445-456, 563-572 Virginia SOL Success 572A-572C	Pythagorean Theorem Battleship https://www.quia.com/ba/81033.html	See MIPs: G.8a - The Pythagorean Relationship (Word) / PDF Version
G.8bc	HMH 585-592, 593-602, 603-612, 613-622	Trigonometry Angles https://campus.mangahigh.com/en-us/px/466/0/0	See MIPs: G.8bc - Special Right Triangles and Right Triangle Trigonometry (Word) /PDF Version
G.10abc	HMH 271-282 Virginia SOL Success 282A-282C		See MIPs: G.10a-c - Angles in Polygons (Word) / PDF Version
G.9	HMH 61-370, 371-382, 383-392, 393-402, 403-416, 445456 Virginia SOL Success 416A-416C, 464A-464C	Properties of quadrilaterals https://jeopardylabs.com/play/quadrilateral-properties-geometry	See MIPs: G.9 - Properties of Quadrilaterals (Word) / PDF Version
G.14abcd	HMH 511-518, 519-528, 537-546, 555-562, 867-874 Virginia SOL Success 874A-874C		See MIPs: G.14 - Similar Solids and Proportional Reasoning (Word) / PDF Version

This page intentionally left blank.

Mapping for Instruction - Fourth Nine Weeks

SOLs

G.11 The student will solve problems, including practical problems, by applying the properties of circles. This will include determining
- **a) angle measures formed by intersecting chords, secants, and/or tangents;**
- **b) lengths of segments formed by intersecting chords, secants, and/or tangents.**
- **c) arc length; and**
- **d) area of a sector.**

G.12 The student will solve problems involving equations of circles.

G.13 The student will use surface area and volume of three-dimensional objects to solve practical problems.

G.14 The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include
- **a) comparing ratios between lengths, perimeters, areas, and volumes of similar figures;**
- **b) determining how changes in one or more dimensions of a figure affect area and/or volume of the figure;**
- **c) determining how changes in area and/or volume of a figure affect one or more dimensions of the figure; and**
- **d) solving problems, including practical problems, about similar geometric figures.**

SOL	Instructional Focus	Vocabulary	Comments	Blocks
G.11a	**Circles:** **- Properties** **- Parts** **- Find angle measures from central and inscribed angles** **- Find measures of angles from quadrilaterals inscribed in a circle**	Circle, center, diameter, chord, radius, degree of a circle, central angle, semicircle, minor arc, major arc, circumference, pi, concentric circle, arc length, area of a circle, sector of a circle, sector area, inscribed angles, quadrilateral, secant, tangent line, point of tangency, intercepted arc, standard equation of a circle		1
G.11ab	**Circles:** **- Find angle measures and segment lengths using tangents and inscribed angles**	(see above)		1
G.11b	**Segment relationships in circles:** **- Find measures of segments formed by chords, secants, and tangents**	(see above)		1.5
G.11a	**Angle relationships in circles:** **- Find measures of angles formed by chords, secants, and tangents**	(see above)		2
G.11cd	**Find length of arcs** **Find areas of sectors**	(see above)		1

G.12	**Equations of Circles:** **- Given a graph or equation, find the center of a circle** **- Given the endpoints of a diameter, find the center of a circle** **- Given the graph or equation, find the radius and diameter** **- Given the endpoints of a diameter, find the radius and diameter** **- Given the center and a point, find the radius and the diameter** **- Given the center and the radius, find points on the circle** **- Write the equation of a circle given:** • **graph** • **center and a point on the circle** • **center and radius or diameter** • **endpoints of a diameter**	(see above)		2
G.13	**Find surface area of cylinders, prisms, pyramids, and cones** **Solve practical problems involving surface area of cylinders, prisms, pyramids, and cones and composite three dimensional figures** **Solve practical problems involving lateral area of circular cylinders, prisms, and regular pyramids** **Determine missing information when given side, area of face, or volume**	Face, edge, vertex, prism, perimeter of the base, right prism, oblique prism, base of a prism, lateral face of a prism, altitude (height) of a prism, lateral area of a prism, surface area of a prism, cylinder, right cylinder, oblique cylinder, base of a cylinder, lateral face of a cylinder, altitude (height) of a cylinder, lateral area of a cylinder, surface area of a cylinder, pyramid, right pyramid, base of a pyramid, lateral face of a pyramid, vertex of a pyramid, altitude (height) of a pyramid, slant height of a pyramid, lateral area of a pyramid, surface area of a pyramid, cone, right cone, base of a cone, altitude (height) of a cone, vertex of a cone, slant height of a cone, lateral area of a cone, surface area of a cone	***The Curriculum Framework now explicitly states that students must solve problems involving lateral area of cylinders, prisms, and regular pyramids.*	1
G.13	**Find volume of cylinders, prisms, pyramids, and cones** **Solve practical problems involving volume of cylinders, prisms, pyramids, and cones and composite three dimensional figures** **Determine missing information when given side, area of face, or volume**	Volume, composite figure		1

G.13	**Spheres and hemispheres:** **Find surface area and volume**	Sphere, radius of a sphere, diameter of a sphere, hemisphere	***The Curriculum Framework now explicitly states that students must determine surface area and volume of hemispheres.*	1
G.14abcd	**Similar solids (surface area and volume)** **- Determine area ratio** **- Determine volume ratio** **- Compare ratios between side lengths, perimeters, areas, and volumes of similar figures/solids** **- Describe how changes in one or more dimensions or measures affect other measures of a figure/solid** **- Solve real world problems**	(see above)		1
			Remediation, Review, Assessment	10
			Total blocks	**22.5**

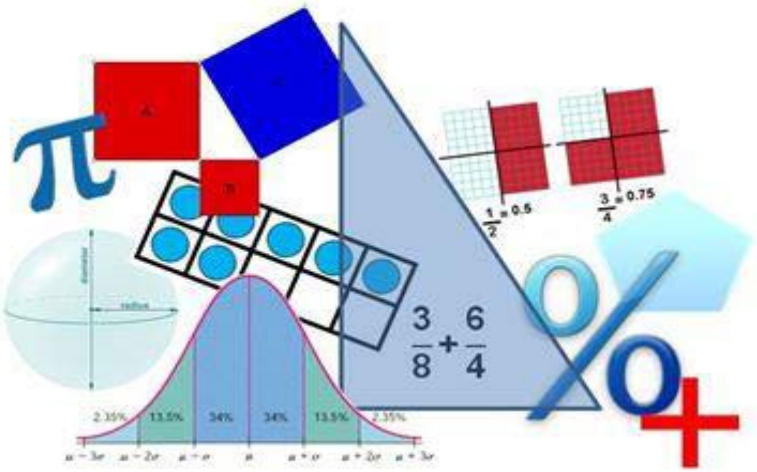
Resources – Fourth Nine Weeks			
SOL	**Textbook**	**Links**	**Supplemental Materials**
G.11 c,d	HMH 659-668, 729-736 737-744	Arc Length and Area of Sector https://campus.mangahigh.com/en-us/px/391/0/0	See MIPs: G.11cd - Arc Length and Area of a Sector (Word) / PDF Version
G.11 a,b	HMH 659-668, 669-678, 679-686, 687-698, 699-710		See MIPs: G.11ab - Angles, Arcs, and Segments (Word) / PDF Version
G.12	HMH 751-758 Virginia SOL Success 758A-758D		See MIPs: G.12 - Circles in the Coordinate Plane (Word) / PDF Version
G.13	HMH 779-790, 791-800, 801-810, 811-818, 833-842, 843854, 855-860	3Dimensional Scavenger Hunt http://www.learningwave.com/abmath/7cups/index.html	See MIPs: G.13 - Surface Area and Volume (Word) / PDF Version

Mathematics

2016 Standards of Learning

Geometry

Curriculum Framework



Board of Education
Commonwealth of Virginia

Superintendent of Public Instruction
Steven R. Staples

Chief Academic Officer/Assistant Superintendent for Instruction
Steven M. Constantino

Office of Mathematics and Governor's Schools
Debra Delozier, Mathematics Specialist
Tina Mazzacane, Mathematics and Science Specialist
Christa Southall, Mathematics Specialist

Acknowledgements
The Virginia Department of Education wishes to express sincere thanks to Michael Bolling, who assisted in the development of the 2016 *Mathematics Standards of Learning* and 2016 *Mathematics Standards of Learning Curriculum Framework*.

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills

This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "… the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

G.1 The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include

a) identifying the converse, inverse, and contrapositive of a conditional statement;

b) translating a short verbal argument into symbolic form; and

c) determining the validity of a logical argument.

Understanding the Standard	Essential Knowledge and Skills
• Inductive reasoning, deductive reasoning, and proof are critical in establishing general claims. • Deductive reasoning is the method that uses logic to draw conclusions based on definitions, postulates, and theorems. • Valid forms of deductive reasoning include the law of syllogism, the law of contrapositive, the law of detachment, and the identification of a counterexample. • Symbolic notation is used to represent logical arguments, including the use of $\rightarrow$, $\leftrightarrow$, $\sim$, $\therefore$, $\wedge$, and $\vee$. • The law of syllogism states that if $p \rightarrow q$ is true and $q \rightarrow r$ is true, then $p \rightarrow r$ is true. • The law of contrapositive states that if $p \rightarrow q$ is true and $\sim q$ is true, then $\sim p$ is true. • The law of detachment states that if $p \rightarrow q$ is true and p is true, then q is true. • A counterexample is used to show an argument is false. • Inductive reasoning is the method of drawing conclusions from a limited set of observations. • Proof is a justification that is logically valid and based on initial assumptions, definitions, postulates, theorems, and/or properties. • Logical arguments consist of a set of premises or hypotheses and a conclusion.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the converse, inverse, and contrapositive of a conditional statement. (a) • Translate verbal arguments into symbolic form using the symbols of formal logic. (b) • Determine the validity of a logical argument using valid forms of deductive reasoning. (c) • Determine that an argument is false using a counterexample. (c)

G.1 The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include

a) identifying the converse, inverse, and contrapositive of a conditional statement;

b) translating a short verbal argument into symbolic form; and

c) determining the validity of a logical argument.

Understanding the Standard	Essential Knowledge and Skills
• When a conditional ($p \rightarrow q$) and its converse ($q \rightarrow p$) are true, the statements can be written as a biconditional, *p iff q*; or *p if and only if q*; or $p \leftrightarrow q$. • Logical arguments that are valid may not be true. Truth and validity are not synonymous. • Exploration of the representation of conditional statements using Venn diagrams may assist in deepening student understanding. • Formal proofs utilize symbols of formal logic to determine validity of a logical argument.	

G.2 The student will use the relationships between angles formed by two lines intersected by a transversal to

a) prove two or more lines are parallel; and

b) solve problems, including practical problems, involving angles formed when parallel lines are intersected by a transversal.

Understanding the Standard	Essential Knowledge and Skills
• Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods. • Parallel lines intersected by a transversal form angles with specific relationships. • Some angle relationships may be used when proving two lines intersected by a transversal are parallel. • If two parallel lines are intersected by a transversal, then: – corresponding angles are congruent; – alternate interior angles are congruent; – alternate exterior angles are congruent; – same-side (consecutive) interior angles are supplementary; and – same-side (consecutive) exterior angles are supplementary. • Deductive proofs can be used to show that two or more lines are parallel. • The construction of the line parallel to a given line through a point not on the line can be justified using the angle relationships formed when two lines are intersected by a transversal.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Prove two or more lines are parallel given angle measurements expressed numerically or algebraically. (a) • Prove two lines are parallel using deductive proofs given relationships between and among angles. (a) • Solve problems by using the relationships between pairs of angles formed by the intersection of two parallel lines and a transversal including corresponding angles, alternate interior angles, alternate exterior angles, same-side (consecutive) interior angles, and same-side (consecutive) exterior angles. (b) • Solve problems, including practical problems, involving intersecting and parallel lines. (b)

G.3 The student will solve problems involving symmetry and transformation. This will include

a) investigating and using formulas for determining distance, midpoint, and slope;

b) applying slope to verify and determine whether lines are parallel or perpendicular;

c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and

d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

Understanding the Standard	Essential Knowledge and Skills
• Symmetry and transformations can be explored with computer software, paper folding, and coordinate methods. • The distance formula is an application of the Pythagorean Theorem. • Geometric figures can be represented in the coordinate plane. • Parallel lines have the same slope. • The product of the slopes of perpendicular lines is −1 unless one of the lines has an undefined slope. • A transformation of a figure, called a preimage, changes the size, shape, and/or position of the figure to a new figure called the image. • Transformations and combinations of transformations can be used to describe movement of objects in a plane. • The image of an object or function graph after an isometric transformation is congruent to the preimage of the object. – A rotation is an isometric transformation in which an image is formed by rotating the preimage about a point called the center of rotation. The center of rotation may or may not be on the preimage. Rotations may be more than 180°. – A reflection is an isometric transformation in which an image is formed by reflecting the preimage over a line called the line of reflection. All corresponding points in the image are equidistant from the line of reflection.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the coordinates of the midpoint or endpoint of a segment, using the midpoint formula. (a) • Use a formula to determine the slope of a line. (a) • Apply the distance formula to determine the length of a line segment when given the coordinates of the endpoints. (a) • Compare the slopes to determine whether two lines are parallel, perpendicular, or neither. (b) • Determine whether a figure has point symmetry, line symmetry, both, or neither. (c) • Given an image and preimage, identify the transformation or combination of transformations that has/have occurred. Transformations include: – a translation; – a reflection over any horizontal or vertical line or the lines $y = x$ or $y = -x$; – a clockwise or counter clockwise rotation of 90°, 180°, 270°, or 360° on a coordinate grid where the center of rotation is limited to the origin; and – a dilation from a fixed point on a coordinate grid. (d)

G.3 The student will solve problems involving symmetry and transformation. This will include

a) investigating and using formulas for determining distance, midpoint, and slope;

b) applying slope to verify and determine whether lines are parallel or perpendicular;

c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and

d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

Understanding the Standard	Essential Knowledge and Skills
– A translation is an isometric transformation in which an image is formed by moving every point on the preimage the same distance in the same direction. • A dilation is a transformation in which an image is formed by enlarging or reducing the preimage proportionally by a scale factor from the center of dilation. The center of dilation may or may not be on the preimage. The image is similar to the preimage.	

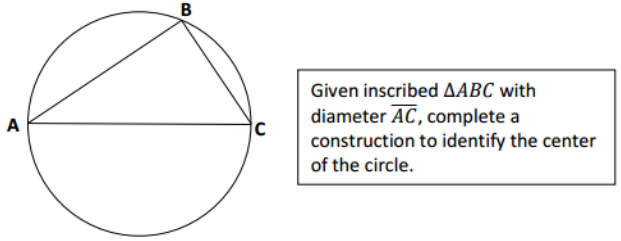
G.4 The student will construct and justify the constructions of

a) a line segment congruent to a given line segment;
b) the perpendicular bisector of a line segment;
c) a perpendicular to a given line from a point not on the line;
d) a perpendicular to a given line at a given point on the line;
e) the bisector of a given angle;
f) an angle congruent to a given angle;
g) a line parallel to a given line through a point not on the line; and
h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Understanding the Standard	Essential Knowledge and Skills
• Construction techniques are used to solve practical problems in engineering, architectural design, and building construction. • Construction techniques include using a straightedge and compass, paper folding, and dynamic geometry software. • Geometric constructions assist in justifying, verifying, and visually reinforcing geometric relationships. • There are multiple methods to most geometric constructions. Students would benefit from experiences with more than one method and should be able to justify each step of geometric constructions. • Individual steps of constructions can be justified using angle relationships, properties of quadrilaterals, congruent triangles, and/or circles. – The construction for a line segment congruent to a given line segment can be justified using properties of a circle. – The construction for the perpendicular bisector of a line segment can be justified using the properties of quadrilaterals or congruent triangles. – The constructions for a perpendicular to a given line from a point on, or not on, the line can be justified using the properties of quadrilaterals or congruent triangles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct and justify the constructions of – a line segment congruent to a given line segment; (a) – the perpendicular bisector of a line segment; (b) – a perpendicular to a given line from a point not on the line; (c) – a perpendicular to a given line at a given point on the line; (d) – the bisector of a given angle; (e) – an angle congruent to a given angle; (f) – a line parallel to a given line through a point not on the given line; (g) and – an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (h)

G.4 The student will construct and justify the constructions of

a) a line segment congruent to a given line segment;
b) the perpendicular bisector of a line segment;
c) a perpendicular to a given line from a point not on the line;
d) a perpendicular to a given line at a given point on the line;
e) the bisector of a given angle;
f) an angle congruent to a given angle;
g) a line parallel to a given line through a point not on the line; and
h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Understanding the Standard	Essential Knowledge and Skills
– The constructions for the bisector of a given angle and an angle congruent to a given angle can be justified using the properties of quadrilaterals or congruent triangles. – The construction for a line parallel to a given line through a point not on the line can be justified using the angle relationships formed when two lines are intersected by a transversal. – The constructions for an equilateral triangle, square, or regular hexagon inscribed in a circle can be justified using properties of circles. • Constructions can be completed within the context of complex figures. Example:  Given inscribed ΔABC with diameter $\overline{AC}$, complete a construction to identify the center of the circle.	

G.5 The student, given information concerning the lengths of sides and/or measures of angles in triangles, will solve problems, including practical problems. This will include

a) ordering the sides by length, given angle measures;
b) ordering the angles by degree measure, given side lengths;
c) determining whether a triangle exists; and
d) determining the range in which the length of the third side must lie.

Understanding the Standard	Essential Knowledge and Skills
• The longest side of a triangle is opposite the largest angle of the triangle and the shortest side is opposite the smallest angle. • In a triangle, the lengths of two sides and the included angle determine the length of the side opposite the angle. • In order for a triangle to exist, the length of each side must be within a range that is determined by the lengths of the other two sides.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given information about the lengths of sides and/or measures of angles in triangles, solve problems, including practical problems. (a, b, c, d) • Order the sides of a triangle by their lengths when given information about the measures of the angles. (a) • Order the angles of a triangle by their measures when given information about the lengths of the sides. (b) • Given the lengths of three segments, determine whether a triangle could be formed. (c) • Given the lengths of two sides of a triangle, determine the range in which the length of the third side must lie. (d)

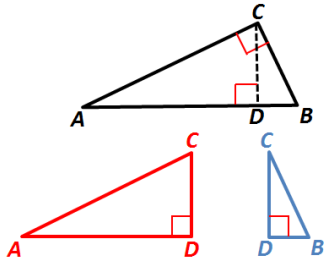
G.6 The student, given information in the form of a figure or statement, will prove two triangles are congruent.

Understanding the Standard	Essential Knowledge and Skills
• Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods. • Congruence has practical applications in a variety of areas, including art, architecture, and the sciences. • Congruence does not depend on the position of the triangles. • Congruent triangles are a result of rigid isometric transformations. • Concepts of logic can demonstrate congruence or similarity. • Congruent figures are also similar, but similar figures are not necessarily congruent. • Corresponding parts of congruent triangles are congruent. • Two triangles can be proven congruent using the following criterion: – Side-Angle-Side (SAS); – Side-Side-Side (SSS); – Angle-Angle-Side (AAS); and – Angle-Side-Angle (ASA). • Two right triangles can be proven congruent using the criteria Hypotenuse-Leg (HL). • Triangle congruency can be explored using geometric constructions such as an angle congruent to a given angle or a line segment congruent to a given line segment. • The construction for the bisector of a given angle can be justified using congruent triangles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Prove two triangles congruent given relationships among angles and sides of triangles expressed numerically or algebraically. • Prove two triangles congruent given representations in the coordinate plane and using coordinate methods (distance formula and slope formula). • Use direct proofs to prove two triangles congruent.

G.6 The student, given information in the form of a figure or statement, will prove two triangles are congruent.

Understanding the Standard	Essential Knowledge and Skills
• The construction for an angle congruent to a given angle can be justified using congruent triangles. • The construction of the perpendicular to a given line from a point on the line can be justified using congruent triangles. • The construction of the perpendicular to a given line from a point not on the line can be justified using congruent triangles.	

G.7 The student, given information in the form of a figure or statement, will prove two triangles are similar.

Understanding the Standard	Essential Knowledge and Skills
• Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods. • Similarity has practical applications in a variety of areas, including art, architecture, and the sciences. • Similarity does not depend on the position of the triangles. • Similar triangles are created using dilations. • Congruent figures are also similar, but similar figures are not necessarily congruent. • Corresponding sides of similar triangles are proportional. • Corresponding angles of similar triangles are congruent. • The altitude in a right triangle creates three similar right triangles.  $\Delta ABC \sim \Delta CBD$ $\Delta ABC \sim \Delta ACD$ $\Delta CBD \sim \Delta ACD$ • Two triangles can be proven similar using the following criterion: – Side-Angle-Side (SAS); – Side-Side-Side (SSS); and – Angle-Angle (AA).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Prove two triangles similar given relationships among angles and sides of triangles expressed numerically or algebraically. • Prove two triangles similar given representations in the coordinate plane and using coordinate methods (distance formula and slope formula). • Use direct proofs to prove triangles similar.

G.8 The student will solve problems, including practical problems, involving right triangles. This will include applying

a) the Pythagorean Theorem and its converse;

b) properties of special right triangles; and

c) trigonometric ratios.

Understanding the Standard	Essential Knowledge and Skills
• The converse of the Pythagorean Theorem can be used to determine if a triangle is a right triangle. • 45°-45°-90° and 30°-60°-90° triangles are special right triangles because their side lengths can be specified as exact values using radicals rather than decimal approximations. • The sine of an acute angle in a right triangle is equal to the cosine of its complement.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve problems, including practical problems, using right triangle trigonometry and properties of special right triangles. (a, b, c) • Determine whether a triangle formed with three given lengths is a right triangle. (a) • Solve for missing lengths in geometric figures, using properties of 45°-45°-90° triangles where rationalizing denominators may be necessary. (b) • Solve for missing lengths in geometric figures, using properties of 30°-60°-90° triangles where rationalizing denominators may be necessary. (b). • Solve problems, including practical problems, involving right triangles with missing side lengths or angle measurements, using sine, cosine, and tangent ratios. (c)

G.9 The student will verify and use properties of quadrilaterals to solve problems, including practical problems.

Understanding the Standard	Essential Knowledge and Skills
• Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods. • Quadrilaterals have a hierarchical nature based on the relationships between their sides, angles, and diagonals. • Properties of quadrilaterals can be used to identify the quadrilateral and to determine the measures of sides and angles. • Given coordinate representations of quadrilaterals, the distance, slope, and midpoint formulas may be used to verify that quadrilaterals have specific properties. • The angle relationships formed when parallel lines are intersected by a transversal can be used to prove properties of quadrilaterals. • Congruent triangles can be used to prove properties of quadrilaterals. • A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Properties of a parallelogram include the following: – opposite sides are congruent; – opposite angles are congruent; – consecutive angles are supplementary; and – diagonals bisect each other. • A rectangle is a quadrilateral with four right angles. Properties of rectangle include the following: – opposite sides are parallel and congruent; and – diagonals are congruent and bisect each other.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve problems, including practical problems, using the properties specific to parallelograms, rectangles, rhombi, squares, isosceles trapezoids, and trapezoids. • Prove that quadrilaterals have specific properties, using coordinate and algebraic methods, such as the distance formula, slope, and midpoint formula. • Prove the properties of quadrilaterals, using direct proofs.

G.9 The student will verify and use properties of quadrilaterals to solve problems, including practical problems.

Understanding the Standard	Essential Knowledge and Skills
• A rhombus is a quadrilateral with four congruent sides. Properties of a rhombus include the following: – all sides are congruent; – opposite sides are parallel; – opposite angles are congruent; – diagonals are perpendicular bisectors of each other; – diagonals bisect opposite angles; and – diagonals divide the rhombus into four congruent right triangles. • A square is a quadrilateral that is a regular polygon with four congruent sides and four right angles. Properties of a square include the following: – opposite sides are parallel; – diagonals are congruent; – diagonals are perpendicular bisectors of each other; and – diagonals divide the square into four congruent 45°-45°-90° triangles. • A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called bases. The nonparallel sides of a trapezoid are called legs. • An isosceles trapezoid has the following properties: – nonparallel sides are congruent; – diagonals are congruent; and – base angles are congruent. • The construction of the perpendicular bisector of a line segment can be justified using the properties of quadrilaterals.	

G.9 The student will verify and use properties of quadrilaterals to solve problems, including practical problems.

Understanding the Standard	Essential Knowledge and Skills
• The construction of the perpendicular to a given line from a point on, or not on, the line can be justified using the properties of quadrilaterals. • The construction of the perpendicular to a given line from a point on the line can be justified using the properties of quadrilaterals. • The construction of a bisector of a given angle can be justified using the properties of quadrilaterals. • The construction of a square inscribed in a circle can be justified using the properties of squares.	

G.10 The student will solve problems, including practical problems, involving angles of convex polygons. This will include determining the

a) sum of the interior and/or exterior angles;
b) measure of an interior and/or exterior angle; and
c) number of sides of a regular polygon.

Understanding the Standard	Essential Knowledge and Skills
• In convex polygons, each interior angle has a measure less than 180°. • In concave polygons, one or more interior angles have a measure greater than 180°. • Two intersecting lines form angles with specific relationships. • An exterior angle is formed by extending a side of a polygon. • The exterior angle and the corresponding interior angle form a linear pair. • The sum of the measures of the interior angles of a convex polygon may be found by dividing the interior of the polygon into nonoverlapping triangles. • Both regular and nonregular polygons can tessellate the plane. • A regular polygon will tessellate the plane if the measure of an interior angle is a factor of 360. • The sum of the measures of the angles around a point in a tessellation is 360°. • Tessellations can be found in art, construction and nature.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve problems, including practical problems, involving angles of convex polygons. (a, b, c) • Determine the sum of the measures of the interior and exterior angles of a convex polygon. (a) • Determine the measure of each interior and exterior angle of a regular polygon. (b) • Determine angle measures of a regular polygon in a tessellation. (b) • Determine the number of sides of a regular polygon, given the measures of interior or exterior angles of the polygon. (c)

G.11 The student will solve problems, including practical problems, by applying properties of circles. This will include determining

a) angle measures formed by intersecting chords, secants, and/or tangents;

b) lengths of segments formed by intersecting chords, secants, and/or tangents;

c) arc length; and

d) area of a sector.

Understanding the Standard	Essential Knowledge and Skills
• All circles are similar. • A chord is a line segment that joins any two points on a circle. A chord is a segment of a secant. • Arcs can be measured in degrees or in units of length. • Applications of the properties of circles may be drawn from architecture, art, and construction. • Properties of circles can be verified using deductive reasoning, algebraic, and coordinate methods. • Inscribed quadrilaterals have opposite angles that are supplementary. • Properties associated with segment lengths can be verified using similar triangles. • The ratio of the central angle to 360° is proportional to the ratio of the arc length to the circumference of the circle. • The ratio of the central angle to 360° is proportional to the ratio of the area of the sector to the area of the circle. • The construction for an inscribed equilateral triangle, square and regular hexagon can be justified using properties of a circle.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve problems, including practical problems, by applying properties of circles. (a, b, c, d) • Determine angle measures and arc measures associated with – two intersecting chords; – two intersecting secants; – an intersecting secant and tangent; – two intersecting tangents; and – central and inscribed angles. (a) • Determine segment lengths associated with: – two intersecting chords; – two intersecting secants; – an intersecting secant and tangent; and – two intersecting tangents. (b) • Calculate the length of an arc of a circle. (c) • Calculate the area of a sector. (d)

G.12 The student will solve problems involving equations of circles.

Understanding the Standard	Essential Knowledge and Skills
• A circle is a locus of points equidistant from a given point, the center. • The distance between any point on the circle and the center is the length of the radius. • Standard form for the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where the coordinates of the center of the circle are (h, k) and r is the length of the radius. • The equation of a circle gives the coordinates of every point, (x, y), on the circle. • The midpoint formula and distance formula are important when determining the equation of a circle. • The equation of a circle with a given center and radius can be derived using the Pythagorean Theorem. • The midpoint of the diameter is the center of the circle.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a graph or the equation of a circle in standard form, identify the coordinates of the center of the circle. • Given the coordinates of the endpoints of a diameter of a circle, determine the coordinates of the center of the circle. • Given a graph or the equation of a circle in standard form, identify the length of the radius or diameter of the circle. • Given the coordinates of the endpoints of the diameter of a circle, determine the length of the radius or diameter of the circle. • Given the coordinates of the center and the coordinates of a point on the circle, determine the length of the radius or diameter of the circle. • Given the coordinates of the center and length of the radius of a circle, identify the coordinates of a point(s) on the circle. • Determine the equation of a circle given: – a graph of a circle with a center with coordinates that are integers; – coordinates of the center and a point on the circle; – coordinates of the center and the length of the radius or diameter; or – coordinates of the endpoints of a diameter.

G.13 The student will use surface area and volume of three-dimensional objects to solve practical problems.

Understanding the Standard	Essential Knowledge and Skills
• A cylinder is a solid figure formed by two congruent parallel faces called bases joined by a curved surface. In this course, cylinders are limited to right circular cylinders. • A cone is a solid figure formed by a face called a base that is joined to a vertex (apex) by a curved surface. In this course, cones are limited to right circular cones. • A prism is a polyhedron that has a congruent pair of parallel bases and faces that are parallelograms. In this course, prisms are limited to right prisms with bases that are triangles, rectangles, or regular hexagons. • A rectangular prism is a polyhedron in which all six faces are rectangles. • A triangular prism is a polyhedron that has a congruent pair of parallel triangular bases and faces that are parallelograms. • A pyramid is a polyhedron with a base that is a polygon and three or more faces that are triangles with a common vertex. • A regular pyramid is a pyramid with a base that is a regular polygon. In this course, pyramids are limited to right regular pyramids with triangular, square, or hexagonal bases. • A regular polygon has congruent sides and congruent interior angles. • Subdivision of polygons may assist in determining the area of regular polygons. • The surface area of a prism or pyramid is the sum of the areas of all its faces. • The surface area of a cylinder, cone, or hemisphere is the sum of the areas of the curved surface and bases. • The surface area of a sphere is the area of the curved surface.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the surface area of cylinders, prisms, pyramids, cones, hemispheres, and spheres, using the appropriate formulas. • Determine the volume of cylinders, prisms, pyramids, cones, hemispheres, and spheres, using the appropriate formulas. • Solve problems including practical problems, involving surface area and volume of cylinders, prisms, pyramids, cones, hemispheres, and spheres, as well as composite three-dimensional figures. • Solve problems, including practical problems, involving the lateral area of circular cylinders, prisms, and regular pyramids. • Given information about a three-dimensional figure such as length of a side, area of a face, or volume, determine missing information.

G.13 The student will use surface area and volume of three-dimensional objects to solve practical problems.

Understanding the Standard	Essential Knowledge and Skills
• The lateral area of a cylinder is the area of the curved surface of the cylinder, not including the parallel bases. • The lateral area of a rectangular-based prism is the sum of the areas of all faces, not including the parallel bases. • The lateral area of a triangular-based prism is the sum of the areas of all faces, not including the triangular-shaped, parallel bases. • The volume of a three-dimensional figure is the number of unit cubes that would fill the figure. • Composite figures consist of two or more three-dimensional figures. The surface area of a composite figure may not be equal to the sum of the surface areas of the individual figures. • Volume and surface area of spheres, cones and cylinders should be considered in terms of π or as a decimal approximation. • Calculators may be used to determine decimal approximations for results.	

G.14 The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include

a) comparing ratios between lengths, perimeters, areas, and volumes of similar figures;

b) determining how changes in one or more dimensions of a figure affect area and/or volume of the figure;

c) determining how changes in area and/or volume of a figure affect one or more dimensions of the figure; and

d) solving problems, including practical problems, about similar geometric figures.

Understanding the Standard	Essential Knowledge and Skills
• A change in one dimension of a figure results in predictable changes in area and/or volume. The resulting figure may or may not be similar to the original figure. • A constant ratio, the scale factor, exists between corresponding dimensions of similar figures. • If the ratio between dimensions of similar figures is a:b then: – The ratio of their areas is a^2:b^2. – The ratio of their volumes is a^3:b^3. • Proportional reasoning is important when comparing attribute measures in similar figures.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare ratios between side lengths, perimeters, areas, and volumes, given two similar figures. (a) • Describe how changes in one or more dimensions affect other derived measures (perimeter, area, surface area, and volume) of a figure. (b) • Describe how changes in one or more measures (perimeter, area, surface area, and volume) affect other measures of a figure. (c) • Solve real-world problems involving measured attributes of similar figures. (d)