Roanoke County Public Schools

AMP 6

(Accelerated Math Program 6)

CURRICULUM GUIDE

2019

Mathematics Curriculum Guide

Revised 2019. Available at www.rcps.us.

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Superintendent	Director of Secondary Instruction	Director of Elementary Instruction	Mathematics Supervisor

Preface

This guide will assist the mathematics teacher in preparing students for the challenges of the twenty-first century. As established by the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics*, educational goals for students are changing. A comprehensive and coherent set of mathematics standards for each and every student from prekindergarten through grade 12, *Principles and Standards* is the first set of rigorous, college and career readiness standards for the 21st century. Students should have many and varied experiences in their mathematical training to help them learn to value mathematics, become confident in their ability to do mathematics, become problem solvers, and learn to communicate and reason mathematically. This guide, along with the available division resources, VDOE resources, professional literature, alternative assessment methods, and in-service activities will assist the mathematics teacher in continuing to integrate these student goals into the curriculum.

Table of Contents

Introduction/General Comments

This curriculum guide follows the 2016 Virginia Math 6 & 7 SOLs as adopted by the Virginia Department of Education. It is extremely important and required that the Sequence of Instruction and Pacing be followed as presented in this guide. The textbooks adopted as a resource for this course are *enVisionmath 2.0* (for Math 6 & 7) by Pearson, 2019 editions.

Students will take three formative assessments during the year (dates to be determined annually). Each teacher-designed test will assess skill levels of the SOLs as presented in the Sequence of Instruction and Pacing and the formative assessment blueprint. The data collected from the formative assessments will help teachers determine students' strengths and weaknesses, and inform instructional decisions.

The Mapping for Instruction is based on specified SOLs which are to be taught in the predetermined order. Note, some SOLs (or their parts) may be taught over multiple 9-week periods.

Refer to the Mathematics 2016 Standards of Learning Math 6 & 7 Curriculum Frameworks during every lesson. It is located at the back of this guide. This will provide valuable information for the teacher (Understanding the Standard) and desired goals for instruction (Essential Knowledge and Skills). Examples of teaching techniques and strategies, definitions, and recommended manipulatives are included in the Curriculum Framework and on the VDOE website under Mathematics Instructional Plans (MIPs) http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/mip/index.shtml.

Resources Overview

Resources for all SOLs
IXL
PowerSchool
BrainPop
Pearsonsuccessnet.com
VDOE
Promethean Planet
Number Talks
Performance Tasks
RCPS Common Assessments
get2math

Sequence of Instruction and Pacing

First Nine Weeks			Second Nine Weeks			Third Nine Weeks			Fourth Nine Weeks		
SOL	**Instructional Focus**	**Blks**	**SOL**	**Instructional Focus**	**Blks**	**SOL**	**Instructional Focus**	**Blks**	**SOL**	**Instructional Focus**	**Blks**
	Algebra Readiness Pretest (mandatory)	1	**6.13** **7.12**	Equations: Represent and solve practical problems with one and two-step linear equations Apply properties to solve one and two-step linear equations	7	**6.8ab** **6.12d** **7.10a-e**	Coordinate plane and functions: Make connections between and among representations of proportional relationships Determine slope Write equation of a line Graph a line Determine y-intercept	8	**7.4ab**	Volume & surface area: Rectangular prisms and cylinders Practical problems	8
6.5c	Decimal operations Practical problems	3	**6.14ab** **7.13**	Inequalities: Represent a practical situation with a one or two-step linear inequality Apply properties to solve one and two-step linear inequalities Graph inequalities and solutions	6	**7.1ab**	Negative exponents and scientific notation	4	**7.8ab** **8.11ab (optional)**	Theoretical and experimental probability Probability of compound events	5
6.5ab	Fractions and mixed numbers: Operations on fractions and mixed numbers Practical problems	6	**6.2ab** **7.1c**	Fractions, decimals, and percents: - Represent ratios - Identify equivalencies - Compare and order rational numbers (positive and negative)	6	**8.9ab (not optional)**	Pythagorean Theorem: Solve for sides Determine if a triangle is a right triangle given three sides	4	**6.10abc** **7.9abc**	Review line plots, pictographs, bar graphs, and stem & leaf plots Circle graphs and Histograms Represent, interpret, and compare data within and among graphs (circle, bar, pictographs, line plots, and histograms)	8

6.4 **7.1d**	Exponents and perfect squares Recognize and represent patterns	2	**6.1** **6.12a-d** **7.3** **8.4 (optional)**	Ratios and proportions: Determine unit rate Determine whether a proportion exists Use proportional reasoning to solve single and multi-step practical problems Percent of increase or decrease	10	**7.6ab**	Quadrilaterals: Compare and contrast quadrilaterals based on their properties Determine unknown side lengths or angle measures	6	**6.11ab**	Mean, median, mode: Mean as the balance point in a line plot Determine effect on measures of center when a single value is added, subtracted, or changed	5
6.3abc **7.1e** **6.6ab** **7.2**	Integers/rational numbers: - Identify - Represent - Compare - Order - Absolute value Operations on integers and rational numbers Practical problems	14	**6.9** **7.5**	Congruence and similar figures Regular polygons Line of symmetry Congruence/similarity of quadrilaterals and triangles	6	**7.7** **8.7a (optional)**	Transformations: Translations & reflections of right triangles and rectangles in the coordinate plane Dilations Scale Factor	4		SOL Review	9
6.6c **7.11**	Properties of real numbers	4				**6.7c** **8.10 (optional)**	Perimeter and area: Triangles and rectangles/squares Parallelograms and trapezoids	4	**8.12abc** **8.13abc** **(optional)**	Boxplots and scatterplots	5
6.6c **6.13** **7.11**	Expressions: Verbal to algebraic Algebraic to verbal Simplify/evaluate	5				**6.7ab** **8.10 (optional)**	Circles: Derive pi (π) Circumference and area of circles Area of composite figures	5			
	Performance Tasks, Remediation, Review, Assessment	10		Performance Tasks, Remediation, Review, Assessment	10		Remediation, Review, Assessment	10		Remediation, Review, Assessment	5
	Total Blocks	**45**		**Total Blocks**	**45**		**Total Blocks**	**45**		**Total Blocks**	**45**

DESMOS CALCULATOR

The Desmos Virginia Scientific Calculator will be used for instruction and assessment.

Mapping for Instruction - First Nine Weeks

SOLs

6.3 The student will
- **a) identify and represent integers;**
- **b) compare and order integers; and**
- **c) identify and describe absolute value of integers.**

6.4 The student will recognize and represent patterns with whole number exponents and perfect squares.

6.5 The student will
- **a) multiply and divide fractions and mixed numbers;***
- **b) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and**
- **c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals.**

6.6 The student will
- **a) add, subtract, multiply, and divide integers;***
- **b) solve practical problems involving operations with integers; and**
- **c) simplify numerical expressions involving integers.***

6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.

7.1 The student will
- **d) determine square roots of perfect squares;* and**
- **e) identify and describe absolute value of rational numbers.**

7.2 The student will solve practical problems involving operations with rational numbers.

7.11 The student will evaluate algebraic expressions for given replacement values of the variables.

***On the state assessment, items measuring this objective are assessed without the use of a calculator.**

SOL	Instructional Focus	Vocabulary	Comments	Blocks
ARI Pretest			*All AMP 6 students must take the pretest to determine their degree of algebra readiness.*	1
6.5c	**Solve multistep practical problems (add, subtract, multiply, divide) with decimals** **Word problems**	Double, twice, triple, half, cut off, equal groups, altogether, in all, total, of, left, left over, divisor, simplest form	***(teach SOL 6.5c first)***	3
6.5ab	**Multiply and divide fractions and mixed numbers** **Solve one-step and multistep practical problems (add, subtract, multiply, divide) with fractions and mixed numbers** **Word problems**	Double, twice, triple, half, cut off, equal groups, altogether, in all, total, of, left, left over, divisor, simplest form, numerator, denominator		6
6.4 **7.1d**	**Exponents (bases and exponents are whole numbers)** **Perfect Squares up to 20^2**	Base, exponent, power, perfect square, factor, exponential form, expanded form, square root	*Identify perfect squares.*	2

		Symbols: square root - $\sqrt{}$		
6.3abc **7.1e**	**Identify/represent integers** **Compare/order integers** **Identify/describe absolute value of integers and rational numbers**	Integer, positive, negative, absolute value, compare, is less than, is greater than, order, least to greatest, greatest to least, ascending, descending, increasing, decreasing, grouping, order of operations, evaluate, solve, simplify, opposite, above, below, real number, rational number, irrational number, whole number, natural number Symbols: <, >, ≤, ≥, =; absolute value - | |		1
6.6a	**Add integers** **Simplify expressions**	Sum, difference, product, quotient, grouping symbols, simplify, evaluate, deposit, withdraw, gain, loss, above sea level, below sea level, absolute value, undefined, order of operations, properties (commutative, associative, distributive, identity, inverse, multiplicative property of zero, substitution	*Emphasize operations with integers.*	2
6.6a	**Subtract integers**	(See above)	*Emphasize operations with integers.*	2
6.6a	**Multiply and divide integers**	(See above)	*Emphasize operations with integers.*	1
6.6b, 7.2	**Practical problems with integers**	(See above)	*Emphasize operations with integers.*	2
6.6c **7.2**	**Use and apply the order of operations with integers and rational numbers (including absolute value)** **Simplify expressions**	Order of operations		3
7.2	**Solve practical problems with rational numbers (add, subtract, multiply, and divide)**		*See Math 7 curriculum framework for instructional support.*	3
6.6c **7.11**	**Properties of Real Numbers**	Additive Identity Property, Multiplicative Identity Property, Additive Inverse Property, Multiplicative Inverse, Associative Property, Commutative Property, Multiplicative Property of Zero, Distributive Property, Properties of Equality (addition, subtraction, multiplication, division), Substitution Property	*Emphasize operations with integers.*	4
6.6c **6.13** **7.11**	**Algebra:** **Identify algebraic vocabulary** **Translate expressions and equations:** - **Verbal to algebraic** - **Algebraic to verbal** **Simplify and evaluate expressions using order of operations and properties of real numbers**	Equation, expression, variable, coefficient, term, constant, like terms, solution set, evaluate, solve, order of operations, properties of the real numbers (see above), replacement value Symbols: parentheses - ( ), brackets - [ ], and absolute value - | |	*See Math 6 curriculum framework for instructional support.*	5
			Performance Tasks, Remediation, Review, Assessment	10
			Total Blocks	**45**

Resources – First Nine Weeks

SOL	Textbook	Links	Supplemental Materials
6.5a,b,c	Math 6 Book: Lesson 1-1 to 1-7	Instructional Videos: http://www.vdoe.whro.org/instruction/math_2011/models_dividing_fractions/DOE_MODELS_FOR_TEACHING_3.swf http://www.vdoe.whro.org/instruction/math_2011/area_model_multiplying_fractions/area_model_multiplying_fractions.swf http://www.thatquiz.org/tq-3/math/fraction/ Battleship: Operations on fractions http://www.quia.com/ba/41637.html Jeopardy: Operations with Fractions http://www.math-play.com/Fractions-Jeopardy/fractions-jeopardy.html	See MIPs: 6.5a - Modeling Division of Fractions (Word) / PDF Version 6.5a - Modeling Multiplication of Fractions (Word) / PDF Version 6.5a - Multiply Fractions and Mixed Numbers (Word) / PDF Version 6.5a - Divide Fractions and Mixed Numbers (Word) / PDF Version 6.5b - Solve Problems Involving Operations with Fractions and Mixed Numbers (Word) / PDF Version 6.5c - Practical Problems Involving Decimals (Word) / PDF Version
6.4 **7.1d**	Math 6 Book: Lesson 3-1 Math 7 Book: Lesson VA-3	Instructional Video: http://vimeo.com/album/1612914/video/23981798 https://www.funbrain.com/games/tic-tac-toe-squares https://www.quia.com/jg/65631.html	See MIPs: 6.4 - Whole Number Exponents and Perfect Squares (Word) / PDF Version 7.1d - Square Roots (Word) / PDF Version
6.3a,b,c **6.6a,b,c** **7.1e** **7.2** **7.11**	(Integers) Math 6 Book: Lesson 2-1 2-2 2-3 VA-1 to VA-4 Math 7 Book: Lesson 1-3 to 1-7 4-1 to 4-5	http://www.vdoe.whro.org/math-strategies2/DOE_MATH_5/DOE_MATH_5.swf http://vimeo.com/album/1612914/video/23979319 http://vimeo.com/album/1612914/video/23978997 Interactive Games and Manipulatives: http://www.crctlessons.com/algebra-game.html http://www.xpmath.com/forums/arcade.php?do=play&gameid=45 http://www.sheppardsoftware.com/mathgames/integers/FS_Integer_subtraction.htm https://my.hrw.com/math06_07/nsmedia/tools/Integer_Chips/Integer_Chips.html	See MIPs: 6.3a - Ground Zero (Word) / PDF Version 6.3b - Compare and Order Integers (Word) / PDF Version 6.3c - Absolute Value of Integers (Word) / PDF Version 6.6a - Operations with Integers (Word) / PDF Version 6.6b - Application of Integer Operations (Word) / PDF Version 6.6c - Order Up (Word) / PDF Version 7.1e - Absolute Value (Word) / PDF Version 7.2 - Solve Problems Involving Operations with Rational Numbers (Word) / PDF Version 7.11 - Evaluating Algebraic Expressions (Word) / PDF Version
7.2 order of operations	Math 7 Book: Lesson 1-5 1-7		See MIPs: 7.2 - Solve Problems Involving Operations with Rational Numbers (Word) / PDF Version

	1-9 1-10		
6.6c **6.13** **7.11**	Math 6 Book: Lesson 3-4 4-1 to 4-5 Math 7 Book: Lesson 1-3 to 1-7 4-1 to 4-4	Instructional Videos: http://www.doe.virginia.gov/instruction/mathematics/resources/videos/index.shtml# http://www.vdoe.whro.org/instruction/math_2011/grade_5_distributive_property/DOE_PROPERTY_4.swf Interactive Games: http://www.math-play.com/Algebraic-Expressions-Millionaire/algebraic-expressions-millionaire.html http://www.superteachertools.us/jeopardyx/jeopardy-review-game.php?gamefile=1464233#.WitH1kpKvIV	See MIPs 6.6c - Order Up (Word) / PDF Version 6.13 - Equation Vocabulary (Word) / PDF Version 6.13 - Modeling One-Step Linear Equations (Word) / PDF Version 6.13 - One Step Equations (Word) /PDF Version 7.11 - Evaluating Algebraic Expressions (Word) / PDF Version

Mapping for Instruction - Second Nine Weeks

SOLs

6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

6.2 The student will
- **a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and**
- **b) compare and order positive rational numbers.***

6.9 The student will determine congruence of segments, angles, and polygons.

6.12 The student will
- **a) represent a proportional relationship between two quantities, including those arising from practical situations;**
- **b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;**
- **c) determine whether a proportional relationship exists between two quantities; and**
- **d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.**

6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.

6.14 The student will
- **a) represent a practical situation with a linear inequality in one variable; and**
- **b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.**

7.1 The student will
- **c) compare and order rational numbers;***

7.3 The student will solve single-step and multistep practical problems, using proportional reasoning.

7.5 The student will solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar quadrilaterals and triangles.

7.12 The student will solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable.

7.13 The student will solve one- and two-step linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and graph the solution on a number line.

8. 4 The student will solve practical problems involving consumer applications.

***On the state assessment, items measuring this objective are assessed without the use of a calculator.**

SOL	Instructional Focus	Vocabulary	Comments	Blocks
6.13 **7.12**	**One-step equations: Represent and solve one-step linear equation in one variable** **Apply properties of real numbers** **Expressions and equations:** • **Verbal to algebraic** • **Algebraic to verbal**	Equation, mathematical sentence, solution, inverse operation, expression, variable expression, verbal expression, algebraic expression, algebraic equation, properties of real numbers: commutative, associative, distributive, identity, and inverse properties.	*Solve one-step linear practical problems.*	3
6.13 **7.12**	**Two-step equations: Represent and solve two-step equations in one variable** **Apply properties of real numbers** **Expressions and equations:** • **Verbal to algebraic** • **Algebraic to verbal**	(see above)	*Solve one-step linear practical problems.*	2
	Multi-Step Equations (Optional)	Multistep Equations		
6.13 **7.12**	**Solve one and two-step equations - practical problems** **Apply properties of real numbers** **Translate expressions and equations:** • **Verbal to algebraic** • **Algebraic to verbal**	(see above)	*Solve one-step linear practical problems.*	2
6.14a	**Inequalities:** **Represent a practical situation with a linear inequality**	Inequality, is greater than, is less than, is greater than or equal to, is less than or equal to, point, closed circle, open circle, solution set	*See 6.14 EKS in the curriculum framework.*	2
6.14b **7.13**	**Solve and graph solutions of one-step linear inequalities** **Represent and identify solutions to inequalities algebraically and graphically (on a number line)**	(see above)	*See 6.14 EKS in the curriculum framework.* *Solve two-step linear inequalities and practical problems.*	1
7.13	**Solve and graph solutions of two-step linear inequalities** **Represent and identify solutions to inequalities algebraically and graphically (on a number line)** **Translate expressions and inequalities:** • **Verbal to algebraic** • **Algebraic to verbal**	(see above)	*Solve two-step linear inequalities and practical problems.*	3

6.2a	**Fractions, Decimals, and Percents:** **Determine equivalencies** **Represent ratios as fractions (proper and improper), mixed numbers, decimals, and percents**	Percent, equal, terminating decimal, repeating decimal, proper fraction, improper fraction, per 100, out of 100, is less than, is greater than, is less than or equal to, is greater than or equal to, equivalent, rational number, mixed number, ratio Symbols: <, >, ≤, ≥, =		3
6.2b **7.1c**	**Fractions, Decimals, and Percents:** **Compare and order positive and negative fractions and decimals in ascending and descending order**	(see above)		3
6.1	**Ratios** **Express ratios with symbols, numbers** **Express ratios with words**	Ratio, relationship, quantities, compare, part to part, part to whole, whole to whole, colon, fraction notation, comparison		2
6.12b	**Determine unit rate**	Unit rate	*See 6.12 EKS for instructional support.*	1
6.12c **7.3**	**Write and solve proportions** **Determine whether or not a proportion exists**	Proportion, equivalent ratios, ratio table, constant, convert, conversion factor, benchmark, percent, tip, tax, discount, speed, recipe conversion, scale drawing, consumer, comparison shopping, US Customary System, product of the means, product of the extremes, cross multiplication, fraction, rate, unit rate, mass, weight, monetary conversion		2
6.12a-d **7.3**	**Use proportional reasoning to solve single and multi-step practical problems:** **Use a ratio table to determine missing values** **Convert US Customary units to metric units given the conversion factor** **Solve scale drawing problems and use scale factors** **Use benchmark (10%) to compute percentages, tips, taxes, and discounts** **Use benchmark (10%) to solve practical problems involving tips, tax, and discounts**	Proportion, equivalent ratios, ratio table, constant, convert, conversion factor, benchmark, percent, tip, tax, discount, speed, recipe conversion, scale drawing, consumer, comparison shopping, US Customary System, product of the means, product of the extremes, cross multiplication, fraction, rate, unit rate, mass, weight, monetary conversion	*See 6.12 EKS for instructional support.*	5

8.4 (optional)	**Percent increase or decrease**	Percent of increase, percent of decrease		
6.9	**Determine congruence (segments, angles, and polygons)** **Identify regular polygons** **Draw lines of symmetry in regular polygons**	Segment, angle, polygon, congruent, non-congruent, regular polygon, line of symmetry, parallel, interior angles Symbols for congruence: hash marks, angle curves Symbol for parallelism: arrows	*Identify regular polygons and draw lines of symmetry in regular polygons.*	2
7.5	**Similar figures: quadrilaterals and triangles** **Identify corresponding sides, angles** **Write similarity statements** **Write proportions**	Similar, similar polygons, corresponding sides, proportional, interior angles, congruent, corresponding angles, hatch or hash marks, similarity, congruence Symbols: ~ , ≅		2
7.5	**Similar figures: Determine missing sides using proportions** **Determine unknown angle measures** **Practical problems**	(see above)	*Determine unknown side lengths and angle measures of similar quadrilaterals or triangles using a proportion.*	2
			Performance Tasks, Remediation, Review, Assessment	10
			Total Blocks	**45**

Resources – Second Nine Weeks

SOL	Textbook	Links	Supplemental Materials
6.13 **7.12**	Math 6 Book: Lesson 3-4 4-1 to 4-5 Math 7 Book: Lesson 5-1 5-2 5-3	Hoop Shoot Game http://www.crctlessons.com/one-step-equations-game.html Instructional Video: http://www.vdoe.whro.org/instruction/math_2011/grade_5_modeling_one-step_equations/DOE_PROPERTY_5.swf Algebra Four Game http://www.shodor.org/interactivate/ activities/AlgebraFour/ (select "One-Step Equations") Hoop Shoot Game http://www.crctlessons.com/two-step-equations-game.html Algebra Four Game http://www.shodor.org/interactivate/ activities/AlgebraFour/ (select "Two-Step Equations") Jeopardy-style game (reviews algebraic expressions, too) http://www.crctlessons.com/CRCT-Game-Algebra/crct-game.html	See MIPs: 6.13 - Equation Vocabulary (Word) / PDF Version 6.13 - Modeling One-Step Linear Equations (Word) / PDF Version 6.13 - One Step Equations (Word) /PDF Version 7.12 - Solving Two-Step Equations (Word) / PDF Version 7.12 - Translating Expressions and Equations (Word) / PDF Version
6.14a,b **7.13**	Math 6 Book: Lesson 4-6 4-7 VA-5 Math 7 Book: Lesson 5-4 to 5-7	https://www.youtube.com/watch?v=R34YS6qViLI (Graphing Inequalities on a number line) https://www.youtube.com/watch?v=smX2wkIUPvQ (Solve one-step inequality with addition or subtraction) Rags to Riches Game http://www.quia.com/rr/374262.html https://www.quia.com/rr/325253.html https://us.sofatutor.com/mathematics/videos/solving-two-step-inequalities https://www.maneuveringthemiddle.com/how-to-teach-one-and-two-step-inequalities/ https://www.onlinemathlearning.com/two-step-inequalities.html	See MIPs: 6-14a - Representing Practical Situations with Inequalities (Word) / PDF Version 6.14b - Solving One Step Inequalites with Addition and Subtraction (Word) / PDF Version 7.13 - Two-step Inequality Practical Problems (Word) / PDF Version
6.2a,b **7.1c**	Math 6 Book: Lesson 2-2 6-1 to 6-6 Math 7 Book: Lesson VA-1	That Quiz: Select "Ordering" and "Both" http://www.thatquiz.org/tq-6/math/identify/fractions/ https://www.mathplayground.com/Decention/index.html (FDP) https://www.mathplayground.com/Triplets/index.html (FDP w/models) https://www.youtube.com/watch?v=5vv--qrUJXo (Comparing and Ordering FDP) https://www.youtube.com/watch?v=WV5VY76Pf5U (Converting fractions to decimals)	See MIPs: 6.2a - Rational Speed Match (Word) / PDF Version 6.2b - Compare and Order Rational Numbers (Word) /PDF Version 7.1c - Ordering Fractions, Decimals, and Percents (Word) /

		Jeopardy Game http://www.math-play.com/Fractions-Decimals-Percents-Jeopardy/fractions-decimals-percents-jeopardy.html Compare and order Jeopardy Game http://www.math-play.com/Comparing-Rational-Numbers/comparing-rational-numbers.html Decention Game http://www.mathplayground.com/Decention/Decention.html	
6.1	Math 6 Book: Lesson 5-1 to 5-7 VA-6 VA-7	http://www.mathsisfun.com/numbers/atio.html (Math is Fun: Ratios)	See MIPs: 6.1 - Field Goals, Balls, and Nets (Word) / PDF Version
6.12a,b,c	Math 6 Book: Lesson 5-2 to 5-10 VA-6 VA-7	https://www.youtube.com/watch?v=USmit5zUGas (Math Antics- Intro to Proportions) https://www.youtube.com/watch?v=liW_ALj4Qj8&t=129s (Unit Rates) https://www.youtube.com/watch?v=JwRYcBUGz5Q (Ratio tables) Instructional Video: http://vimeo.com/album/1612914/video/23981751	See MIPs: 6.12ab - Ratio Tables and Unit Rates (Word) / PDF Version 6.12cd - Identifying and Representing Proportional Relationships (Word) / PDF Version
7.3 **8.4**	Math 7 Book: Lesson 2-1 to 2-6 3-1 to3-6	Quia Jeopardy: Ratios, Rates and Proportions http://www.quia.com/cb/158527.html http://www.themathlab.com/Algebra/basics/blow%20'em%20up%20cartoons.htm Percent Shopping http://www.mathplayground.com/percent_shopping.html Math at the Mall http://www.mathplayground.com/mathatthemall1.html Going Out to Lunch (Activity 5)/Shopping for Groceries (Activity 10): http://www.doe.virginia.gov/instruction/mathematics/resources/videos/handouts/scientific_calculator_manual.pdf	See MIPs: 7.3 - Sales Tax, Tip and Discount (Word) / PDF Version 7.3 - Conversions (Word) / PDF Version 7.3 - Proportions (Word) / PDF Version 8.4 - Consumer Applications – Taxes, Tips, and Simple Interest (Word) / PDF Version 8.4 - The Scoop-on-Ice-Cream Planning (Word) / PDF Version 8.4 - Percent of Increase or Decrease (Word) / PDF Version 8.4 - Do You Like to Spend Money? (Word) / PDF Version
6.9 **7.5**	Math 7 Book: Lesson 8-3 8-4 VA-10	That quiz: Choose "Similar" http://www.thatquiz.org/tq-A/math/triangle/ https://www.khanacademy.org/math/geometry/hs-geo-similarity/hs-geo-solving-similar-triangles/v/similarity-example-problems https://www.khanacademy.org/math/geometry/hs-geo-similarity/hs-geo-solving-similar-triangles/e/solving_similar_triangles_1 http://www.math.com/school/subject1/lessons/S1U2L4GL.html https://www.mathsisfun.com/geometry/triangles-similar.html	See MIPs: 6.9 - Side to Side (Word) / PDF Version 7.5 - Similar Figures (Word) / PDF Version 7.5 - Missing Measurements (Word) /PDF Version Similar and Congruent Figures Videos: https://youtu.be/h7ZAQfNh8XA https://youtu.be/1O-ieOZ5y6s https://youtu.be/Tlq9amS9hy4 https://youtu.be/kLkM2zIWbKU https://youtu.be/7zz087jWaQY https://youtu.be/vvHZXTWLtwI https://youtu.be/6gvj69UAkWg

Mapping for Instruction - Third Nine Weeks

SOLs

6.7 The student will

- **a) derive π (pi);**
- **b) solve problems, including practical problems, involving circumference and area of a circle; and**
- **c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles.**

6.8 The student will

- **a) identify the components of the coordinate plane; and**
- **b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.**

6.12 The student will

- **d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.**

7.1 The student will

- **a) investigate and describe the concept of negative exponents for powers of ten;**
- **b) compare and order numbers greater than zero written in scientific notation;***

7.6 The student will

- **a) compare and contrast quadrilaterals based on their properties; and**
- **b) determine unknown side lengths or angle measures of quadrilaterals.**

7.7 The student will apply translations and reflections of right triangles or rectangles in the coordinate plane.

7.10 The student will

- **a) determine the slope, m, as rate of change in a proportional relationship between two quantities and write an equation in the form $y = mx$ to represent the relationship;**
- **b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in $y = mx$ form where m represents the slope as rate of change;**
- **c) determine the y-intercept, b, in an additive relationship between two quantities and write an equation in the form $y = x + b$ to represent the relationship;**
- **d) graph a line representing an additive relationship between two quantities given the y-intercept and an ordered pair, or given the equation in the form $y = x + b$, where b represents the y-intercept; and**
- **e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.**

8.7 The student will

- **a) given a polygon, apply transformations, to include translations, reflections, and dilations, in the coordinate plane;**

8.9 The student will

- **a) verify the Pythagorean Theorem; and**
- **b) apply the Pythagorean Theorem.**

8.10 The student will solve area and perimeter problems, including practical problems, involving composite plane figures.

SOL	Instructional Focus	Vocabulary	Comments	Blocks
6.8ab	**Coordinate Plane:** **Identify parts of coordinate graph** **Identify/graph ordered pairs**	Horizontal, vertical, x-axis, y-axis, coordinate plane, ordered pair, perpendicular, origin, quadrant, I, II, III, IV, intersecting, coordinates, counterclockwise, clockwise, up, down, left, right, polygon		1
7.10a	**Slope, Functions, and Graphs:** **Determine slope** **Write an equation of a line in the form $y = mx$**	Slope, $y = mx$, $y = x + b$, constant of proportionality, constant ratio, proportional, unit rate, proportional relationship, ordered pair, point, graph, rate of change, equation, vertical change, horizontal change, table, line, slope triangle, x-value, y-value, input, output, y-intercept, independent and dependent variables	*See Curriculum Framework for SOL 7.10a-e.*	3
7.10b	**Graph a line:** **Given slope and ordered pair** **Given y = mx**	(see above)	*See Curriculum Framework for SOL 7.10a-e.*	
7.10c	**Determine y-intercept** **Write equations in the form y = x + b**	(see above)	*See Curriculum Framework for SOL 7.10a-e.*	3
7.10d	**Graph a line:** **Given y-intercept and an ordered pair** **Given y = x + b**	(see above)	*See Curriculum Framework for SOL 7.10a-e.*	
6.12d **7.10e**	**Make connections from graphs of proportional relationships**	Equivalent, proportional relationship, constant of proportionality, quantity, table, unit rate, value, ratio table, constant, rate Slope, $y = mx$, $y = x + b$, constant ratio, proportional, proportional relationship, ordered pair, point, graph, rate of change, equation, vertical change, horizontal change, line, slope triangle, x-value, y-value, input, output, y-intercept.	*See 6.12 EKS for instructional support.* *See Curriculum Framework for SOL 7.10a-e.*	1
7.1a	**Negative exponents for powers of ten (convert to fraction and decimal)**	Negative exponents, percent, ratio, scientific notation, integer, rational number, terminating or repeating decimal, factor, proper fraction, improper fractions, mixed number, equivalent, perfect square, square root, absolute value Symbols: square root - $\sqrt{\ }$; abs value - | |		1
7.1b	**Scientific notation (write)** **Decimal equivalents**	(see above)		2
7.1b	**Scientific notation (compare and order)**	(see above)		1
8.9ab **(not optional)**	**Pythagorean Theorem:** **Determine if a triangle is a right triangle given three sides** **Solve for a side given two sides** **Practical problems**	Pythagorean theorem, right triangle, leg, hypotenuse, converse; $a^2 + b^2 = c^2$	*Students in PreAlgebra will cover this content (not optional) but teachers may decide when they want to teach it.*	4

7.6a	**Quadrilaterals:** **Compare/contrast properties of parallelogram, rectangle, square, rhombus, and trapezoid** **Sort/classify quadrilaterals based on properties**	Polygon, plane figure, line segment, quadrilateral, parallel sides, symmetry, diagonal, vertices, bisect, parallelogram, rectangle, square, rhombus, trapezoid, bases, legs, isosceles trapezoid		3
7.6b	**Determine side lengths and angle measures of quadrilaterals based on properties**	(see above)	*Determine unknown side lengths and angle measures of quadrilaterals using properties of quadrilaterals.*	3
7.7	**Translations (right triangles and rectangles):** **Identify coordinates of an image that has been translated**	Transformation, preimage, image, translation, reflection, line of reflection, corresponding points, equidistant, *x*-axis, *y*-axis, right triangles, rectangles, vertical, horizontal, coordinate plane, coordinates		2
7.7	**Reflections (right triangles and rectangles):** **Identify coordinates of an image that has been reflected over x- or y-axis** **Identify coordinates of an image that has been translated, then reflected (over x- and y-axis) or reflected (over x- and y-axis) and then translated** **Sketch images that have been reflected, translated, or a combination of both**	(see above)		2
8.7a (optional)	**Dilations:** **Center of dilation is the origin (0,0)** **Scale factors of ¼, ½, 2, 3, 4**	(see above)	*Dilations from Math 8.7a are optional. Rotations are taught in Geometry (G.3d).*	
6.7c	**Solve problems involving perimeter and area of triangles, rectangles, and squares** **Practical problems**	Ratio, circumference, diameter, distance, plane figure, circle, square, rectangle, triangle, area, perimeter, base height, radius, center Formulas for perimeter and area of triangles, rectangles, and squares		4
8.10 (optional)	**Area of parallelograms and trapezoids**	Formulas for area of parallelograms and trapezoids	*This content will also be covered in Geometry (G.13).*	
6.7a	**Derive pi (π) using concrete materials or computer models**	Ratio, pi, circumference, diameter, distance, plane figure, circle, square, rectangle, triangle, area, perimeter, base height, radius, center Formulas for circumference and area of circles		2
6.7b	**Solve problems involving circumference and area of circles** **Practical problems**	(see above)		3

8.10 (optional)	**Area of Composite Figures**	Composite figures Formulas for circumference, perimeter, and area of plane figures, semi-circle	*See curriculum framework for SOL 8.10.*	
			Performance Tasks, Remediation, Review, Assessment	10
			Total Blocks	**45**

Resources – Third Nine Weeks

SOL	Textbook	Links	Supplemental Materials
6.8ab	Math 6 Book: Lesson 2-4	Coordinate Plane Battleship: http://www.educationworld.com/a_ysl/archives/06-1/lesson001.shtml https://www.youtube.com/watch?v=r16I6LB2YbQ (Intro to Coordinate Plane) https://www.mathplayground.com/space_graph.html (Plotting points on the Coordinate Plane)	See MIPs: 6.8ab - What's the Point? (Word) / PDF Version
7.10abcd	Math 7 Book: Lesson VA-4 VA-5 VA-6	https://campus.mangahigh.com/en-us/px/238/0/0 http://www.shodor.org/interactivate/activities/SlopeSlider/ https://www.mathgames.com/skill/8.32-find-slope-from-two-points http://www.bbc.co.uk/bitesize/ks3/maths/algebra/graphs/activity/ https://www.quia.com/rr/79713.html http://www.mathwarehouse.com/algebra/linear_equation/interactive-slope.php https://phet.colorado.edu/en/simulation/graphing-lines	See MIPs: 7.10ab - Discover Slope (m) (Word) / PDF Version 7.10cd - Discover y-intercept (b) (Word) / PDF Version Slope Videos https://youtu.be/qnMaWTmdbKk https://youtu.be/zihsQC0IUd8 https://youtu.be/wuigT2tyFzQ https://youtu.be/vos-Ja4b3cg https://youtu.be/WkspBxrzuZo https://youtu.be/IL3UCuXrUzE
6.12d **7.10a-e**	Math 6 Book: Lesson 5-2 to 5-10 VA-6 VA-7 Math 7 Book: Lesson VA-4 VA-5 VA-6	Instructional Video: http://vimeo.com/album/1612914/video/23981751	See MIPs: 6.12cd - Identifying and Representing Proportional Relationships (Word) / PDF Version 7.10ab - Discover Slope (m) (Word) / PDF Version 7.10cd - Discover y-intercept (b) (Word) / PDF Version 7.10e - Making Connections (Word) / PDF Version
7.1b	Math 7 Book: Lesson VA-2	Battleship game http://www.quia.com/ba/78951.html http://www.math-play.com/Scientific-Notation-Concentration/Scientific-Notation-Concentration.html Powers of 10 Video: http://powersof10.com/film	See MIPs: 7.1b - Scientific Notation / PDF Version
8.9a,b	Math 8 Book: Lesson 7-1 7-2 7-3 7-4	Jeopardy: Pythagorean Theorem http://www.quia.com/cb/278769.html Pythagorean Word Problems http://www.regentsprep.org/Regents/math/ALGEBRA/AT1/PracPyth.htm	See MIPs: 8.9 - Pythagorean Theorem (Word) / PDF Version IXL Math 8 under “Pythagorean theorem” - Pythagorean theorem: find the length of the hypotenuse - Pythagorean theorem: find the missing leg length - Pythagorean theorem: word problems

		inspecting a Deck (Activity 6): http://www.doe.virginia.gov/instruction/matheatics/resources/videos/handouts/scientific_calculator_ manual.pdf	- Converse of the Pythagorean theorem: is it a right triangle? Roanoke County Math 8 Performance Task: *Pythagorean Theorem* Kahoot titled "Pythagoream Theorem" by aktravis (10 questions) Pythagorean Theorem fence activity
7.6ab	Math 7 Book: Lesson 8-2 VA-8	Quadrilateral Quest http://teams.lacoe.edu/documentation/classrooms/Amy/geometry/6-8/activities/quad_quest/quad_quest.html Quadrilateral Game http://www.phschool.com/atschool/cmp2/active_math/site/Grade6/Quad/index.html Instructional Video: http://www.vdoe.whro.org/math-strategies2/DOE_MATH_9/DOE_MATH_9.swf https://www.mathsisfun.com/geometry/quadrilaterals-interactive.html https://www.mathsisfun.com/quadrilaterals.html http://www.softschools.com/math/geometry/quadrilaterals/ http://www.snappymaths.com/other/shapeandspace/2dshapes/interactive/quadrilaterals/quadrilaterals.htm https://www.pbslearningmedia.org/resource/mgbh.math.g.linkquadri/constructing-quadrilaterals/#.WjqcNDdrzIU	See MIPs: 7.6a - Classifying Quadrilaterals (Word) / PDF Version 7.6b - Quadrilaterals – Measures of Sides and Angles (Word) / PDF Version Quadrilateral Videos https://youtu.be/rh93AfCyOik https://youtu.be/k7oLm94kQEE https://youtu.be/_wfTsmmteVE https://youtu.be/kzJ6l-Hr7lc https://youtu.be/3lQ6fjww9ow https://youtu.be/kJzGlmCGioI
7.7 **8.7a (optional)**	Math 7 Book: Lesson VA-9	http://www.shodor.org/interactivate/activities/Transmographer/ https://www.mathgames.com/skill/8.17-identify-reflections-rotations-and-translations https://www.turtlediary.com/game/translation-reflection-rotation.html https://www.mangahigh.com/en-us/games/transtar http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/TranslateShapesShoot.htm http://www.sciencekids.co.nz/gamesactivities/math/transformation.html	See MIPs: 7.7 - Translation and Reflection (Word) / PDF Version 8.7ab - Transformations (Word) / PDF Version Transformation Videos https://youtu.be/NKtJd1hkI9k Transformations "Gangnam Style" song https://youtu.be/XdjH_EWhCZ0 Translating a Triangle https://youtu.be/j87gj_KH9pA Translations Mash Up Math https://youtu.be/j1X_UIOvEwA Reflections https://youtu.be/ouNp8FtgiEE Reflections Mash Up Math
6.7abc **8.10 (optional)**	Math 6 Book: Lesson 2-6 7-1 to 7-4 VA-8 VA-9	Rags to Riches: Review Area, Perimeter and circumference http://www.quia.com/rr/141971.html https://www.youtube.com/watch?v=8-cazxAL_tU (Intro to Pi) Everything you Wanted to Know about Area and Perimeter. Select "Area", then choose Level 3 http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/perimeter_and_area/index.html	See MIPs: 6.7ab - Going the Distance (Word) / PDF Version 6.7c - Practical Problems Involving Area and Perimeter (Word) / PDF Version 8.10 - Composite Figures: Area and Perimeter (Word) / PDF Version

Mapping for Instruction - Fourth Nine Weeks

SOLs

6.10 The student, given a practical situation, will
- **a) represent data in a circle graph;**
- **b) make observations and inferences about data represented in a circle graph; and**
- **c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.**

6.11 The student will
- **a) represent the mean of a data set graphically as the balance point; and**
- **b) determine the effect on measures of center when a single value of a data set is added, removed, or changed.**

7.4 The student will
- **a) describe and determine the volume and surface area of rectangular prisms and cylinders; and**
- **b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders.**

7.8 The student will
- **a) determine the theoretical and experimental probabilities of an event; and**
- **b) investigate and describe the difference between the experimental probability and theoretical probability of an event.**

7.9 The student, given data in a practical situation, will
- **a) represent data in a histogram;**
- **b) make observations and inferences about data represented in a histogram; and**
- **c) compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.**

Optional (but highly encouraged)

8.11 The student will
- **a) compare and contrast the probability of independent and dependent events; and**
- **b) determine probabilities for independent and dependent events.**

8.12 The student will
- **a) represent numerical data in boxplots;**
- **b) make observations and inferences about data represented in boxplots; and**
- **c) compare and analyze two data sets using boxplots.**

8.13 The student will
- **a) represent data in scatterplots;**
- **b) make observations about data represented in scatterplots; and**
- **c) use a drawing to estimate the line of best fit for data represented in a scatterplot.**

SOL	Instructional Focus	Vocabulary	Comments	Blocks
7.4ab	**Determine volume of rectangular prisms and cylinders using concrete objects, diagrams, and formulas** **Determine surface area of rectangular prisms and cylinders using concrete objects, nets, diagrams, and formulas** **Determine whether volume or surface area should be applied to solve practical problems** **Practical problems involving volume and surface area**	Polyhedron, polygons, rectangular prism, cylinder, face, surface area, volume, nets, two-dimensional, three-dimensional, square units, cubic units, capacity, base, height, pi		8
7.8ab	**Probability:** **Determine theoretical probability**	Event, equally likely, ratio, equivalent fraction, decimal, percent, theoretical probability, experimental probability, simulation, experiment, Law of Large Numbers		5

7.8ab	**Probability:** **Determine experimental probability** **Describe change in experimental probability as number of trials increases**	(see above)		
7.8ab	**Probability:** **Describe difference between theoretical and experimental probability of the same event**	(see above)		
8.11ab (optional)	**Probability of Compound Events**	Independent and dependent events		
6.10ab	**Collect, organize, and represent data in a circle graph** **Make observations and inferences about data in circle graphs**	Data, relationship, part to whole, percent, frequency, category, construct, analyze, survey, categorical data, numerical data, line plot, bar graph, pictograph, scale, compare, predict, inference, collect, organize, represent, observations, table, title, scale, key, data categories, labels		2
7.9ab	**Graphs: Histograms** **Create histograms** **Make observations about histograms**	Histogram, numerical data, data points, range of values, class or bin, frequency, title, labels, consecutive intervals, equal intervals, frequency distribution, table, *x*-axis, *y*-axis, vertical bar, comparisons, predictions, inferences, data analysis, patterns, trends, categorical data, line plot, clusters, gaps, range, mode, extreme data values, circle graphs, stem and leaf plots		2
3.15ab **4.14abc** **5. 16**	**Review: Line plots, pictographs, bar graphs, and stem & leaf plots**	Data, relationship, part to whole, percent, frequency, category, construct, analyze, survey, categorical data, numerical data, line plot, bar graph, pictograph, scale, compare, predict, inference, collect, organize, represent, observations, table, title, scale, key, data categories, labels		2
6.10c **7.9c**	**Compare data in circle graphs with same data in bar graphs, pictographs, line plots** **Compare data in histogram to same data in line plots, circle graphs, and stem & leaf plots**	(see above)		2
6.11ab	**Mean as the balance point in a line plot** **Calculate mean, median, mode** **Determine effect on measures of center when a single value is added, subtracted, or changed**	Measures of center, data set, mean, median, mode, balance point, average, values, outlier, descriptor, range	*See Curriculum Framework for SOL 6.11b; effects on measures of center when a single value is added, subtracted, or changed.*	5
	SOL Review			9
8.12abc **8.13abc**	**Enrichment Activities:** **Boxplots** **Scatterplots**	Boxplot, scatterplot, line of best fit		5
			Performance Tasks, Remediation, Review, Assessment	5
			Total Blocks	**45**

Resources – Fourth Nine Weeks

SOL	Textbook	Links	Supplemental Materials
7.4ab	Math 7 Book: Lesson 8-7 8-8 8-9 VA-11 VA-12 VA-13	That quiz http://www.thatquiz.org/tq-4/math/geometry/ Instructional Video: http://www.vdoe.whro.org/math-strategies/FLA_DOE_8/FLA_DOE_8.swf https://www.learner.org/interactives/geometry/area_surface.html https://www.learner.org/interactives/geometry/area_volume.html https://www.onlinemathlearning.com/volume-games.html http://www.mrsburkhartsclass.com/volume--surface-area-games	See MIPs: 7.4ab - Volume and Surface Area of Rectangular Prisms and Cylinders (Word) / PDF Version Surface Area and Volume Videos: https://youtu.be/XYlqJpKcgfc (SA of Rect Prism) https://youtu.be/ZJ-VMcbLTaU (SA of All 3D shapes) https://youtu.be/Bb_XJ7UPDlM (SA and Volume of Cylinders) https://youtu.be/E8tuMaDxgJM (Volume of Rect Prism) https://youtu.be/JijhDDJvExo (Volume song)
7.8ab **8.11ab**	Math 7 Book: Lesson 7-1 7-2 7-3 7-4	Adjustable Spinner http://illuminations.nctm.org/activitydetail.aspx?ID=79 Racing Game (2 dice) http://www.shodor.org/interactivate/activities/RacingGameWithTwoDie/ Coin Tossing Activity: http://nlvm.usu.edu/en/nav/frames_asid_305_g_3_t_5.html?from=topic_t_5.html http://interactivesites.weebly.com/probability.html http://www.shodor.org/interactivate/activities/ExpProbability/ http://www.free-training-tutorial.com/probability-games.html http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.STAT&ID2=AB.MATH.JR.STAT.PROB&lesson=html/video_interactives/probability/probabilityInteractive.html https://illuminations.nctm.org/adjustablespinner/ http://mrnussbaum.com/probfair/ https://www.mathplayground.com/probability.html	See MIPs: 7.8ab - What are the Chances? (Word) / PDF Version 8.11 - Probability (Word) / PDF Version http://www.thatquiz.org/tq-d/math/probability/ Independent Events: http://staff.argyll.epsb.ca/jreed/math8/strand4/4203.htm Candy? Probably (Activity 15): http://www.doe.virginia.gov/instruction/mathematics/resources/videos/handouts/scientific_calculator_manual.pdf
6.10ab	Math 6 Book: Lesson 8-1 VA-11	That quiz - higher order thinking questions related to circle graphs http://www.thatquiz.org/tq-5/math/graphs/ https://www.youtube.com/watch?v=KzXZfv9anpU (circle graphs) https://www.youtube.com/watch?v=x5GxZ2Ezsmc (interpreting circle graphs) https://www.youtube.com/watch?v=9ldcKQhUCG0	See MIPs: 6.10abc - May I have Fries with That? (Word) / PDF Version

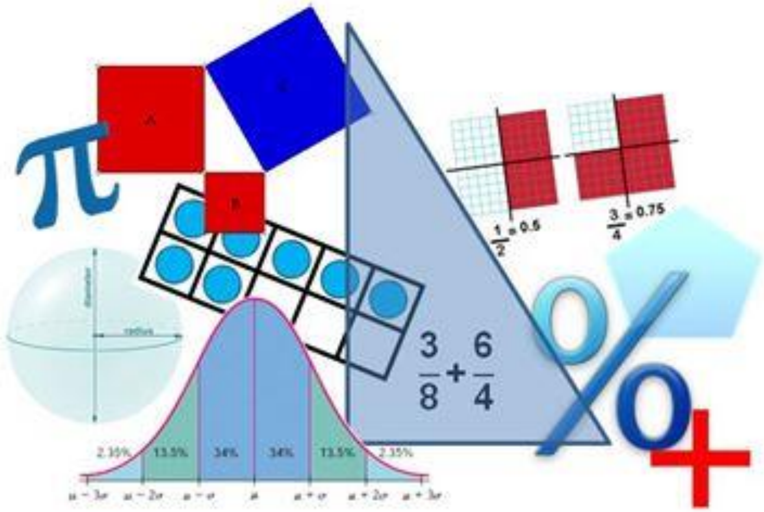
		(interpreting circle graphs)	
7.9ab	Math 7 Book: Lesson 8-5 8-6 VA-7 6-1 6-2 6-3 6-4	https://www.mathgames.com/skill/6.130-create-histograms	See MIPs: 7.9abc - Numbers in a Name (Word) / PDF Version 7.9c - All Graphs are Not the Same (Word) / PDF Version
6.10c **7.9c**	Math 7 Book: Lesson 8-5 8-6 VA-7 6-1 6-2 6-3 6-4	Graph Creator http://nces.ed.gov/nceskids/createagraph/	See MIPs: 6.10abc - May I have Fries with That? (Word) / PDF Version 7.9abc - Numbers in a Name (Word) / PDF Version 7.9c - All Graphs are Not the Same (Word) / PDF Version Stem-Leaf Plots and Line Plots Histograms - https://youtu.be/RWP2q4wyRds
6.11ab	Math 6 Book: Lesson 8-2 VA-12 8-5 (preAP) 8-6 (preAP) 8-7 (preAP)	Mean as the Balance Point http://www.vdoe.whro.org/instruction/math_2011/mean_as_balance_point/DOE_MEAN_2.swf Who are you calling a Nerd? (Activity 11): http://www.doe.virginia.gov/instruction/mathematics/resources/videos/handouts/scientific_calculator_manual.pdf	See MIPs: 6.11a - Balancing Act (Word) / PDF Version 6.11b - Effects on Measures of Center (Word) /PDF Version
8.12abc **8.13abc**			See MIPs: 8.12 - Representing Data Using Boxplots (Word) / PDF Version 8.13ab - Constructing and Analyzing Scatterplots (Word) / PDF Version 8.13c - Scatterplots: Estimating the Line of Best Fit (Word) / PDF Version

Mathematics

2016 Standards of Learning

Grade 6

Curriculum Framework



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The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills
This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and to see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "… the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> “Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Mathematics instruction in grades six through eight continues to focus on the development of number sense, with emphasis on rational and real numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to many middle school mathematics topics.

Students develop an understanding of integers and rational numbers using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.

Students develop an understanding of real numbers and the properties of operations on real numbers through experiences with rational and irrational numbers and apply the order of operations.

Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

Understanding the Standard

- A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities. Ratios are used in practical situations when there is a need to compare quantities.
- In the elementary grades, students are taught that fractions represent a part-to-whole relationship. However, fractions may also express a measurement, an operator (multiplication), a quotient, or a ratio. Examples of fraction interpretations include:
  - Fractions as parts of wholes: $\frac{3}{4}$ represents three parts of a whole, where the whole is separated into four equal parts.
  - Fractions as measurement: the notation $\frac{3}{4}$ can be interpreted as three one-fourths of a unit.
  - Fractions as an operator: $\frac{3}{4}$ represents a multiplier of three-fourths of the original magnitude.
  - Fractions as a quotient: $\frac{3}{4}$ represents the result obtained when three is divided by four.
  - Fractions as a ratio: $\frac{3}{4}$ is a comparison of 3 of a quantity to the whole quantity of 4.
- A ratio may be written using a colon (*a*:*b*), the word *to* (*a* to *b*), or fraction notation $\left(\frac{a}{b}\right)$.
- The order of the values in a ratio is directly related to the order in which the quantities are compared.
  - Example: In a certain class, there is a ratio of 3 girls to 4 boys (3:4).
    Another comparison that could represent the relationship between these quantities is the ratio of 4 boys to 3 girls (4:3). Both ratios give the same information about the number of girls and boys in the class, but they are distinct ratios. When you switch the order of comparison (girls to boys vs. boys to girls), there are different ratios being expressed.
- Fractions may be used when determining equivalent ratios.
  - Example: The ratio of girls to boys in a class is 3:4, this can be interpreted as:

    $$\text{number of girls} = \frac{3}{4} \cdot \text{number of boys.}$$

    In a class with 16 boys, number of girls = $\frac{3}{4} \cdot (16) = 12$ girls.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Represent a relationship between two quantities using ratios.
- Represent a relationship in words that makes a comparison by using the notations $\frac{a}{b}$, *a*:*b*, and *a to b.*
- Create a relationship in words for a given ratio expressed symbolically.

6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

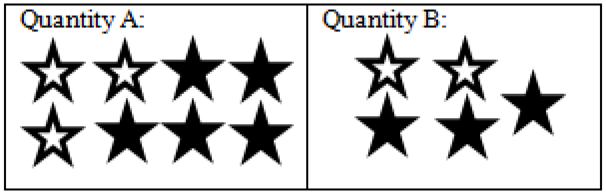
Understanding the Standard	Essential Knowledge and Skills
− Example: A similar comparison could compare the ratio of boys to girls in the class as being 4:3, which can be interpreted as: number of boys = $\frac{4}{3} \cdot$ number of girls. In a class with 12 girls, number of boys = $\frac{4}{3} \cdot (12) = 16$ boys. • A ratio can compare two real-world quantities (e.g., miles per gallon, unit rate, and circumference to diameter of a circle). • Ratios may or may not be written in simplest form. • A ratio can represent different comparisons within the same quantity or between different quantities.	

Ratio	Comparison
part-to-whole (within the same quantity)	compare part of a whole to the entire whole
part-to-part (within the same quantity)	compare part of a whole to another part of the same whole
whole-to-whole (different quantities)	compare all of one whole to all of another whole
part-to-part (different quantities)	compare part of one whole to part of another whole

6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

Understanding the Standard	Essential Knowledge and Skills

– Examples: Given Quantity A and Quantity B, the following comparisons could be expressed.



Ratio	Example	Ratio Notation(s)
part-to-whole (within the same quantity)	compare the number of unfilled stars to the total number of stars in Quantity A	3:8; 3 to 8; or $\frac{3}{8}$
part-to-part [1] (within the same quantity)	compare the number of unfilled stars to the number of filled stars in Quantity A	3:5 or 3 to 5
whole-to-whole [1] (different quantities)	compare the number of stars in Quantity A to the number of stars in Quantity B	8:5 or 8 to 5
part-to-part [1] (different quantities)	compare the number of unfilled stars in Quantity A to the number of unfilled stars in Quantity B	3:2 or 3 to 2

[1] Part-to-part comparisons and whole-to-whole comparisons are ratios that are not typically represented in fraction notation except in certain contexts, such as determining whether two different ratios are equivalent.

6.2 The student will

a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and

b) compare and order positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard

- Fractions, decimals and percents can be used to represent part-to-whole ratios.
  - Example: The ratio of dogs to the total number of pets at a grooming salon is 5:8. This implies that 5 out of every 8 pets being groomed is a dog. This part-to-whole ratio could be represented as the fraction $\frac{5}{8}$ ($\frac{5}{8}$ of all pets are dogs), the decimal 0.625 (0.625 of the number of pets are dogs), or as the percent 62.5% (62.5% of the pets are dogs).
- Fractions, decimals, and percents are three different ways to express the same number. Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent.
- Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, base ten blocks, fraction circles, number lines, colored counters, cubes, decimal squares, shaded figures, shaded grids, or calculators).
- *Percent* means “per 100” or how many “out of 100”; *percent* is another name for *hundredths*.
- A number followed by a percent symbol (%) is equivalent to a fraction with that number as the numerator and with 100 as the denominator (e.g., 30% = $\frac{30}{100} = \frac{3}{10}$; 139% = $\frac{139}{100}$).
- Percents can be expressed as decimals (e.g., 38% = $\frac{38}{100}$ = 0.38; 139% = $\frac{139}{100}$ = 1.39).
- Some fractions can be rewritten as equivalent fractions with denominators of powers of 10, and can be represented as decimals or percents (e.g., $\frac{3}{5} = \frac{6}{10} = \frac{60}{100}$ = 0.60 = 60%). Fractions, decimals, and percents can be represented by using an area model, a set model, or a measurement model. For example, the fraction $\frac{1}{3}$ is shown below using each of the three models.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Represent ratios as fractions (proper or improper), mixed numbers, decimals, and/or percents. (a)
- Determine the decimal and percent equivalents for numbers written in fraction form (proper or improper) or as a mixed number, including repeating decimals. (a)
- Represent and determine equivalencies among decimals, percents, fractions (proper or improper), and mixed numbers that have denominators that are 12 or less or factors of 100. (a)
- Compare two percents using pictorial representations and symbols (<, ≤, ≥, >, =). (b)
- Order no more than four positive rational numbers expressed as fractions (proper or improper), mixed numbers, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less or factors of 100). Ordering may be in ascending or descending order. (b)

6.2 The student will

a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and

b) compare and order positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills

• Percents are used to solve practical problems including sales, data description, and data comparison.
• The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of positive rational numbers are: $\sqrt{25}$, 0.275, $\frac{1}{4}$, 82, 75%, $\frac{22}{5}$, $4.\overline{59}$.
• Students are not expected to know the names of the subsets of the real numbers until grade eight.
• Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., 3 $\frac{5}{8}$).
• Strategies using 0, $\frac{1}{2}$ and 1 as benchmarks can be used to compare fractions.
  – Example: Which is greater, $\frac{4}{7}$ or $\frac{3}{9}$? $\frac{4}{7}$ is greater than $\frac{1}{2}$ because 4, the numerator, represents more than half of 7, the denominator. The denominator tells the number of parts that make the whole. $\frac{3}{9}$ is less than $\frac{1}{2}$ because 3, the numerator, is less than half of 9, the denominator, which tells the number of parts that make the whole. Therefore, $\frac{4}{7} > \frac{3}{9}$.
• When comparing two fractions close to 1, use the distance from 1 as your benchmark.
  – Example: Which is greater, $\frac{6}{7}$ or $\frac{8}{9}$? $\frac{6}{7}$ is $\frac{1}{7}$ away from 1 whole. $\frac{8}{9}$ is $\frac{1}{9}$ away from 1 whole. Since, $\frac{1}{9} < \frac{1}{7}$, then $\frac{6}{7}$ is a greater distance away from 1 whole than $\frac{8}{9}$. Therefore, $\frac{6}{7} < \frac{8}{9}$.	

6.2 **The student will**

a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and

b) compare and order positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Some fractions such as $\frac{1}{8}$, have a decimal representation that is a terminating decimal (e. g., $\frac{1}{8} = 0.125$) and some fractions such as $\frac{2}{9}$, have a decimal representation that does not terminate but continues to repeat (e. g., $\frac{2}{9}$ = 0.222…). The repeating decimal can be written with ellipses (three dots) as in 0.222… or denoted with a bar above the digits that repeat as in $0.\overline{2}$.	

6.3 **The student will**
a) identify and represent integers;
b) compare and order integers; and
c) identify and describe absolute value of integers.

Understanding the Standard	Essential Knowledge and Skills
• The set of integers includes the set of whole numbers and their opposites {…-2, -1, 0, 1, 2, …}. Zero has no opposite and is an integer that is neither positive nor negative. • Integers are used in practical situations, such as temperature (above/below zero), deposits/withdrawals in a checking account, golf (above/below par), time lines, football yardage, positive and negative electrical charges, and altitude (above/below sea level). • Integers should be explored by modeling on a number line and using manipulatives, such as two-color counters, drawings, or algebra tiles. • The opposite of a positive number is negative and the opposite of a negative number is positive. • Positive integers are greater than zero. • Negative integers are less than zero. • A negative integer is always less than a positive integer. • When comparing two negative integers, the negative integer that is closer to zero is greater. • An integer and its opposite are the same distance from zero on a number line. – Example: the opposite of 3 is −3 and the opposite of −10 is 10. • On a conventional number line, a smaller number is always located to the left of a larger number (e.g.,–7 lies to the left of –3, thus –7 < –3; 5 lies to the left of 8 thus 5 is less than 8) • The absolute value of a number is the distance of a number from zero on the number line regardless of direction. Absolute value is represented using the symbol | |(e.g., $\|-6\| = 6$ and $\|6\| = 6$). • The absolute value of zero is zero.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model integers, including models derived from practical situations. (a) • Identify an integer represented by a point on a number line. (a) • Compare and order integers using a number line. (b) • Compare integers, using mathematical symbols ($<, \leq, >, \geq, =$). (b) • Identify and describe the absolute value of an integer. (c)

6.4 The student will recognize and represent patterns with whole number exponents and perfect squares.

Understanding the Standard	Essential Knowledge and Skills
• The symbol $\cdot$ can be used in grade six in place of "x" to indicate multiplication. • In exponential notation, the base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In 8^3, 8 is the base and 3 is the exponent (e.g., $8^3 = 8 \cdot 8 \cdot 8$). • Any real number other than zero raised to the zero power is 1. Zero to the zero power (0^0) is undefined. • A perfect square is a whole number whose square root is an integer (e.g., $36 = 6 \cdot 6 = 6^2$). Zero (a whole number) is a perfect square. • Perfect squares may be represented geometrically as the areas of squares the length of whose sides are whole numbers (e.g., $1 \cdot 1$, $2 \cdot 2$, $3 \cdot 3$, etc.). This can be modeled with grid paper, tiles, geoboards and virtual manipulatives. • The examination of patterns in place value of the powers of 10 in grade six leads to the development of scientific notation in grade seven.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize and represent patterns with bases and exponents that are whole numbers. • Recognize and represent patterns of perfect squares not to exceed 20^2, by using grid paper, square tiles, tables, and calculators. • Recognize powers of 10 with whole number exponents by examining patterns in place value.

The computation and estimation strand in grades six through eight focuses on developing conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring an understanding as to why procedures work and make sense.

Students develop and refine estimation strategies based on an understanding of number concepts, properties and relationships. The development of problem solving, using operations with integers and rational numbers, builds upon the strategies developed in the elementary grades. Students will reinforce these skills and build on the development of proportional reasoning and more advanced mathematical skills.

Students learn to make sense of the mathematical tools available by making valid judgments about the reasonableness of answers. Students will balance the ability to make precise calculations through the application of the order of operations with knowing when calculations may require estimation to obtain appropriate solutions to practical problems.

6.5 The student will

a) multiply and divide fractions and mixed numbers;*
b) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and
c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor. • When the numerator and denominator have no common factors other than 1, then the fraction is in simplest form. • Addition and subtraction are inverse operations as are multiplication and division. • Models for representing multiplication and division of fractions may include arrays, paper folding, repeated addition, repeated subtraction, fraction strips, fraction rods, pattern blocks, and area models. • It is helpful to use estimation to develop computational strategies. – Example: $2\frac{7}{8} \cdot \frac{3}{4}$ is about $\frac{3}{4}$ of 3, so the answer is between 2 and 3. • When multiplying a whole number by a fraction such as $3 \cdot \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. • When multiplying a fraction by a fraction such as $\frac{2}{3} \cdot \frac{3}{4}$, we are asking for part of a part. • When multiplying a fraction by a whole number such as $\frac{1}{2} \cdot 6$, we are trying to determine a part of the whole. • A multistep problem is a problem that requires two or more steps to solve. • Different strategies can be used to estimate the result of computations and judge the reasonableness of the result. – Example: What is an approximate answer for 2.19 ÷ 0.8? The answer is around 2 because 2.19 ÷ 0.8 is about 2 ÷ 1 = 2.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate/model multiplication and division of fractions (proper or improper) and mixed numbers using multiple representations. (a) • Multiply and divide fractions (proper or improper) and mixed numbers. Answers are expressed in simplest form. (a) • Solve single-step and multistep practical problems that involve addition and subtraction with fractions (proper or improper) and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form. (b) • Solve single-step and multistep practical problems that involve multiplication and division with fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form. (b) • Solve multistep practical problems involving addition, subtraction, multiplication and division with decimals. Divisors are limited to a three-digit number, with decimal divisors limited to hundredths. (c)

6.5 The student will

a) multiply and divide fractions and mixed numbers;*

b) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and

c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Understanding the placement of the decimal point is important when determining quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, and by 0.06. • Solving multistep problems in the context of practical situations enhances interconnectedness and proficiency with estimation strategies. • Examples of practical situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, and sharing the cost of a pizza or the prize money from a contest.	

6.6 The student will
a) add, subtract, multiply, and divide integers;*
b) solve practical problems involving operations with integers; and
c) simplify numerical expressions involving integers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard

- The set of integers is the set of whole numbers and their opposites (e.g., …-3, -2, -1, 0, 1, 2, 3…). Zero has no opposite and is neither positive nor negative.
- Integers are used in practical situations, such as temperature changes (above/below zero), balance in a checking account (deposits/withdrawals), golf, time lines, football yardage, and changes in altitude (above/below sea level).
- Concrete experiences in formulating rules for adding, subtracting, multiplying, and dividing integers should be explored by examining patterns using calculators, using a number line, and using manipulatives, such as two-color counters, drawings, or by using algebra tiles.
- Sums, differences, products and quotients of integers are either positive, negative, undefined or zero. This may be demonstrated through the use of patterns and models.
- The order of operations is a convention that defines the computation order to follow in simplifying an expression. Having an established convention ensures that there is only one correct result when simplifying an expression.
- The order of operations is as follows:
  - First, complete all operations within grouping symbols.[1] If there are grouping symbols within other grouping symbols, do the innermost operation first.
  - Second, evaluate all exponential expressions.
  - Third, multiply and/or divide in order from left to right.
  - Fourth, add and/or subtract in order from left to right.

  [1] Parentheses ( ), absolute value | | (e.g., $|3(-5+2)|$), and the division bar (e.g., $\frac{3+4}{5+6}$) should be treated as grouping symbols.
- Expressions are simplified using the order of operations and applying the properties of real numbers. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard):

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Model addition, subtraction, multiplication and division of integers using pictorial representations or concrete manipulatives. (a)
- Add, subtract, multiply, and divide two integers. (a)
- Solve practical problems involving addition, subtraction, multiplication, and division with integers. (b)
- Use the order of operations and apply the properties of real numbers to simplify numerical expressions involving more than two integers. Expressions should not include braces { } or brackets [ ], but may contain absolute value bars | **|.** Simplification will be limited to three operations, which may include simplifying a whole number raised to an exponent of 1, 2 or 3. (c)

6.6 The student will

a) add, subtract, multiply, and divide integers;*

b) solve practical problems involving operations with integers; and

c) simplify numerical expressions involving integers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
– Commutative property of addition: $a + b = b + a$. – Commutative property of multiplication: $a \cdot b = b \cdot a$. – Associative property of addition: $(a + b) + c = a + (b + c)$. – Associative property of multiplication: $(ab)c = a(bc)$. – Subtraction and division are neither commutative nor associative. – Distributive property (over addition/subtraction): $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$. – Identity property of addition (additive identity property): $a + 0 = a \text{ and } 0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \text{ and } 1 \cdot a = a$. – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division. – Inverse property of addition (additive inverse property): $a + (-a) = 0 \text{ and } (-a) + a = 0$. – Multiplicative property of zero: $a \cdot 0 = 0 \text{ and } 0 \cdot a = 0$ – Substitution property: If $a = b$ then b can be substituted for a in any expression, equation or inequality. • The power of a number represents repeated multiplication of the number (e.g., $8^3 = 8 \cdot 8 \cdot 8$). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In the example, 8 is the base, and 3 is the exponent. • Any number, except zero, raised to the zero power is 1. Zero to the zero power (0^0) is undefined.	

Measurement and geometry in the middle grades provide a natural context and connection among many mathematical concepts. Students expand informal experiences with geometry and measurement in the elementary grades and develop a solid foundation for further exploration of these concepts in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

Students develop measurement skills through exploration and estimation. Physical exploration to determine length, weight/mass, liquid volume/capacity, and angle measure are essential to develop a conceptual understanding of measurement. Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, and geometry software provide experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, square-based pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

Students explore and develop an understanding of the Pythagorean Theorem. Understanding how the Pythagorean Theorem can be applied in practical situations has a far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one.)

Level 2: Analysis. Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three.)

Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

Level 4: Deduction. Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

6.7 The student will

a) derive π (pi);

b) solve problems, including practical problems, involving circumference and area of a circle; and

c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles.

Understanding the Standard	Essential Knowledge and Skills
• The value of pi (π) is the ratio of the circumference of a circle to its diameter. Thus, the circumference of a circle is proportional to its diameter. • The calculation of determining area and circumference may vary depending upon the approximation for pi. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi (π) button on a calculator. • Experiences in deriving the formulas for area, perimeter, and volume using manipulatives such as tiles, one-inch cubes, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and their use. • Perimeter is the path or distance around any plane figure. The perimeter of a circle is called the circumference. • The circumference of a circle is about three times the measure of its diameter. • The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle. • The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve. • The area of a circle is computed using the formula $A = \pi r^2$, where r is the radius of the circle. • The perimeter of a square whose side measures s can be determined by multiplying 4 by s ($P = 4s$), and its area can be determined by squaring the length of one side ($A = s^2$). • The perimeter of a rectangle can be determined by computing the sum of twice the length and twice the width ($P = 2l + 2w$, or $P = 2(l + w)$), and its area can be determined by computing the product of the length and the width ($A = lw$). • The perimeter of a triangle can be determined by computing the sum of the side lengths ($P = a + b + c$), and its area can be determined by computing $\frac{1}{2}$ the product of base and the height ($A = \frac{1}{2}bh$).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. (a) • Solve problems, including practical problems, involving circumference and area of a circle when given the length of the diameter or radius. (b) • Solve problems, including practical problems, involving area and perimeter of triangles and rectangles.(c)

6.8 The student will

a) identify the components of the coordinate plane; and

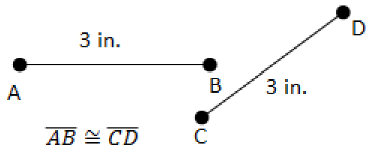
b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.

Understanding the Standard	Essential Knowledge and Skills
• In a coordinate plane, the coordinates of a point are typically represented by the ordered pair (*x*, *y*), where *x* is the first coordinate and *y* is the second coordinate. • Any given point is defined by only one ordered pair in the coordinate plane. • The grid lines on a coordinate plane are perpendicular. • The axes of the coordinate plane are the two intersecting perpendicular lines that divide it into its four quadrants. The *x*-axis is the horizontal axis and the *y*-axis is the vertical axis. • The quadrants of a coordinate plane are the four regions created by the two intersecting perpendicular lines (*x*- and *y*-axes). Quadrants are named in counterclockwise order. The signs on the ordered pairs for quadrant I are (+,+); for quadrant II, (–,+); for quadrant III, (–, –); and for quadrant IV, (+,–). • In a coordinate plane, the origin is the point at the intersection of the *x*-axis and *y*-axis; the coordinates of this point are (0, 0). • For all points on the *x*-axis, the *y*-coordinate is 0. For all points on the *y*-axis, the *x*-coordinate is 0. • The coordinates may be used to name the point. (e.g., the point (2, 7)). It is not necessary to say “the point whose coordinates are (2, 7).” The first coordinate tells the location or distance of the point to the left or right of the *y*-axis and the second coordinate tells the location or distance of the point above or below the *x*-axis. For example, (2, 7) is two units to the right of the *y*-axis and seven units above the *x*-axis. • Coordinates of points having the same *x*-coordinate are located on the same vertical line. For example, (2, 4) and (2, -3) are both two units to the right of the *y*-axis and are vertically seven units from each other. • Coordinates of points having the same *y*-coordinate are located on the same horizontal line. For example, (-4, -2) and (2, -2) are both two units below the *x*-axis and are horizontally six units from each other.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and label the axes, origin, and quadrants of a coordinate plane. (a) • Identify the quadrant or the axis on which a point is positioned by examining the coordinates (ordered pair) of the point. Ordered pairs will be limited to coordinates expressed as integers. (a) • Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. (b) • Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. (b) • Relate the coordinates of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line. Ordered pairs will be limited to coordinates expressed as integers. (b) • Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate. Ordered pairs will be limited to coordinates expressed as integers. Apply these techniques in the context of solving practical and mathematical problems. (b)

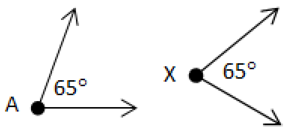
6.9 The student will determine congruence of segments, angles, and polygons.

Understanding the Standard

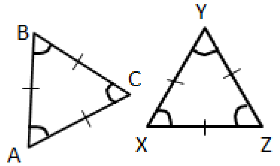
- The symbol for congruency is ≅ .
- Congruent figures have exactly the same size and the same shape. Line segments are congruent if they have the same length. Angles are congruent if they have the same measure. Congruent polygons have an equal number of sides, and all the corresponding sides and angles are congruent.
  - Examples:



$\overline{AB} \cong \overline{CD}$



∠A ≅ ∠X



$\Delta ABC \cong \Delta XYZ$

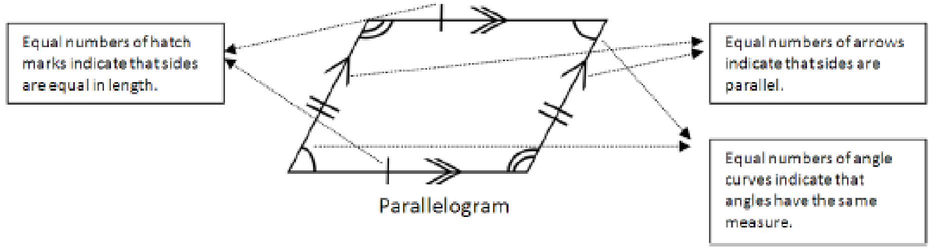
- A polygon is a closed plane figure composed of at least three line segments that do not cross.
- A regular polygon has congruent sides and congruent interior angles.
- The number of lines of symmetry of a regular polygon is equal to the number of sides of the polygon.
- A line of symmetry divides a figure into two congruent parts, each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines.
- Noncongruent figures may have the same shape but not the same size.
- Students should be familiar with geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides. An equal number of hatch (hash) marks indicate that those sides are equal in length. An equal number of arrows indicate that those sides are parallel. An equal number of angle curves indicate that those angles have the same measure. See the diagram below.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify regular polygons.
- Draw lines of symmetry to divide regular polygons into two congruent parts.
- Determine the congruence of segments, angles, and polygons given their properties.
- Determine whether polygons are congruent or noncongruent according to the measures of their sides and angles.

6.9 The student will determine congruence of segments, angles, and polygons.

Understanding the Standard	Essential Knowledge and Skills
 • The determination of the congruence or noncongruence of two figures can be accomplished by placing one figure on top of the other or by comparing the measurements of all corresponding sides and angles. • Construction of congruent line segments, angles, and polygons helps students understand congruency.	

In the middle grades, students develop an awareness of the power of data analysis and the application of probability through fostering their natural curiosity about data and making predictions.

The exploration of various methods of data collection and representation allows students to become effective at using different types of graphs to represent different types of data. Students use measures of center and dispersion to analyze and interpret data.

Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability. Through experiments and simulations, students build on their understanding of the Fundamental Counting Principle from elementary mathematics to learn more about probability in the middle grades.

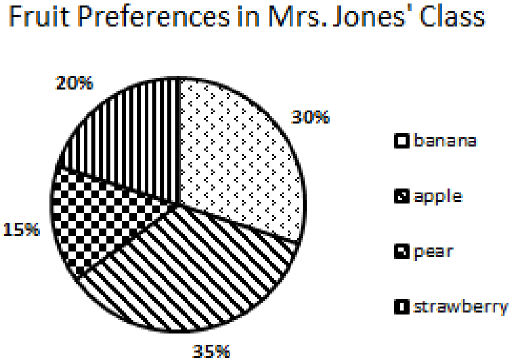
6.10 The student, given a practical situation, will

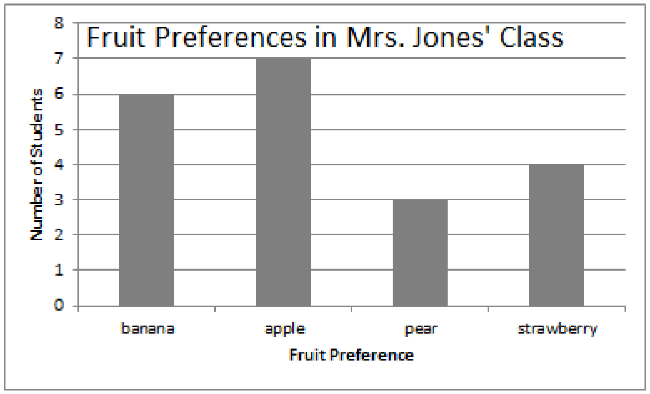
a) represent data in a circle graph;
b) make observations and inferences about data represented in a circle graph; and
c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.

Understanding the Standard

- Circle graphs are used for data showing a relationship of the parts to the whole.
  - Example: the favorite fruit of 20 students in Mrs. Jones class was recorded in the table. Compare the same data displayed in both a circle graph and a bar graph.

Fruit Preference	# of students
banana	6
apple	7
pear	3
strawberry	4





- Circle graphs can represent percent or frequency.
- Circle graphs are not useful for representing data with large numbers of categories.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect, organize and represent data in a circle graph. The number of data values should be limited to allow for comparisons that have denominators of 12 or less or those that are factors of 100 (e.g., in a class of 20 students, 7 choose apples as a favorite fruit, so the comparison is 7 out of 20, $\frac{7}{20}$, or 35%). (a)
- Make observations and inferences about data represented in a circle graph. (b)
- Compare data represented in a circle graph with the same data represented in bar graphs, pictographs, and line plots. (c)

6.10 The student, given a practical situation, will
a) represent data in a circle graph;
b) make observations and inferences about data represented in a circle graph; and
c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.

Understanding the Standard

- Teachers should be reasonable about the selection of data values. The number of data values can affect how a circle graph is constructed (e.g., 10 out of 25 would be 40%, but 7 out of 9 would be $77.\overline{7}\%$, making the construction of a circle graph more complex). Students should have experience constructing circle graphs, but a focus should be placed on the analysis of circle graphs.
- Students are not expected to construct circle graphs by multiplying the percentage of data in a category by 360° in order to determine the central angle measure. Limiting comparisons to fraction parameters noted will assist students in constructing circle graphs.
- To collect data for any problem situation, an experiment can be designed, a survey can be conducted, or other data-gathering strategies can be used. The data can be organized, displayed, analyzed, and interpreted to solve the problem.
- Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data.
- Different types of graphs can be used to display categorical data. The way data are displayed often depends on what someone is trying to communicate.
  - A line plot is used for categorical and discrete numerical data and is used to show frequency of data on a number line. It is a simple way to organize data.
    Example:

Essential Knowledge and Skills

6.10 The student, given a practical situation, will

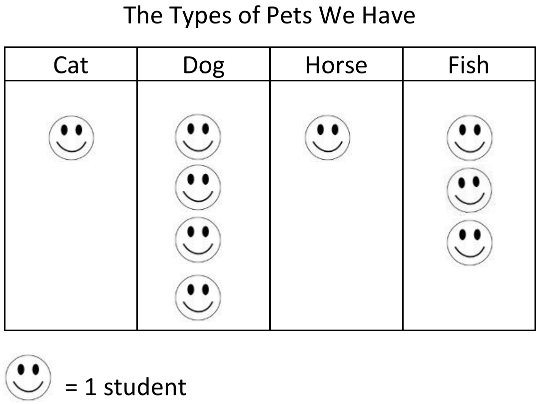
a) represent data in a circle graph;

b) make observations and inferences about data represented in a circle graph; and

c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.

Understanding the Standard	Essential Knowledge and Skills

- A bar graph is used for categorical and discrete numerical data (e.g., number of months or number of people with a particular eye color) and is used to show comparisons.
- A pictograph is mainly used to show categorical data. Pictographs are used to show frequency and compare items. However, the use of partial pictures can give misleading information.
  - Example:



- A circle graph is used for categorical and discrete numerical data. Circle graphs are used to show a relationship of the parts to a whole.
- All graphs must include a title, percent or number labels for data categories, and a key. A key is essential to explain how to read the graph. A title is essential to explain what the graph represents.
- A scale should be chosen that is appropriate for the data values being represented.
- Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions.
- The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inferences).

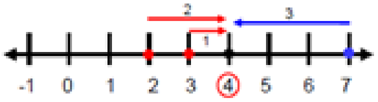
6.11 The student will
a) represent the mean of a data set graphically as the balance point; and
b) determine the effect on measures of center when a single value of a data set is added, removed, or changed.

Understanding the Standard

- Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data.
- Measures of center are types of averages for a data set. They represent numbers that describe a data set. Mean, median, and mode are measures of center that are useful for describing the average for different situations.
  - Mean may be appropriate for sets of data where there are no values much higher or lower than those in the rest of the data set.
  - Median is a good choice when data sets have a couple of values much higher or lower than most of the others.
  - Mode is a good descriptor to use when the set of data has some identical values, when data is non-numeric (categorical) or when data reflects the most popular item.
- Mean can be defined as the point on a number line where the data distribution is balanced. This requires that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances from the mean of all the data points below the mean. This is the concept of mean as the balance point.
  - Example: Given the data set:

    2, 3, 4, 7

    The mean value of 4 can be represented on a number line as the balance point:



- The mean can also be found by calculating the numerical average of the data set.
- In grade five mathematics, mean is defined as fair share.
- Defining mean as the balance point is a prerequisite for understanding standard deviation, which is addressed in high school level mathematics.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Represent the mean of a set of data graphically as the balance point represented in a line plot. (a)
- Determine the effect on measures of center when a single value of a data set is added, removed, or changed. (b)

6.11 The student will

a) represent the mean of a data set graphically as the balance point; and

b) determine the effect on measures of center when a single value of a data set is added, removed, or changed.

Understanding the Standard	Essential Knowledge and Skills
• The median is the middle value of a data set in ranked order. If there are an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the numerical average of the two middle values. • The mode is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there is more than one value that occurs most often, all these most-frequently-occurring values are modes. When there are exactly two modes, the data set is bimodal.	

Patterns, functions and algebra become a larger mathematical focus in the middle grades as students extend their knowledge of patterns developed in the elementary grades.

Students make connections between the numeric concepts of ratio and proportion and the algebraic relationships that exist within a set of equivalent ratios. Students use variable expressions to represent proportional relationships between two quantities and begin to connect the concept of a constant of proportionality to rate of change and slope. Representation of relationships between two quantities using tables, graphs, equations, or verbal descriptions allow students to connect their knowledge of patterns to the concept of functional relationships. Graphing linear equations in two variables in the coordinate plane is a focus of the study of functions which continues in high school mathematics.

Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality, domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades. Students learn to solve equations by using concrete materials. They expand their skills from one-step to multistep equations and inequalities through their application in practical situations.

6.12 **The student will**

a) represent a proportional relationship between two quantities, including those arising from practical situations;

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard

- A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities.
- Equivalent ratios arise by multiplying each value in a ratio by the same constant value. For example, the ratio of 4:2 would be equivalent to the ratio 8:4, since each value in the first ratio could be multiplied by 2 to obtain the second ratio.
- A proportional relationship consists of two quantities where there exists a constant number (constant of proportionality) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.
- Proportional thinking requires students to thinking multiplicatively, versus additively. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e.., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, because context can help students to see the relationship. Students will explore algebraic representations of additive relationships in grade seven.
  - Example:

Additive relationship:

x	y
2 $\xrightarrow{+8}$	10
3 $\xrightarrow{+8}$	11
4 $\xrightarrow{+8}$	12
5 $\xrightarrow{+8}$	13

Multiplicative relationship:

x	y
2 $\xrightarrow{\cdot 5}$	10
3 $\xrightarrow{\cdot 5}$	15
4 $\xrightarrow{\cdot 5}$	20
5 $\xrightarrow{\cdot 5}$	25

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio. (a)
- Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a practical situation. (a)
- Identify the unit rate of a proportional relationship represented by a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (b)
- Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate. Unit rates are limited to positive values. (b)
- Determine whether a proportional relationship exists between two quantities, when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (c)
- Determine whether a proportional relationship exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. (c)

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;

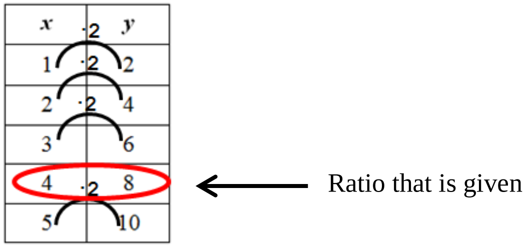
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard

  - ○ In the additive relationship, *y* is the result of adding 8 to *x.*
  - ○ In the multiplicative relationship, *y* is the result of multiplying 5 times *x*.
  - ○ The ordered pair (2, 10) is a quantity in both relationships; however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.
- Students have had experiences with tables of values (input/output tables that are additive and multiplicative) in elementary grades.
- A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios. A constant exists that can be multiplied by the measure of one quantity to get the measure of the other quantity for every ratio pair. The same proportional relationship exists between each pair of quantities in a ratio table.
  - − Example: Given that the ratio of *y* to *x* in a proportional relationship is 8:4, create a ratio table that includes three additional equivalent ratios.



x	y
1	2
2	4
3	6
4	8
5	10

Ratio that is given

Students have had experience with tables of values (input/output tables) in elementary grades and the concept of a ratio table should be connected to their prior knowledge of representing number patterns in tables.

Essential Knowledge and Skills

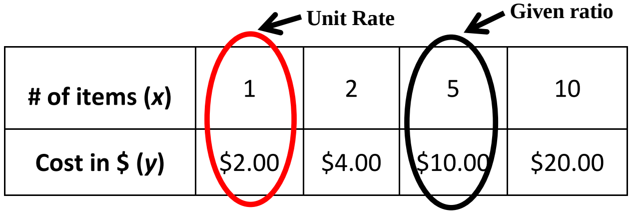
- Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values. (d)

6.12 **The student will**

- **a) represent a proportional relationship between two quantities, including those arising from practical situations;**
- **b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;**
- **c) determine whether a proportional relationship exists between two quantities; and**
- **d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.**

Understanding the Standard

- A rate is a ratio that involves two different units and how they relate to each other. Relationships between two units of measure are also rates (e.g., inches per foot).
- A unit rate describes how many units of the first quantity of a ratio correspond to one unit of the second quantity.
  - Example: If it costs $10 for 5 items at a store (a ratio of 10:5 comparing cost to the number of items), then the unit rate would be $2.00/per item (a ratio of 2:1 comparing cost to number of items).



# of items (*x*)	1	2	5	10
Cost in $ (*y*)	$2.00	$4.00	$10.00	$20.00

- Any ratio can be converted into a unit rate by writing the ratio as a fraction and then dividing the numerator and denominator each by the value of the denominator. Example: It costs $8 for 16 gourmet cookies at a bake sale. What is the price per cookie (unit rate) represented by this situation?

$$\frac{8}{16} = \frac{8 \div 16}{16 \div 16} = \frac{0.5}{1}$$

  So, it would cost $0.50 per cookie, which would be the unit rate.

  - Example: $\frac{8}{16}$ and 40 to 10 are ratios, but are not unit rates. However, $\frac{0.5}{1}$ and 4 to 1 are unit rates.
- Students in grade six should build a conceptual understanding of proportional relationships and unit rates before moving to more abstract representations and complex computations in higher grade levels. Students are not expected to use formal calculations for slope and unit rates (e.g., slope formula) in grade six.

Essential Knowledge and Skills

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard

- Example of a proportional relationship:

  Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza. This ratio table represents the cost (*y*) per number of pizzas ordered (*x*).

x number of pizzas	1	2	3	4
y total cost	8	16	24	32

In this relationship, the ratio of *y* (cost in $) to *x* (number of pizzas) in each ordered pair is the same:

$$\frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4}$$

- Example of a non-proportional relationship:

  Uptown Pizza sells medium pizzas for $7 each but charges a $3 delivery fee per order. This table represents the cost per number of pizzas ordered.

x number of pizzas	1	2	3	4
y total cost	10	17	24	31

The ratios represented in the table above are not equivalent.

In this relationship, the ratio of *y* to *x* in each ordered pair is not the same:

$$\frac{10}{1} \neq \frac{17}{2} \neq \frac{24}{3} \neq \frac{31}{4}$$

Essential Knowledge and Skills

6.12 The student will

- **a) represent a proportional relationship between two quantities, including those arising from practical situations;**
- **b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;**
- **c) determine whether a proportional relationship exists between two quantities; and**
- **d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.**

Understanding the Standard

Other non-proportional relationships will be studied in later mathematics courses.

- Proportional relationships can be described verbally using the phrases “for each,” “for every,” and “per.”
- Proportional relationships involve collections of pairs of equivalent ratios that may be graphed in the coordinate plane. The graph of a proportional relationship includes ordered pairs (x, y) that represent pairs of values that may be represented in a ratio table.
- Proportional relationships can be expressed using verbal descriptions, tables, and graphs.
  - Example: (verbal description) To make a drink, mix 1 liter of syrup with 3 liters of water. If x represents how many liters of syrup are in the mixture and y represents how many liters of water are in the mixture, this proportional relationship can be represented using a ratio table:

Syrup (liters) x	1	2	3	4
Water (liters) y	3	6	9	12

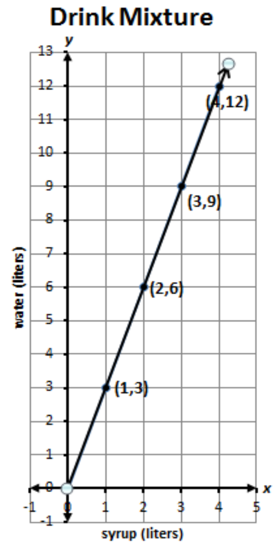
The ratio of the amount of water (y) to the amount of syrup (x) is 3:1. Additionally, the proportional relationship may be graphed using the ordered pairs in the table.

Essential Knowledge and Skills

6.12 The student will

- **a) represent a proportional relationship between two quantities, including those arising from practical situations;**
- **b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;**
- **c) determine whether a proportional relationship exists between two quantities; and**
- **d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.**

Understanding the Standard



- The representation of the ratio between two quantities may depend upon the order in which the quantities are being compared.
  - Example: In the mixture example above, we could also compare the ratio of the liters of syrup per liters of water, as shown:

Water (liters) x	3	6	9	12
Syrup (liters) y	1	2	3	4

In this comparison, the ratio of the amount of syrup (y) to the amount of water (x) would be 1:3.

Essential Knowledge and Skills

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;

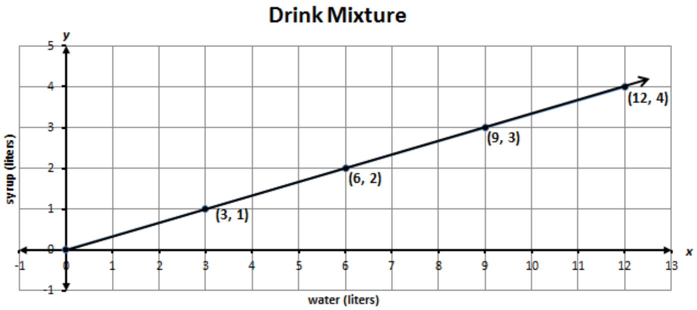
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

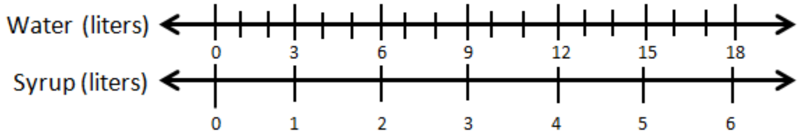
Understanding the Standard

The graph of this relationship could be represented by:



Students should be aware of how the order in which quantities are compared affects the way in which the relationship is represented as a table of equivalent ratios or as a graph.

- Double number line diagrams can also be used to represent proportional relationships and create collections of pairs of equivalent ratios.
  - Example:



In this proportional relationship, there are three liters of water for each liter of syrup represented on the number lines.

Essential Knowledge and Skills

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;

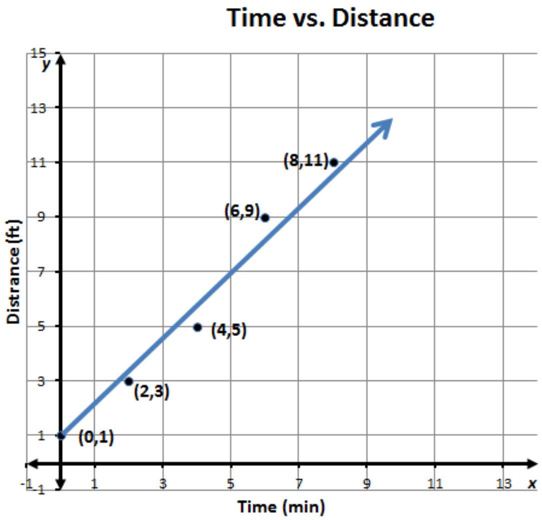
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard

- A graph representing a proportional relationship includes ordered pairs that lie in a straight line that, if extended, would pass through (0, 0), creating a pattern of horizontal and vertical increases. The context of the problem and the type of data being represented by the graph must be considered when determining whether the points are to be connected by a straight line on the graph.
  - Example of the graph of a non-proportional relationship:



The relationship of distance (y) to time (x) is non-proportional. The ratio of y to x for each ordered pair is not equivalent. That is,

$$\frac{11}{8} \neq \frac{9}{6} \neq \frac{5}{4} \neq \frac{3}{2} \neq \frac{1}{0}$$

Essential Knowledge and Skills

6.12 The student will

- **a) represent a proportional relationship between two quantities, including those arising from practical situations;**
- **b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;**
- **c) determine whether a proportional relationship exists between two quantities; and**
- **d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.**

Understanding the Standard

The points of the graph do not lie in a straight line. Additionally, the line does not pass through the point (0, 0), thus the relationship of *y* to *x* cannot be considered proportional.

- Practical situations that model proportional relationships can typically be represented by graphs in the first quadrant, since in most cases the values for *x* and *y* are not negative.
- Unit rates are not typically negative in practical situations involving proportional relationships.
- A unit rate could be used to find missing values in a ratio table.
  - Example: A store advertises a price of $25 for 5 DVDs. What would be the cost to purchase 2 DVDs? 3 DVDs? 4 DVDs?

# DVDs	1	2	3	4	5
Cost	$5	?	?	?	$25

The ratio of $25 per 5 DVDs is also equivalent to a ratio of $5 per 1 DVD, which would be the unit rate for this relationship. This unit rate could be used to find the missing value of the ratio table above. If we multiply the number of DVDs by a constant multiplier of 5 (the unit rate) we obtain the total cost. Thus, 2 DVDs would cost $10, 3 DVDs would cost $15, and 4 DVDs would cost $20.

Essential Knowledge and Skills

6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
• A one-step linear equation may include, but not be limited to, equations such as the following: $2x = 5$; $y - 3 = -6$; $\frac{1}{5}x = -3$; $a - (-4) = 11$. • A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable. • An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an “equal sign (=)” (e.g., $\frac{3}{4}$, $5x$, $140 - 38.2$, $18 \cdot 21$, $5 + x$.) • An expression that contains a variable is a variable expression. A variable expression is like a phrase: As a phrase does not have a verb, so an expression does not have an “equal sign (=)”. An expression cannot be solved. • A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. Example, the verbal expression “a number multiplied by 5” could be represented by the variable expression “$n \cdot 5$” or “$5n$.” • An algebraic expression is a variable expression that contains at least one variable (e.g., $x - 3$). • A verbal sentence is a complete word statement (e.g., “The sum of a number and two is five” could be represented by “$n + 2 = 5$”). • An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., $2x = 7$). • A term is a number, variable, product, or quotient in an expression of sums and/or differences. In $7x^2 + 5x - 3$, there are three terms, $7x^2$, $5x$, and 3. • A coefficient is the numerical factor in a term. Example: in the term $3xy^2$, 3 is the coefficient; in the term z, 1 is the coefficient. • An equation is a mathematical sentence stating that two expressions are equal. • A variable is a symbol used to represent an unknown quantity.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Identify examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient. • Represent and solve one-step linear equations in one variable, using a variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale. • Apply properties of real numbers and properties of equality to solve a one-step equation in one variable. Coefficients are limited to integers and unit fractions. Numeric terms are limited to integers. • Confirm solutions to one-step linear equations in one variable. • Write verbal expressions and sentences as algebraic expressions and equations. • Write algebraic expressions and equations as verbal expressions and sentences. • Represent and solve a practical problem with a one-step linear equation in one variable.

6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
• The solution to an equation is a value that makes it a true statement. Many equations have one solution and are represented as a point on a number line. Solving an equation or inequality involves a process of determining which value(s) from a specified set, if any, make the equation or inequality a true statement. Substitution can be used to determine whether a given value(s) makes an equation or inequality true. • Properties of real numbers and properties of equality can be used to solve equations, justify equation solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard). – Commutative property of addition: $a + b = b + a$. – Commutative property of multiplication: $a \cdot b = b \cdot a$. – Subtraction and division are neither commutative nor associative. – Identity property of addition (additive identity property): $a + 0 = a \; and \; 0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \; and \; 1 \cdot a = a$. – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division. – Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$). – Inverse property of addition (additive inverse property): $a + (-a) = 0 \; and \; (-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \; and \; \frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0 \; and \; 0 \cdot a = 0$. – Division by zero is not a possible mathematical operation. It is undefined. – Addition property of equality: If $a = b$, then $a + c = b + c$.	

6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
– Subtraction property of equality: If $a = b$, then $a - c = b - c$. – Multiplication property of equality: If $a = b$, then $a \cdot c = b \cdot c$. – Division property of equality: If $a = b \; and \; c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. – Substitution property: If $a = b$ then b can be substituted for a in any expression, equation or inequality.	

6.14 The student will

a) represent a practical situation with a linear inequality in one variable; and

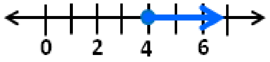
b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.

Understanding the Standard

- The solution set to an inequality is the set of all numbers that make the inequality true.
- Inequalities can represent practical situations.

  Example: Jaxon works at least 4 hours per week mowing lawns. Write an inequality representing this situation and graph the solution.

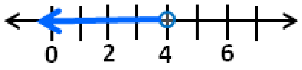
  $x \geq 4$ or $4 \leq x$



  Students might then be asked: “Would Jaxon ever work 3 hours in a week? 6 hours?

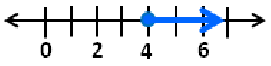
- The variable in an inequality may represent values that are limited by the context of the problem or situation. Example: if the variable represents all children in a classroom who are taller than 36 inches, the variable will be limited to have a minimum and maximum value based on the heights of the children. Students are not expected to represent these situations with a compound inequality (e.g., $36 < x < 70$) but only recognize that the values satisfying the single inequality ($x > 36$) will be limited by the context of the situation.
- Inequalities using the < or > symbols are represented on a number line with an open circle on the number and a shaded line over the solution set.

  Example: When graphing $x < 4$, use an open circle above the 4 to indicate that the 4 is not included.



- Inequalities using the ≤ or ≥ symbols are represented on a number line with a closed circle on the number and shaded line in the direction of the solution set.

  Example: When graphing $x \geq 4$ fill in the circle above the 4 to indicate that the 4 is included.



- It is important for students to see inequalities written with the variable before the inequality symbol and after. Example: $x > 5$ is not the same relationship as $5 > x$. However, $x > 5$ is the same relationship as $5 < x$.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to

- Given a verbal description, represent a practical situation with a one-variable linear inequality. (a)
- Apply properties of real numbers and the addition or subtraction property of inequality to solve a one-step linear inequality in one variable, and graph the solution on a number line. Numeric terms being added or subtracted from the variable are limited to integers. (b)
- Given the graph of a linear inequality with integers, represent the inequality two different ways (e.g., $x < -5$ or $-5 > x$) using symbols. (b)
- Identify a numerical value(s) that is part of the solution set of a given inequality. (a, b)

6.14 The student will

a) represent a practical situation with a linear inequality in one variable; and

b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.

Understanding the Standard	Essential Knowledge and Skills
• A one-step linear inequality may include, but not be limited to, inequalities such as the following: $2 + x > 5$; $y - 3 \le -6$; a -(-4) ≥ 11. • Solving an equation or inequality involves a process of determining which value(s) from a specified set, if any, make the equation or inequality a true statement. Substitution can be used to determine whether a given value(s) makes an equation or inequality true. • Properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard): – Commutative property of addition: $a + b = b + a$. – Commutative property of multiplication: $a \cdot b = b \cdot a$. – Subtraction and division are neither commutative nor associative. – Identity property of addition (additive identity property): $a + 0 = a \; and \; 0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \; and \; 1 \cdot a = a$. – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division. – Inverses are numbers that combine with other numbers and result in identity elements (e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$). – Inverse property of addition (additive inverse property): $a + (-a) = 0 \; and \; (-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \; and \; \frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0 \; and \; 0 \cdot a = 0$.	

6.14 The student will

a) represent a practical situation with a linear inequality in one variable; and

b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.

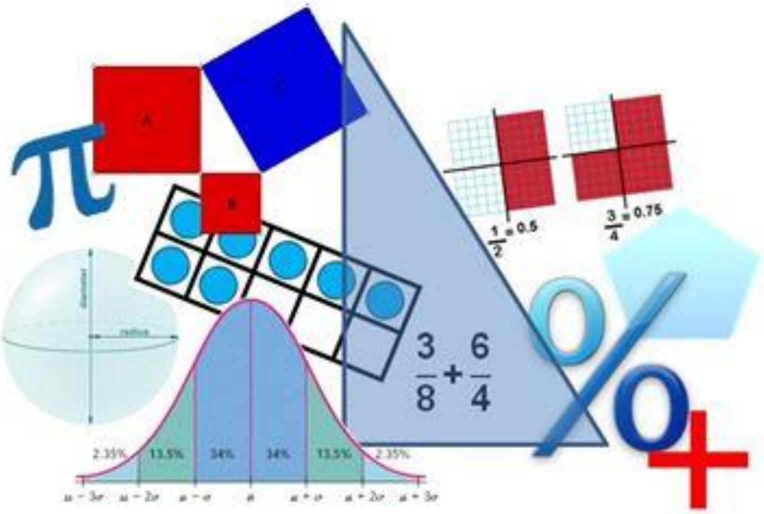
Understanding the Standard	Essential Knowledge and Skills
– Addition property of inequality: If $a < b$, then $a + c < b + c$; if $a > b$, then $a + c > b + c$ (this property also applies to $\leq and \geq$). – Subtraction property of inequality: If $a < b$, then $a - c < b - c$; if $a > b$, then $a - c > b - c$ (this property also applies to $\leq and \geq$). – Substitution property: If $a = b$ then b can be substituted for a in any expression, equation or inequality.	

Mathematics

2016 Standards of Learning

Grade 7

Curriculum Framework



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NOTICE

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for *Standards of Learning* assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the *Standards of Learning*. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and to see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "… the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> “Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that requires students to think critically, to reason, to develop problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Mathematics instruction in grades six through eight continues to focus on the development of number sense, with emphasis on rational and real numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to many middle school mathematics topics.

Students develop an understanding of integers and rational numbers using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation.
Flexible thinking about rational number representations is encouraged when students solve problems.

Students develop an understanding of real numbers and the properties of operations on real numbers through experiences with rational and irrational numbers and apply the order of operations.

Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) compare and order numbers greater than zero written in scientific notation;*
c) compare and order rational numbers;*
d) determine square roots of perfect squares;* and
e) identify and describe absolute value of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Negative exponents for powers of 10 are used to represent numbers between 0 and 1. (e.g., $10^{-3} = \frac{1}{10^3} = 0.001$). • Negative exponents for powers of 10 can be investigated through patterns such as: $10^{2} = 100$ $10^{1} = 10$ $10^{0} = 1$ $10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$ $10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$ • Percent means “per 100” or how many “out of 100”; percent is another name for hundredths. • A percent is a ratio in which the denominator is 100. A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., $\frac{3}{5} = \frac{60}{100} = 0.60 = 60\%$). • Scientific notation should be used whenever the situation calls for use of very large or very small numbers. • A number written in scientific notation is the product of two factors — a decimal greater than or equal to 1 but less than 10, and a power of 10 (e.g., $3.1 \times 10^5 = 310{,}000$ and $2.85 \times 10^{-4} = 0.000285$). • The set of integers includes the set of whole numbers and their opposites, {…-2, -1, 0, 1, 2…}. Zero has no opposite and is neither positive nor negative. • The opposite of a positive number is negative and the opposite of a negative number is positive.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize powers of 10 with negative exponents by examining patterns. (a) • Represent a power of 10 with a negative exponent in fraction and decimal form. (a) • Convert between numbers greater than 0 written in scientific notation and decimals. (b) • Compare and order no more than four numbers greater than 0 written in scientific notation. Ordering may be in ascending or descending order. (b) • Compare and order no more than four rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions and mixed numbers may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place. Ordering may be in ascending or descending order. (c) • Identify the perfect squares from 0 to 400. (d) • Determine the positive square root of a perfect square from 0 to 400. (d) • Demonstrate absolute value using a number line. (e)

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) compare and order numbers greater than zero written in scientific notation;*
c) compare and order rational numbers;*
d) determine square roots of perfect squares;* and
e) identify and describe absolute value of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are: $\sqrt{25}$, $\frac{1}{4}$, -2.3, 82, 75%, $4.\overline{59}$. • Rational numbers may be expressed as positive and negative fractions or mixed numbers, positive and negative decimals, integers and percents. • Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). Fractions can be positive or negative. • Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, base ten blocks, fraction circles, colored counters, cubes, decimal squares, shaded figures, shaded grids, number lines and calculators). • Negative numbers lie to the left of zero and positive numbers lie to the right of zero on a number line. • Smaller numbers always lie to the left of larger numbers on the number line. • A perfect square is a whole number whose square root is an integer. Zero (a whole number) is a perfect square. (e.g., $36 = 6 \cdot 6 = 6^2$).	• Determine the absolute value of a rational number. (e) • Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle to solve practical problems. (e)

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) compare and order numbers greater than zero written in scientific notation;*
c) compare and order rational numbers;*
d) determine square roots of perfect squares;* and
e) identify and describe absolute value of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• A square root of a number is a number which, when multiplied by itself, produces the given number (e.g., $\sqrt{121}$ is 11 since $11 \cdot 11 = 121$). • The symbol $\sqrt{\ }$ may be used to represent a non-negative (principal) square root. Students in grade 8 mathematics will explore the negative square root of a number, denoted $-\sqrt{\ }$. • The square root of a number can be represented geometrically as the length of a side of a square. • Squaring a number and taking a square root are inverse operations. • The absolute value of a number is the distance from 0 on the number line regardless of direction. Distance is positive (e.g., $\left\|-\frac{1}{2}\right\| = \frac{1}{2}$). • The absolute value of zero is zero.	

The computation and estimation strand in grades six through eight focuses on developing conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring an understanding as to why procedures work and make sense.

Students develop and refine estimation strategies based on an understanding of number concepts, properties and relationships. The development of problem solving, using operations with integers and rational numbers, builds upon the strategies developed in the elementary grades. Students will reinforce these skills and build on the development of proportional reasoning and more advanced mathematical skills.

Students learn to make sense of the mathematical tools available by making valid judgments of the reasonableness of answers. Students will balance the ability to make precise calculations through the application of the order of operations with knowing when calculations may require estimation to obtain appropriate solutions to practical problems.

7.2 The student will solve practical problems involving operations with rational numbers.

Understanding the Standard	Essential Knowledge and Skills
• The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where a and b are integers and b does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are: $\sqrt{25}$, $\frac{1}{4}$, -2.3, 82, 75%, $4.\overline{59}$. • Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). A fraction can have a positive or negative value. • Solving problems in the context of practical situations enhances interconnectedness and proficiency with estimation strategies. Practical problems involving rational numbers in grade seven provide students the opportunity to use problem solving to apply computation skills involving positive and negative rational numbers expressed as integers, fractions, and decimals, along with the use of percents within practical situations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve practical problems involving addition, subtraction, multiplication, and division with rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place.

7.3 The student will solve single-step and multistep practical problems, using proportional reasoning.

Understanding the Standard

- A proportion is a statement of equality between two ratios. A proportion can be written as $\frac{a}{b} = \frac{c}{d}$, a:b = c:d, or a is to b as c is to d.
- Equivalent ratios arise by multiplying each value in a ratio by the same constant value. For example, the ratio of 3:2 would be equivalent to the ratio 6:4 because each of the values in 3:2 can be multiplied by 2 to get 6:4.
- A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios.
- A proportion can be solved by determining the product of the means and the product of the extremes. For example, in the proportion a:b = c:d, a and d are the extremes and b and c are the means. If values are substituted for a, b, c, and d such as 5:12 = 10:24, then the product of extremes ($5 \cdot 24$) is equal to the product of the means ($12 \cdot 10$).
- In a proportional relationship, two quantities increase multiplicatively. One quantity is a constant multiple of the other.
- A proportion is an equation which states that two ratios are equal. When solving a proportion, the ratios may first be written as fractions.
    - Example: A recipe for oatmeal cookies calls for 2 cups of flour for every 3 cups of oatmeal. How much flour is needed for a larger batch of cookies that uses 9 cups of oatmeal? To solve this problem, the ratio of flour to oatmeal could be written as a fraction in the proportion used to determine the amount of flour needed when 9 cups of oatmeal is used. To use a proportion to solve for the unknown cups of flour needed, solve the proportion: $\frac{2}{3} = \frac{x}{9}$.
      To use a table of equivalent ratios to find the unknown amount, create the table:

flour (cups)	2	4	?
oatmeal (cups)	3	6	9

      To complete the table, we must create an equivalent ratio to 2:3; just as 4:6 is equivalent to 2:3, then 6 cups of flour to 9 cups of oatmeal would create an equivalent ratio.
- A proportion can be solved by determining equivalent ratios.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Given a proportional relationship between two quantities, create and use a ratio table to determine missing values.
- Write and solve a proportion that represents a proportional relationship between two quantities to find a missing value.
- Apply proportional reasoning to convert units of measurement within and between the U.S. Customary System and the metric system when given the conversion factor.
- Apply proportional reasoning to solve practical problems, including scale drawings. Scale factors shall have denominators no greater than 12 and decimals no less than tenths**.**
- Using 10% as a benchmark, compute 5%, 10%, 15%, or 20% of a given whole number.
- Using 10% as a benchmark, compute 5%, 10%, 15%, or 20% in a practical situation such as tips, tax, and discounts.
- Solve problems involving tips, tax, and discounts. Limit problems to only one percent computation per problem.

7.3 The student will solve single-step and multistep practical problems, using proportional reasoning.

Understanding the Standard	Essential Knowledge and Skills
• A rate is a ratio that compares two quantities measured in different units. A unit rate is a rate with a denominator of 1. Examples of rates include miles/hour and revolutions/minute. • Proportions are used in everyday contexts, such as speed, recipe conversions, scale drawings, map reading, reducing and enlarging, comparison shopping, tips, tax, and discounts, and monetary conversions. • A multistep problem is a problem that requires two or more steps to solve. • Proportions can be used to convert length, weight (mass), and volume (capacity) within and between measurement systems. For example, if 1 inch is about 2.54 cm, how many inches are in 16 cm? $\frac{1\ inch}{2.54\ cm} = \frac{x\ inch}{16\ cm}$ $2.54x = 1 \cdot 16$ $2.54x = 16$ $x = \frac{16}{2.54}$ $x = 6.299$ or about 6.3 inches • Examples of conversions may include, but are not limited to: − Length: between feet and miles; miles and kilometers − Weight: between ounces and pounds; pounds and kilograms − Volume: between cups and fluid ounces; gallons and liters • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • When converting measurement units in practical situations, the precision of the conversion factor used will be based on the accuracy required within the context of the problem. For example, when converting from miles to kilometers, we may use a conversion factor of 1 mile ≈ 1.6 km or 1 mile ≈ 1.609 km, depending upon the accuracy needed.	

7.3 The student will solve single-step and multistep practical problems, using proportional reasoning.

Understanding the Standard	Essential Knowledge and Skills
• Estimation may be used prior to calculating conversions to evaluate the reasonableness of a solution. • A percent is a ratio in which the denominator is 100. • Proportions can be used to represent percent problems as follows: $\frac{percent}{100} = \frac{part}{whole}$	

Measurement and geometry in the middle grades provide a natural context and connection among many mathematical concepts. Students expand informal experiences with geometry and measurement in the elementary grades and develop a solid foundation for further exploration of these concepts in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

Students develop measurement skills through exploration and estimation. Physical exploration to determine length, weight/mass, liquid volume/capacity, and angle measure are essential to develop a conceptual understanding of measurement. Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, and geometry software provide experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, square-based pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

Students explore and develop an understanding of the Pythagorean Theorem. Understanding how the Pythagorean Theorem can be applied in practical situations has a far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one.)

Level 2: Analysis. Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three.)

Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

Level 4: Deduction. Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

7.4 The student will

a) describe and determine the volume and surface area of rectangular prisms and cylinders; and

b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders.

Understanding the Standard

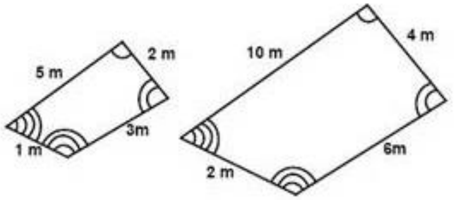
- A polyhedron is a solid figure whose faces are all polygons.
- A rectangular prism is a polyhedron in which all six faces are rectangles. A rectangular prism has eight vertices and 12 edges.
- A cylinder is a solid figure formed by two congruent parallel faces called bases joined by a curved surface. In this grade level, cylinders are limited to right circular cylinders.
- A face is any flat surface of a solid figure.
- The surface area of a prism is the sum of the areas of all 6 faces and is measured in square units.
- The volume of a three-dimensional figure is a measure of capacity and is measured in cubic units.
- Nets are two-dimensional representations of a three-dimensional figure that can be folded into a model of the three-dimensional figure.
- A rectangular prism can be represented on a flat surface as a net that contains six rectangles — two that have measures of the length and width of the base, two others that have measures of the length and height, and two others that have measures of the width and height. The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$).
- A cylinder can be represented on a flat surface as a net that contains two circles (the bases of the cylinder) and one rectangular region (the curved surface of the cylinder) whose length is the circumference of the circular base and whose width is the height of the cylinder. The surface area of the cylinder is the sum of the area of the two circles and the rectangle representing the curved surface ($SA = 2\pi r^2 + 2\pi rh$).
- The volume of a rectangular prism is computed by multiplying the area of the base, B, (length times width) by the height of the prism ($V = lwh = Bh$).
- The volume of a cylinder is computed by multiplying the area of the base, B, (πr^2) by the height of the cylinder ($V = \pi r^2 h = Bh$).
- The calculation of determining surface area and volume may vary depending upon the approximation for pi. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi button on the calculator.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Determine the surface area of rectangular prisms and cylinders using concrete objects, nets, diagrams, and formulas. (a)
- Determine the volume of rectangular prisms and cylinders using concrete objects, diagrams, and formulas. (a)
- Determine if a practical problem involving a rectangular prism or cylinder represents the application of volume or surface area. (b)
- Solve practical problems that require determining the surface area of rectangular prisms and cylinders. (b)
- Solve practical problems that require determining the volume of rectangular prisms and cylinders. (b).

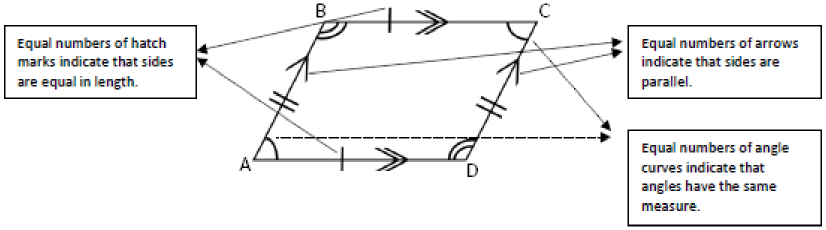
7.5 The student will solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar quadrilaterals and triangles.

Understanding the Standard	Essential Knowledge and Skills
• Similar polygons have corresponding sides that are proportional and corresponding interior angles that are congruent. • Similarity has practical applications in a variety of areas, including art, architecture, and the sciences. • Similarity does not depend on the position or orientation of the figures. • Congruent polygons have the same size and shape. Corresponding angles and sides are congruent. • Congruent polygons are similar polygons for which the ratio of the corresponding sides is 1:1. However, similar polygons are not necessarily congruent. • The symbol ~ is used to represent similarity. For example, $\Delta ABC \sim \Delta DEF$. • The symbol ≅ is used to represent congruence. For example, $\angle A \cong \angle B$ • Similarity statements can be used to determine corresponding parts of similar figures such as: Given: $\Delta ABC \sim \Delta DEF$ $\angle A$ corresponds to $\angle D$ $\overline{AB}$ corresponds to $\overline{DE}$ • A proportion representing corresponding sides of similar figures can be written as $\frac{a}{b} = \frac{c}{d}$. – Example: Given two similar quadrilaterals with corresponding angles labeled, write a proportion involving corresponding sides.  $\frac{5}{10} = \frac{2}{4}$ or $\frac{5}{10} = \frac{3}{6}$ or $\frac{1}{2} = \frac{2}{4}$ are some of the ways to express the proportional relationships that exist.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify corresponding sides and corresponding congruent angles of similar quadrilaterals and triangles. • Given two similar quadrilaterals or triangles, write similarity statements using symbols. • Write proportions to express the relationships between the lengths of corresponding sides of similar quadrilaterals and triangles. • Solve a proportion to determine a missing side length of similar quadrilaterals or triangles. • Given angle measures in a quadrilateral or triangle, determine unknown angle measures in a similar quadrilateral or triangle.

7.5 The student will solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar quadrilaterals and triangles.

<table>
<tr><th>Understanding the Standard</th><th>Essential Knowledge and Skills</th></tr>
<tr><td>

- The traditional notation for marking congruent angles is to use a curve on each angle. Denote which angles are congruent with the same number of curved lines. For example, if $\angle A$ is congruent to $\angle C$, then both angles will be marked with the same number of curved lines.



- Congruent sides are denoted with the same number of hatch (or hash) marks on each congruent side. For example, a side on a polygon with 2 hatch marks is congruent to the side with 2 hatch marks on a congruent polygon or within the same polygon.

</td><td></td></tr>
</table>

7.6 The student will
a) compare and contrast quadrilaterals based on their properties; and
b) determine unknown side lengths or angle measures of quadrilaterals.

Understanding the Standard

- A polygon is a closed plane figure composed of at least three line segments that do not cross.
- A quadrilateral is a polygon with four sides.
- Properties of quadrilaterals include: number of parallel sides, angle measures, number of congruent sides, lines of symmetry, and the relationship between the diagonals.
- A diagonal is a segment in a polygon that connects two vertices but is not a side.
- To bisect means to divide into two equal parts.
- A line of symmetry divides a figure into two congruent parts each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines.
- A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Properties of a parallelogram include the following:
  - opposite sides are parallel and congruent;
  - opposite angles are congruent; and
  - diagonals bisect each other and one diagonal divides the figure into two congruent triangles.
- Parallelograms, with the exception of rectangles and rhombi, have no lines of symmetry. A rectangle and a rhombus have two lines of symmetry, with the exception of a square which has four lines of symmetry.
- A rectangle is a quadrilateral with four right angles. Properties of a rectangle include the following:
  - opposite sides are parallel and congruent;
  - all four angles are congruent and each angle measures 90°; and
  - diagonals are congruent and bisect each other.
- A square is a quadrilateral that is a regular polygon with four congruent sides and four right angles. Properties of a square include the following:
  - opposite sides are congruent and parallel;
  - all four angles are congruent and each angle measures 90°; and
  - diagonals are congruent and bisect each other at right angles.

Essential Knowledge and Skills

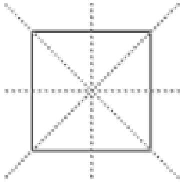
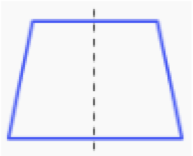
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Compare and contrast properties of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid. (a)
- Sort and classify quadrilaterals, as parallelograms, rectangles, trapezoids, rhombi, and/or squares based on their properties. (a)
- Given a diagram, determine an unknown angle measure in a quadrilateral, using properties of quadrilaterals. (b)
- Given a diagram determine an unknown side length in a quadrilateral using properties of quadrilaterals. (b)

7.6 The student will

a) compare and contrast quadrilaterals based on their properties; and

b) determine unknown side lengths or angle measures of quadrilaterals.

Understanding the Standard	Essential Knowledge and Skills
• A rhombus is a quadrilateral with four congruent sides. Properties of a rhombus include the following: – all sides are congruent; – opposite sides are parallel; – opposite angles are congruent; and – diagonals bisect each other at right angles. • A square has four lines of symmetry. The diagonals of a square coincide with two of the lines of symmetry that can be drawn. Example: Square with lines of symmetry shown:  • A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called bases. The nonparallel sides of a trapezoid are called legs. • An isosceles trapezoid has legs of equal length and congruent base angles. An isosceles trapezoid has one line of symmetry. Example: Isosceles trapezoid with line of symmetry shown:  • A chart, graphic organizer, or Venn diagram can be made to organize quadrilaterals according to properties such as sides and/or angles. • Quadrilaterals can be classified by the number of parallel sides: parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has one pair of parallel sides; other quadrilaterals have no parallel sides.	

7.6 The student will
a) compare and contrast quadrilaterals based on their properties; and
b) determine unknown side lengths or angle measures of quadrilaterals.

Understanding the Standard	Essential Knowledge and Skills
• Quadrilaterals can be classified by the measures of their angles: a rectangle and a square have four 90° angles; a trapezoid may have zero or two 90° angles. • Quadrilaterals can be classified by the number of congruent sides: a rhombus and a square have four congruent sides; a parallelogram and a rectangle each have two pairs of congruent sides, and an isosceles trapezoid has one pair of congruent sides. • A square is a special type of both a rectangle and a rhombus, which are special types of parallelograms, which are special types of quadrilaterals. • Any figure that has the properties of more than one subset of quadrilaterals can belong to more than one subset. • The sum of the measures of the interior angles of a quadrilateral is 360°. Properties of quadrilaterals can be used to find unknown angle measures in a quadrilateral.	

7.7 The student will apply translations and reflections of right triangles or rectangles in the coordinate plane.

Understanding the Standard	Essential Knowledge and Skills
• A transformation of a figure called the preimage changes the size, shape, or position of the figure to a new figure called the image. • Translations and reflections do not change the size or shape of a figure (e.g., the preimage and image are congruent figures). Translations and reflections change the position of a figure. • A translation is a transformation in which an image is formed by moving every point on the preimage the same distance in the same direction. • A reflection is a transformation in which an image is formed by reflecting the preimage over a line called the line of reflection. All corresponding points in the image and preimage are equidistant from the line of reflection. • The image of a polygon is the resulting polygon after the transformation. The preimage is the polygon before the transformation. • A transformation of preimage point *A* can be denoted as the image *A′* (read as "A prime"). • The preimage of a figure that has been translated and then reflected over the *x*- or *y*-axis may result in a different transformation than the preimage of a figure that has been reflected over the *x*- or *y*-axis and then translated.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or rectangle that has been translated either vertically, horizontally, or a combination of a vertical and horizontal translation. • Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or a rectangle that has been reflected over the *x*- or *y*-axis. • Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or rectangle that has been translated and reflected over the *x*- or *y*-axis or reflected over the *x*- or *y*-axis and then translated. • Sketch the image of a right triangle or rectangle that has been translated vertically, horizontally, or a combination of both. • Sketch the image of a right triangle or rectangle that has been reflected over the *x*- or *y*-axis. • Sketch the image of a right triangle or rectangle that has been translated and reflected over the *x*- or *y*-axis or reflected over the *x*- or *y*-axis and then translated.

In the middle grades, students develop an awareness of the power of data analysis and the application of probability through fostering their natural curiosity about data and making predictions.

The exploration of various methods of data collection and representation allows students to become effective at using different types of graphs to represent different types of data. Students use measures of center and dispersion to analyze and interpret data.

Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability. Through experiments and simulations, students build on their understanding of the Fundamental Counting Principle from elementary mathematics to learn more about probability in the middle grades.

7.8 The student will
a) determine the theoretical and experimental probabilities of an event; and
b) investigate and describe the difference between the experimental probability and theoretical probability of an event.

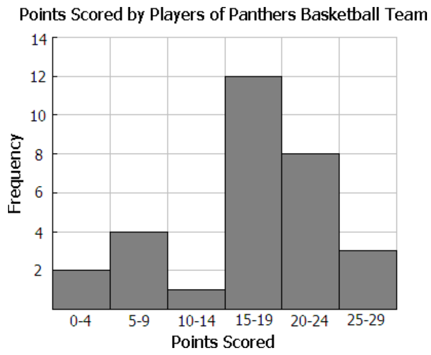
Understanding the Standard	Essential Knowledge and Skills
• In general, if all outcomes of an event are equally likely, the probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes in the sample space. • The probability of an event occurring can be represented as a ratio or equivalent fraction, decimal, or percent. • The probability of an event occurring is a ratio between 0 and 1. − A probability of 0 means the event will never occur. − A probability of 1 means the event will always occur. • The theoretical probability of an event is the expected probability and can be determined with a ratio. • If all outcomes of an event are equally likely, the theoretical probability of an event = $\frac{\text{number of possible favorable outcomes}}{\text{total number of possible outcomes}}$ • The experimental probability of an event is determined by carrying out a simulation or an experiment. • The experimental probability of an event = $\frac{\text{number of times desired outcomes occur}}{\text{number of trials in the experiment}}$ • In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability (Law of Large Numbers).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the theoretical probability of an event. (a) • Determine the experimental probability of an event. (a) • Describe changes in the experimental probability as the number of trials increases. (b) • Investigate and describe the difference between the probability of an event found through experiment or simulation versus the theoretical probability of that same event. (b)

7.9 The student, given data in a practical situation, will

- **a) represent data in a histogram;**
- **b) make observations and inferences about data represented in a histogram; and**
- **c) compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.**

Understanding the Standard

- A histogram is a graph that provides a visual interpretation of numerical data by indicating the number of data points that lie within a range of values, called a class or a bin. The frequency of the data that falls in each class or bin is depicted by the use of a bar. Every element of the data set is not preserved when representing data in a histogram.
- All graphs must include a title and labels that describe the data.
- Numerical data that can be characterized using consecutive intervals are best displayed in a histogram.
- Teachers should be reasonable about the selection of data values. Students should have experiences constructing histograms, but a focus should be placed on the analysis of histograms.
- A histogram is a form of bar graph in which the categories are consecutive and equal intervals. The length or height of each bar is determined by the number of data elements (frequency) falling into a particular interval.



- A frequency distribution shows how often an item, a number, or range of numbers occurs. It can be used to construct a histogram.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect, organize, and represent data in a histogram. (a)
- Make observations and inferences about data represented in a histogram. (b)
- Compare data represented in histograms with the same data represented in line plots, circle graphs, and stem-and-leaf plots. (c)

7.9 **The student, given data in a practical situation, will**

- **a) represent data in a histogram;**
- **b) make observations and inferences about data represented in a histogram; and**
- **c) compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.**

Understanding the Standard

Number of Cappuccinos Made per Hour at the Cafe

Number of Cups of Coffee	Tally	Frequency
0 - 3	𝍷𝍷	2
4 – 7	𝍷𝍷𝍷	3
8 – 11	𝍸 𝍷𝍷𝍷	8
12 – 15	𝍷𝍷𝍷	3
16 – 19	𝍷𝍷	2

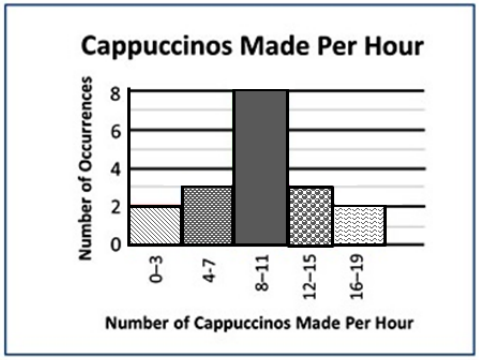
To construct a histogram:

- Organize collected data into a table. Create one column for data range categories (bins), divided into equal intervals that will include all of your data (for example, 0-10, 11-20, 21-30), and another column for frequency.
  - Bins should be all the same size.
  - Bins should include all of the data.
  - Boundaries for bins should reflect the data values being represented.
  - Determine the number of bins based upon the data.
  - If possible, the number of bins created should be a factor the number of data values (e.g., a histogram representing 20 data values might have 4 or 5 bins).
- Create a graph. Mark the data range intervals on the *x*-axis (horizontal axis) with no space between the categories. Mark frequency on the *y*-axis (vertical axis), also in equal intervals.
- Plot the data. For each data range category (bin), draw a horizontal line at the appropriate frequency or marker. Then, create a vertical bar for that category reaching up to the marked frequency. Do this for each data range category (bin).

Essential Knowledge and Skills

7.9 **The student, given data in a practical situation, will**
- **a) represent data in a histogram;**
- **b) make observations and inferences about data represented in a histogram; and**
- **c) compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.**

Understanding the Standard	Essential Knowledge and Skills



- Note: histograms may be drawn so that the bars are horizontal. To do this, interchange the *x*- and *y*-axis. Mark the data range intervals (bins) on the *y*-axis and the frequency on the *x*-axis. Draw the bars horizontally.

- Comparisons, predictions and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. Data analysis helps describe data, recognize patterns or trends, and make predictions.
- There are two types of data: categorical and numerical. Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. While students need to be aware of the differences, they do not have to know the terms for each type of data.
- Different types of graphs can be used to display categorical data. The way data is displayed is often dependent on what someone is trying to communicate.
- A line plot provides an ordered display of all values in a data set and shows the frequency of data on a number line. Line plots are used to show the spread of the data, to include clusters (groups of data points) and gaps (large spaces between data points), and quickly identify the range, mode, and any extreme data values.

7.9 The student, given data in a practical situation, will
a) represent data in a histogram;
b) make observations and inferences about data represented in a histogram; and
c) compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.

Understanding the Standard	Essential Knowledge and Skills
• A circle graph is used for categorical and discrete numerical data. Circle graphs are used to show a relationship of the parts to a whole. Every element of the data set is not preserved when representing data in a circle graph. • A stem and leaf plot is used for discrete numerical data and is used to show frequency of data distribution. A stem and leaf plot displays the entire data set and provides a picture of the distribution of data. • Different situations or contexts warrant different types of graphs, and it helps to have a good knowledge of what graphs are available. Students can determine which graph makes the most sense to use based on the type of data provided and which graph can help them answer questions most easily. • Comparing different types of representations (charts and graphs) provide students an opportunity to learn how different graphs can show different things about the same data. Following construction of graphs, students benefit from discussions around what information each graph provides. • The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inference).	

Patterns, functions and algebra become a larger mathematical focus in the middle grades as students extend their knowledge of patterns developed in the elementary grades.

Students make connections between the numeric concepts of ratio and proportion and the algebraic relationships that exist within a set of equivalent ratios. Students use variable expressions to represent proportional relationships between two quantities and begin to connect the concept of a constant of proportionality to rate of change and slope. Representation of relationships between two quantities using tables, graphs, equations, or verbal descriptions allow students to connect their knowledge of patterns to the concept of functional relationships. Graphing linear equations in two variables in the coordinate plane is a focus of the study of functions which continues in high school mathematics.

Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality, domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades. Students learn to solve equations by using concrete materials. They expand their skills from one-step to multistep equations and inequalities through their application in practical situation.

7.10 The student will

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y = mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y = x + b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y = x + b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- When two quantities, *x* and *y*, vary in such a way that one of them is a constant multiple of the other, the two quantities are "proportional". A model for that situation is $y = mx$ where m is the slope or rate of change. Slope may also represent the unit rate of a proportional relationship between two quantities, also referred to as the constant of proportionality or the constant ratio of *y* to *x*.
- The slope of a proportional relationship can be determined by finding the unit rate.

  Example: The ordered pairs (4, 2) and (6, 3) make up points that could be included on the graph of a proportional relationship. Determine the slope, or rate of change, of a line passing through these points. Write an equation of the line representing this proportional relationship.

x	*y*
4	2
6	3

  The slope, or rate of change, would be $\frac{1}{2}$ or 0.5 since the *y*-coordinate of each ordered pair would result by multiplying $\frac{1}{2}$ times the *x*-coordinate. This would also be the unit rate of this proportional relationship. The ratio of *y* to *x* is the same for each ordered pair. That is, $\frac{y}{x} = \frac{2}{4} = \frac{3}{6} = \frac{1}{2} = 0.5$

  The equation of a line representing this proportional relationship of *y* to *x* is $y = \frac{1}{2}x$ or $y = 0.5x$.
- The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change of the line.

$$\text{slope} = \frac{change\ in\ y}{change\ in\ x} = \frac{vertical\ change}{horizontal\ change}$$

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Determine the slope, *m*, as rate of change in a proportional relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form $y = mx$ to represent the relationship. Slope will be limited to positive values. (a)
- Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope, *m*, as rate of change. Slope will be limited to positive values. (b)
- Graph a line representing a proportional relationship between two quantities given the equation of the line in the form $y = mx$, where *m* represents the slope as rate of change. Slope will be limited to positive values. (b)
- Determine the *y*-intercept, *b*, in an additive relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form $y = x + b$, $b \neq 0$, to represent the relationship. (c)

7.10 The student will

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y* = *mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- The graph of the line representing a proportional relationship will include the origin (0, 0).
- A proportional relationship between two quantities can be modeled given a practical situation. Representations may include verbal descriptions, tables, equations, or graphs. Students may benefit from an informal discussion about independent and dependent variables when modeling practical situations. Grade eight mathematics formally addresses identifying dependent and independent variables.
  - Example (using a table of values): Cecil walks 2 meters every second (verbal description). If *x* represents the number of seconds and *y* represents the number of meters he walks, this proportional relationship can be represented using a table of values:

x (seconds)	1	2	3	4
y (meters)	2	4	6	8

This proportional relationship could be represented using the equation $y = 2x$, since he walks 2 meters for each second of time. That is, $\frac{y}{x} = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = 2$, the unit rate (constant of proportionality) is 2 or $\frac{2}{1}$. The same constant ratio of *y* to *x* exists for every ordered pair. This proportional relationship could be represented by the following graph:

Essential Knowledge and Skills

- Graph a line representing an additive relationship ($y = x + b$, $b \neq 0$) between two quantities, given an ordered pair on the line and the *y*-intercept (*b*). The *y*-intercept (*b*) is limited to integer values and slope is limited to 1. (d)
- Graph a line representing an additive relationship between two quantities, given the equation in the form $y = x + b$, $b \neq 0$. The *y*-intercept (*b*) is limited to integer values and slope is limited to 1. (d)
- Make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs. (e)

7.10 The student will

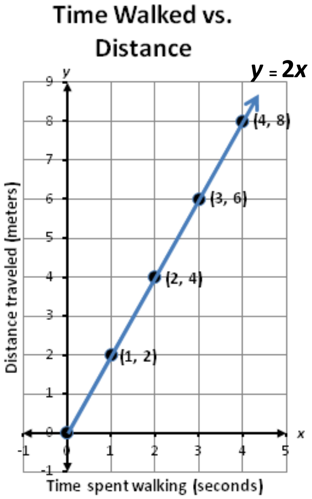
a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form $y = mx$ to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in $y = mx$ form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b*, in an additive relationship between two quantities and write an equation in the form $y = x + b$ to represent the relationship;

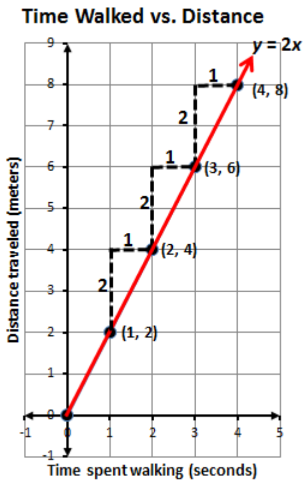
d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form $y = x + b$, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard	Essential Knowledge and Skills
 • A graph of a proportional relationship can be created by graphing ordered pairs generated in a table of values (as shown above), or by observing the rate of change or slope of the relationship and using slope triangles to graph ordered pairs that satisfy the relationship given. − Example (using slope triangles): Cecil walks 2 meters every second. If *x* represents the number of seconds and *y* represents the number of meters he walks, this proportional relationship can be represented graphically using slope triangles.	

7.10 The student will

- **a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form $y = mx$ to represent the relationship;**
- **b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in $y = mx$ form where *m* represents the slope as rate of change.**
- **c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form $y = x + b$ to represent the relationship;**
- **d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form $y = x + b$, where *b* represents the *y*-intercept; and**
- **e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.**

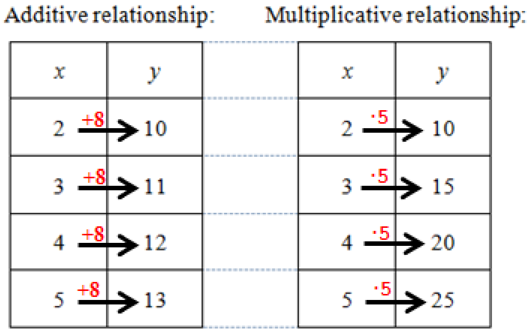
Understanding the Standard	Essential Knowledge and Skills
 The rate of change from (1, 2) to (2, 4) is 2 units up (the change in *y*) and 1 unit to the right (the change in *x*), $\frac{2}{1}$ or 2. Thus, the slope of this line is 2. Slope triangles can be used to generate points on a graph that satisfy this relationship.	

7.10 The student will

- **a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;**
- **b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form where *m* represents the slope as rate of change.**
- **c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;**
- **d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and**
- **e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.**

Understanding the Standard	Essential Knowledge and Skills

- Proportional thinking requires students to thinking multiplicatively. However, the relationship between two quantities is not always proportional. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e.., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, since context can help students to see the relationship.
  - Example:



Additive relationship:

x	*y*
$2 \xrightarrow{+8} 10$	
$3 \xrightarrow{+8} 11$	
$4 \xrightarrow{+8} 12$	
$5 \xrightarrow{+8} 13$	

Multiplicative relationship:

x	*y*
$2 \xrightarrow{\cdot 5} 10$	
$3 \xrightarrow{\cdot 5} 15$	
$4 \xrightarrow{\cdot 5} 20$	
$5 \xrightarrow{\cdot 5} 25$	

In the additive relationship, *y* is the result of adding 8 to *x.*
In the multiplicative relationship, *y* is the result of multiplying 5 times *x*.
The ordered pair (2, 10) is a quantity in both relationships, however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.

7.10 The student will

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- Two quantities, x and y, have an additive relationship when a constant value, b, exists where $y = x + b$, where $b \neq 0$. An additive relationship is not proportional and its graph does not pass through (0, 0). Note that b can be a positive value or a negative value. When b is negative, the right side of the equation could be written using a subtraction symbol (e.g., if b is −5, then the equation $y = x - 5$ could be used).
  - Example: Thomas is four years older than his sister, Amanda (verbal description). The following table shows the relationship between their ages at given points in time.

Amanda's Age	4	5	6	11
Thomas' Age	8 +4	9 +4	10 +4	15 +4

The equation that represents the relationship between Thomas' age and Amanda's age is $y = x + 4$. A graph of the relationship between their ages is shown below:

Essential Knowledge and Skills

7.10 The student will

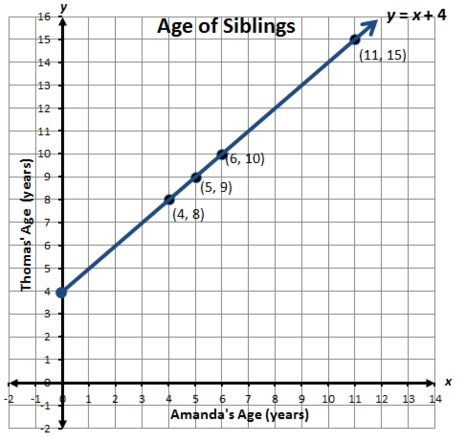
a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y* = *mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard	Essential Knowledge and Skills
 • Graphing a line given an equation can be addressed using different methods. One method involves determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. Another method involves using slope triangles to determine points on the line. − Example: Graph the equation $y = x - 1$. In order to graph the equation, we can create a table of values by substituting arbitrary values for *x* to determine coordinating values for *y*:	

7.10 **The student will**

- **a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;**
- **b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form where *m* represents the slope as rate of change.**
- **c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;**
- **d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and**
- **e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.**

Understanding the Standard

x	x - 1	y
-1	(-1) - 1	-2
0	(0) - 1	-1
1	(1) - 1	0
2	(2) - 1	1

These values can then be plotted as the points (-1, -2), (0, -1), (1, 0), and (2, 1) on a graph.

An equation written in $y = x + b$ form provides information about the graph. If the equation is $y = x - 1$, then the slope, *m*, of the line is 1 or $\frac{1}{1}$ and the point where the line crosses the *y*-axis can be located at (0, -1). We also know,

$$\text{slope} = m = \frac{change\ in\ y\text{-}value}{change\ in\ x\text{-}value} = \frac{+1}{+1}\ or\ \frac{-1}{-1}$$

So we can plot some other points on the graph using this relationship between *y* and *x* values.

A table of values can be used to determine the graph of a line. The *y*-intercept is located on the *y*-axis which is where the *x*-coordinate is 0. The change in each *y*-value compared to the corresponding *x*-value can be verified by the patterns in the table of values.

	x	y	
+1	-1	-2	+1
+1	0	-1	+1
+1	1	0	+1
	2	1	

Essential Knowledge and Skills

7.11 The student will evaluate algebraic expressions for given replacement values of the variables.

Understanding the Standard

- To evaluate an algebraic expression, substitute a given replacement value for a variable and apply the order of operations. For example, if $a = 3$ and $b = -2$ then $5a + b$ can be evaluated as: 5(3) + (-2) and simplified using the order of operations to equal 15 + (-2) which equals 13.
- Expressions are simplified by using the order of operations.
- The order of operations is a convention that defines the computation order to follow in simplifying an expression. It ensures that there is only one correct value. The order of operations is as follows:
  - First, complete all operations within grouping symbols[1]. If there are grouping symbols within other grouping symbols, do the innermost operations first.
  - Second, evaluate all exponential expressions.
  - Third, multiply and /or divide in order from left to right.
  - Fourth, add and /or subtract in order from left to right.

  [1] Parentheses ( ), brackets [ ], and the division bar should be treated as grouping symbols.
- Expressions are simplified using the order of operations and applying the properties of real numbers. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard).
  - Commutative property of addition: $a + b = b + a$.
  - Commutative property of multiplication: $a \cdot b = b \cdot a$.
  - Associative property of addition: $(a + b) + c = a + (b + c)$.
  - Associative property of multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
  - Subtraction and division are neither commutative nor associative.
  - Distributive property (over addition/subtraction): $a \cdot (b + c) = a \cdot b + a \cdot c \; and$ $a \cdot (b - c) = a \cdot b - a \cdot c$.
  - The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Represent algebraic expressions using concrete materials and pictorial representations. Concrete materials may include colored chips or algebra tiles.
- Use the order of operations and apply the properties of real numbers to evaluate expressions for given replacement values of the variables. Exponents are limited to 1, 2, 3, or 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [ ] and absolute value | |. Square roots are limited to perfect squares. Limit the number of replacements to no more than three per expression.

7.11 The student will evaluate algebraic expressions for given replacement values of the variables.

Understanding the Standard	Essential Knowledge and Skills
– Identity property of addition (additive identity property): $a + 0 = a \text{ and } 0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \text{ and } 1 \cdot a = a$. – Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$). – Inverse property of addition (additive inverse property): $a + (-a) = 0 \text{ and } (-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \text{ and } \frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0 \text{ and } 0 \cdot a = 0$. – Division by zero is not a possible mathematical operation. It is undefined. – Substitution property: If $a = b$, then *b* can be substituted for *a* in any expression, equation, or inequality.	

7.12 The student will solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
• An equation is a mathematical sentence that states that two expressions are equal. • The solution to an equation is the value(s) that make it a true statement. Many equations have one solution and can be represented as a point on a number line. • A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable. • The inverse operation for addition is subtraction, and the inverse operation for multiplication is division. • A two-step equation may include, but not be limited to equations such as the following: $2x + \frac{1}{2} = -5$; $-25 = 7.2x + 1$; $\frac{x-7}{-3} = 4$; $\frac{3}{4}x - 2 = 10$. • An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an “equal sign (=)” (e.g., $\frac{3}{4}$, $5x$, $140 - 38.2$, $18 \cdot 21$, $5 + x$). • An expression that contains a variable is a variable expression. A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an “equal sign (=).” An expression cannot be solved. • A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. For example, the verbal expression “a number multiplied by 5” could be represented by “$n \cdot 5$” or “$5n$”. • An algebraic expression is a variable expression that contains at least one variable (e.g., $2x – 3$). • A verbal sentence is a complete word statement (e.g., “The sum of twice a number and two is fifteen.” could be represented by “$2n + 2 = 15$”). • An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., $2x – 8 = 7$). • Properties of real numbers and properties of equality can be applied when solving equations, and justifying solutions. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard):	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent and solve two-step linear equations in one variable using a variety of concrete materials and pictorial representations. • Apply properties of real numbers and properties of equality to solve two-step linear equations in one variable. Coefficients and numeric terms will be rational. • Confirm algebraic solutions to linear equations in one variable. • Write verbal expressions and sentences as algebraic expressions and equations. • Write algebraic expressions and equations as verbal expressions and sentences. • Solve practical problems that require the solution of a two-step linear equation.

7.12 The student will solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
– Commutative property of addition: $a + b = b + a$. – Commutative property of multiplication: $a \cdot b = b \cdot a$. – Subtraction and division are not commutative. – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division. – Identity property of addition (additive identity property): $a + 0 = a \; and \; 0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \; and \; 1 \cdot a = a$. – Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$). – Inverse property of addition (additive inverse property): $a + (-a) = 0 \; and \; (-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \; and \; \frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0 \; and \; 0 \cdot a = 0$. – Division by zero is not a possible mathematical operation. It is undefined. – Substitution property: If $a = b$, then *b* can be substituted for *a* in any expression, equation, or inequality. – Addition property of equality: If $a = b$, then $a + c = b + c$. – Subtraction property of equality: If $a = b$, then $a - c = b - c$. – Multiplication property of equality: If $a = b$, then $a \cdot c = b \cdot c$. – Division property of equality: If $a = b \; and \; c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.	

7.13 The student will solve one- and two-step linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and graph the solution on a number line.

Understanding the Standard

- A one-step inequality may include, but not be limited to, inequalities such as the following: $2x > 5$; $y - \frac{2}{3} \leq -6$; $\frac{1}{5}x < -3$; $a - (-4) \geq \frac{11}{2}$.
- A two-step inequality may include, but not be limited to inequalities such as the following: $2x + 1 < -25$; $2x + \frac{1}{2} \geq -5$; $-25 > 7.2x + 1$; $\frac{x-7}{-3} \leq 4$; $\frac{3}{4}x - 2 \leq 10$.
- The solution set to an inequality is the set of all numbers that make the inequality true.
- The inverse operation for addition is subtraction, and the inverse operation for multiplication is division.
- The procedures for solving inequalities are the same as those to solve equations except for the case when an inequality is multiplied or divided on both sides by a negative number. Then the inequality sign is changed from less than to greater than, or greater than to less than.
- When both expressions of an inequality are multiplied or divided by a negative number, the inequality symbol reverses (e.g., $-3x < 15$ is equivalent to $x > -5$).
- Solutions to inequalities can be represented using a number line.
- In an inequality, there can be more than one value for the variable that makes the inequality true. There can be many solutions. (i.e., $x + 4 > -3$ then the solution is $x > -7$. This means that x can be any number greater than -7. A few solutions might be -6.5, -3, 0, 4, 25, etc.)
- Properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard).
  - Commutative property of addition: $a + b = b + a$.
  - Commutative property of multiplication: $a \cdot b = b \cdot a$.
  - Subtraction and division are not commutative.
  - The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Apply properties of real numbers and the multiplication and division properties of inequality to solve one-step inequalities in one variable, and the addition, subtraction, multiplication, and division properties of inequality to solve two-step inequalities in one variable. Coefficients and numeric terms will be rational.
- Represent solutions to inequalities algebraically and graphically using a number line.
- Write verbal expressions and sentences as algebraic expressions and inequalities.
- Write algebraic expressions and inequalities as verbal expressions and sentences.
- Solve practical problems that require the solution of a one- or two-step inequality.
- Identify a numerical value(s) that is part of the solution set of a given inequality.

7.13 The student will solve one- and two-step linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and graph the solution on a number line.

Understanding the Standard	Essential Knowledge and Skills
– Identity property of addition (additive identity property): $a + 0 = a\ and\ 0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a\ and\ 1 \cdot a = a$. – Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$). – Inverse property of addition (additive inverse property): $a + (-a) = 0\ and\ (-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1\ and\ \frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0\ and\ 0 \cdot a = 0$. – Division by zero is not a possible mathematical operation. It is undefined. – Substitution property: If $a = b$, then *b* can be substituted for *a* in any expression, equation, or inequality. – Addition property of inequality: If $a < b$, then $a + c < b + c$; if $a > b$, then $a + c > b + c$. – Subtraction property of inequality: If $a < b$, then $a - c < b - c$; if $a > b$, then $a - c > b - c$.. – Multiplication property of inequality: If $a < b\ and\ c > 0$, then $a \cdot c < b \cdot c$; if $a > b\ and\ c > 0$, then $a \cdot c > b \cdot c$. – Multiplication property of inequality (multiplication by a negative number): If $a < b\ and\ c < 0$, then $a \cdot c > b \cdot c$; if $a > b\ and\ c < 0$, then $a \cdot c < b \cdot c$. – Division property of inequality: If $a < b\ and\ c > 0$, then $\frac{a}{c} < \frac{b}{c}$; if $a > b\ and\ c > 0$, then $\frac{a}{c} > \frac{b}{c}$. – Division property of inequality (division by a negative number): If $a < b\ and\ c < 0$, then $\frac{a}{c} > \frac{b}{c}$; if $a > b\ and\ c < 0$, then $\frac{a}{c} < \frac{b}{c}$.	