Roanoke County Public Schools

Math 6

Curriculum Guide

2019

Mathematics Curriculum Guide

Revised 2019. Available at www.rcps.us.

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Preface

This guide will assist the mathematics teacher in preparing students for the challenges of the twenty-first century. As established by the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics*, educational goals for students are changing. A comprehensive and coherent set of mathematics standards for each and every student from prekindergarten through grade 12, *Principles and Standards* is the first set of rigorous, college and career readiness standards for the 21st century. Students should have many and varied experiences in their mathematical training to help them learn to value mathematics, become confident in their ability to do mathematics, become problem solvers, and learn to communicate and reason mathematically. This guide, along with the available division resources, VDOE resources, professional literature, alternative assessment methods, and in-service activities will assist the mathematics teacher in continuing to integrate these student goals into the curriculum.

Table of Contents

Introduction/General Comments

This curriculum guide follows the 2016 Virginia Math 6 SOLs as adopted by the Virginia Department of Education. It is extremely important and required that the Sequence of Instruction and Pacing be followed as presented in this guide. The textbook adopted as a resource for this course is *enVisionmath 2.0* (for Math 6) by Pearson, 2019 edition.

Students will take three formative assessments during the year (dates to be determined annually). Each teacher-designed test will assess skill levels of the SOLs as presented in the Sequence of Instruction and Pacing and the formative assessment blueprint. The data collected from the formative assessments will help teachers determine students' strengths and weaknesses, and inform instructional decisions.

The Mapping for Instruction is based on specified SOLs which are to be taught in the predetermined order. Note, some SOLs (or their parts) may be taught over multiple 9-week periods.

Refer to the Mathematics 2016 Standards of Learning Math 6 Curriculum Framework during every lesson. It is located at the back of this guide. This will provide valuable information for the teacher (Understanding the Standard) and desired goals for instruction (Essential Knowledge and Skills). Examples of teaching techniques and strategies, definitions, and recommended manipulatives are included in the Curriculum Framework and on the VDOE website under Mathematics Instructional Plans (MIPs) http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/mip/index.shtml.

Resources Overview

Resources for all SOLs
http://www.doe.virginia.gov/instruction/mathematics/resources/videos/index.shtml
iXL
PowerSchool
BrainPop
Pearsonsuccessnet.com
VDOE
Promethean Planet
Number Talks
Performance Tasks
quizizz.com
Kahoot.com
https://vocaroo.com (Voice Recording)
http://www.readwritethink.org/files/resources/interactives/comic/ (Comic Strip Creator)
https://webqr.com/ (qr reader for laptop)

Sequence of Instruction and Pacing											
First Nine Weeks			**Second Nine Weeks**			**Third Nine Weeks**			**Fourth Nine Weeks**		
SOL	**Instructional Focus**	**Blks**	**SOL**	**Instructional Focus**	**Blks**	**SOL**	**Instructional Focus**	**Blks**	**SOL**	**Instructional Focus**	**Blks**
	Classroom Introduction Algebra Readiness Pretest (mandatory)	2	**6.6b** **6.13**	Represent and solve equations in one variable using concrete materials Focus on algebraic vocabulary Confirm solutions Expressions: Verbal to Algebraic Algebraic to Verbal Represent and solve practical problems with one-step linear equations Apply properties to solve one-step linear equations	16	**6.2ab**	Fractions, Decimals, and Percents: Represent ratios Determine equivalencies Compare and order **(Suggestion: test/quiz here)**	12	**3.15ab** **4.14abc** **5.16abc** **6.10abc**	Review: Line Plots, Bar Graphs & Pictographs Create circle graphs: • Represent data • Interpret data • Compare data to bar graphs, pictographs, and line plots	10-15
6.4	Exponents and Perfect Squares Recognize and represent patterns	6	**6.14ab**	Solve inequalities: Represent a practical situation with a one-step linear inequality Apply properties to solve one-step linear inequalities Add and subtract Graph inequalities Confirm solutions **(Suggestion: test/quiz here)**	8	**6.12abcd**	Proportions: Practical problems Determine unit rate Determine whether a proportion exists Make connections between and among representations **(Suggestion: test/quiz here)**	15	**6.11ab**	Mean, median, mode Mean as the balance point in a line plot Determine effect on measures of center when a single value is added, subtracted, or changed	10-15

5.7 **6.6c**	Review order of operations and simplifying – positive values only (no negative numbers) **(Suggestion: test/quiz here)**	4	**6.1**	Ratios	5	**6.7abc**	Perimeter and area of triangles and rectangles/squares Derive Pi Circumference and area of circles **(Suggestion: test/quiz here)**	15	**6.9**	Congruence Regular polygons Line of Symmetry **(Suggestion: test/quiz here)**	10-15
6.3abc	Integers: - Identify - Represent - Compare - Order - Absolute value	8	**6.5abc**	Add, subtract, multiply and divide fractions & mixed numbers (single and multi-step) Practical problems w/ fractions, mixed numbers, and decimals **(Suggestion: test/quiz here)**	10						
6.6abc	Integer Operations: Add, subtract, multiply, divide Order of operations Simplify expressions Practical problems **(Suggestion: test/quiz here)**	17									
6.8ab	Coordinate Plane	3									
										Algebra Readiness Posttest (may use SOL test as posttest)	
	Remediation, Review, Assessment	5		Remediation, Review, Assessment	6		Remediation, Review, Assessment	3		Remediation, Review, Assessment	10-20
	Total Blocks	**45**		**Total Blocks**	**45**		**Total Blocks**	**45**		**Total Blocks**	**45**

DESMOS CALCULATOR

The Desmos Virginia Scientific Calculator will be used for instruction and assessment.

Mapping for Instruction - First Nine Weeks

SOLs

5.7 (Review) The student will simplify whole number numerical expressions using the order of operations.

6.3 The student will
- **a) identify and represent integers;**
- **b) compare and order integers; and**
- **c) identify and describe absolute value of integers.**

6.4 The student will recognize and represent patterns with whole number exponents and perfect squares.

6.6 The student will
- **a) add, subtract, multiply, and divide integers; (no calculator)**
- **b) solve practical problems involving operations with integers; and**
- **c) simplify numerical expressions involving integers. (no calculator)**

6.8 The student will
- **a) identify the components of the coordinate plane; and**
- **b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.**

SOL	Instructional Focus	Vocabulary	Comments	Blocks
ARI pretest	**Identify areas of weakness in Algebra Readiness** **Classroom introduction**		*All Math 6 students must take the pretest to determine their degree of algebra readiness.*	2
6.4	**Exponents (bases and exponents are whole numbers)** **Perfect Squares up to 20^2**	Base, exponent, power, perfect square, factor, exponential form, expanded form, square root Symbols: square root - $\sqrt{\ }$		6
5.7 **6.6 c**	**Order of operations (review without integers)**	Grouping symbols, parenthesis, brackets, sum, difference, product, quotient, simplified, expression, evaluate, solve		4
6.3 a, b, c	**Identify/represent integers** **Compare/order integers** **Identify/describe absolute value**	Integer, positive, negative, absolute value, compare, is less than, is greater than, order, least to greatest, greatest to least, ascending, descending, increasing, decreasing, grouping, order of operations, evaluate, solve, simplify, opposite, above, below, zero Symbols: $<$, $>$, $\leq$, $\geq$, $=$; absolute value - $\lvert\ \rvert$		8
6.6 a, b, c	**Integer Operations:** **Add, subtract, multiply, and divide** **Practical problems**	Sum, difference, product, quotient, grouping symbols, simplify, evaluate, deposit, withdraw, gain, loss, above sea level, below sea level, absolute value, undefined, order of operations, properties (commutative, associative, distributive,		17

	Simplify expressions	identity, inverse, multiplicative property of zero, substitution		
6.8 a, b	**Coordinate Plane:** **Identify parts of coordinate graph** **Identify/graph ordered pairs**	Horizontal, vertical, x-axis, y-axis, coordinate plane, ordered pair, perpendicular, origin, quadrant, I, II, III, IV, intersecting, coordinates, counterclockwise, clockwise, up, down, left, right, polygon		3
			Remediation, Review, Assessment	5
			Total Blocks	**45**

Resources – First Nine Weeks			
SOL	**Textbook**	**Links**	**Supplemental Materials**
6.4	Lesson 3-1	Exponent Notes/Practice Problems: https://www.mathgoodies.com/lessons/vol3/exponents (Math Goodies: Exponents) https://www.ezschool.com/play/math/timed-evaluation---find-the-value-of-the-exponent-before-the-alien-lands/796 (EZSchool: Alien Exponents Game) http://www.mathslice.com/squareroot_ws.php (Math Slice: Exponents) Perfect Squares: http://illuminations.nctm.org/Lesson.aspx?id=3089 (Illuminations: In Search of Perfect Squares) http://www.crctlessons.com/Perfect-Squares/perfect-squares-game.html (CRCT Lessons: Perfect Squares Game) http://www.xpmath.com/forums/arcade.php?do=play&gameid=88 (XPMath: Square Root Cannon Game) http://braingenie.ck12.org/skills/105177 (BrainGenie: Evaluating Algebraic Expressions) https://www.youtube.com/watch?v=TVeKiUacqHY (Exponent and Perfect Square lesson) https://www.youtube.com/watch?v=1dqkoi6Pd8M (Exponent and Perfect Square lesson)	MIPs: 6.4 - Whole Number Exponents and Perfect Squares /PDF Version Tiles, grid paper, geoboards, calculator keys: x^2 , y^x, √, tables
5.7 **6.6c**	Lesson 3-3 3-5	GEMDAS	MIPs: 6.6c - Order Up /PDF Version
6.3 a, b, c	Lesson 2-1 2-2 2-3	https://www.youtube.com/watch?v=x0E4vxLydNY (PBS Math Club- What is an integer?) https://www.youtube.com/watch?v=TlZVPfvTLXA (Khan Academy- identifying integers on a number line https://www.youtube.com/watch?v=dQCNC97Y-mw (Absolute Value) https://www.youtube.com/watch?v=Oq2KoAGrY64 (Comparing and Ordering Integers)	MIPs: 6.3a - Ground Zero / PDF Version 6.3b - Compare and Order Integers / PDF Version 6.3c - Absolute Value of Integers / PDF Version Number lines, two-color counters, drawings, algebra tiles, calculators

6.6 a, b, c	Lesson 3-3 VA-1 VA-2 VA-3 VA-4	https://www.youtube.com/watch?v=_BgblvF90UE (Math Antics- Adding and Subtracting Integers) https://www.youtube.com/watch?v=16ePaaLFxlg (Operations with Integers) https://www.mathplayground.com/ASB_IntegerWarp.html (Multiplying Integers)	MIPs: 6.6a - Operations with Integers / PDF Version 6.6b - Application of Integer Operations / PDF Version 6.6c - Order Up / PDF Version Two-color counters, number lines, calculator* *6.6b only
6.8 a, b	Lesson 2-4	https://www.youtube.com/watch?v=r16I6LB2YbQ (Intro to Coordinate Plane) https://www.mathplayground.com/space_graph.html (Plotting points on the Coordinate Plane)	MIPs: 6.8ab - What's the Point? / PDF Version Graph paper, rulers

Mapping for Instruction - Second Nine Weeks

SOLs

6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

6.5 The student will
- **a) multiply and divide fractions and mixed numbers; (no calculator)**
- **b) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and**
- **c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals.**

6.6 The student will
- **b) solve practical problems involving operations with integers; and**

6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.

6.14 The student will
- **a) represent a practical situation with a linear inequality in one variable; and**
- **b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.**

SOL	Instructional Focus	Vocabulary	Comments	Blocks
6.6b **6.13**	**Algebra:** **Represent and solve one-step linear equation in one variable** **Identify algebraic vocabulary** **Translate expressions and equations:** - **Verbal to algebraic** - **Algebraic to verbal** **Solve one-step linear equations** **Apply properties to solve equations** **Practical problems**	Expression, variable expression, algebraic expression, verbal sentence, algebraic equation, term, coefficient, variable, equation		16
6.14 a, b	**Inequalities:** **Represent a practical situation with a linear inequality** **Solve and graph solutions of one-step linear inequalities involving adding and subtracting** **Practical problems**	Is greater than, is less than, is greater than or equal to, is less than or equal to, point, closed circle, open circle, solution set	*See 6.14 EKS in the curriculum framework.*	8
6.1	**Ratios** **Express ratios with symbols, numbers** **Represent a ratio using correct notation** **Express ratios with words**	Ratio, relationship, quantities, compare, part to part, part to whole, whole to whole, colon, fraction notation, comparison, ratio notation $\frac{a}{b}$, *a:b*, *a to b*		5

6.5 c	**Solve multistep practical problems (add, subtract, multiply, divide) with decimals**	Double, twice, triple, half, cut off, equal groups, altogether, in all, total, of, left, left over, divisor, simplest form, numerator, denominator	*(teach SOL 6.5c first)*	3
6.5 a, b	**Multiply and divide fractions and mixed numbers** **Solve one-step and multistep practical problems (add, subtract, multiply, divide) with fractions and mixed numbers**	Double, twice, triple, half, cut off, equal groups, altogether, in all, total, of, left, left over, divisor, simplest form, numerator, denominator		7
			Remediation, Review, Assessment	6
			Total Blocks	**45**

Resources – Second Nine Weeks			
SOL	**Textbook**	**Links**	**Supplemental Materials**
6.6b **6.13**	Lesson 3-3 VA-1 VA-2 VA-3 VA-4 Lesson 3-4 4-1 to 4-5 Lesson 3-4 4-1 4-2 4-3 4-4 4-5	https://www.youtube.com/watch?v=JrolgDL5v3w (Solving one-step equations) https://www.youtube.com/watch?v=sP_lJyo1U00 (Solving one-step equations) https://www.youtube.com/watch?v=yHqGxeU4oyM (Intro to Algebra terms)	MIPs: 6.6b - Application of Integer Operations / PDF Version 6.13 - Equation Vocabulary / PDF Version 6.13 - Modeling One-Step Linear Equations / PDF Version 6.13 - One Step Equations / PDF Version Colored chips, algebra tiles, weights on a balanced scale, number lines, arrows Models, balance scale, algebra tiles, algebra blocks, hands on equations
6.14 a, b	Lesson 4-6 4-7 VA-5	https://www.youtube.com/watch?v=R34YS6qViLI (Graphing Inequalities on a number line) https://www.youtube.com/watch?v=smX2wkIUPvQ (Solve one-step inequality with addition or subtraction) https://www.quia.com/rr/325253.html	MIPs: 6-14a - Representing Practical Situations with Inequalities / PDF Version 6.14b - Solving One Step Inequalities with Addition and Subtraction Number lines, peppermints, life savers, arrows
6.1	Lesson 5-1 to 5-7 VA-6 VA-7	http://www.mathsisfun.com/numbers/atio.html (Math is Fun: Ratios)	MIPs: 6.1 - Field Goals, Balls, and Nets
6.5 a, b, c	Lesson 1-1 to 1-7	https://www.kroger.com/ Variety of restaurant links	MIPs: 6.5a - Modeling Division of Fractions (Word) / PDF Version 6.5a - Modeling Multiplication of Fractions (Word) / PDF Version 6.5a - Multiply Fractions and Mixed Numbers (Word) / PDF Version 6.5a - Divide Fractions and Mixed Numbers (Word) / PDF Version 6.5b - Solve Problems Involving Operations with Fractions and Mixed Numbers (Word) / PDF Version 6.5c - Practical Problems Involving Decimals / PDF Version Arrays, paper folding, fraction strips, fraction rods, pattern blocks, area models, menus, grocery ads

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Mapping for Instruction - Third Nine Weeks

SOLs

6.2 The student will
- **a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents (no calculator); and**
- **b) compare and order positive rational numbers. (no calculator)**

6.7 The student will
- **a) derive π (pi);**
- **b) solve problems, including practical problems, involving circumference and area of a circle; and**
- **c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles.**

6.12 The student will
- **a) represent a proportional relationship between two quantities, including those arising from practical situations;**
- **b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;**
- **c) determine whether a proportional relationship exists between two quantities; and**
- **d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.**

SOL	Instructional Focus	Vocabulary	Comments	Blocks
6.2 a, b	**Equivalencies among fractions, mixed numbers, decimals, and percents** **Order (ascending and descending) and compare rational numbers**	Percent, equal, terminating decimal, repeating decimal, proper fraction, improper fraction, per 100, out of 100, is less than, is greater than, is less than or equal to, is greater than or equal to, equivalent, rational number, mixed number, ratio Symbols: $<$, $>$, $\leq$, $\geq$, $=$		12
6.12 a, b, c, d	**Represent proportions** **Determine unit rate** **Find missing value in ratio table** **Determine if proportion exists** **Make connections between proportions** **Make equivalent ratios** **Practical problems**	Equivalent, proportional relationship, constant of proportionality, quantity, multiplicative, table, unit rate, value, ratio table, constant, rate	*See 6.12 EKS for instructional support.*	15
6.7 c, a, b	**Derive pi (π)** **Solve problems involving circumference and area of circles (include practical problems)** **Solve problems involving perimeter and area of triangles, rectangles, and squares (include practical problems)**	Ratio, circumference, diameter, distance, plane figure, circle, square, rectangle, triangle, area, perimeter, base height, radius, center Formulas for perimeter, circumference, and area	*Teach perimeter and area first.*	15
			Remediation, Review, Assessment	3
			Total Blocks	**45**

Resources – Third Nine Weeks			
SOL	**Textbook**	**Links**	**Supplemental Materials**
6.2 a, b	Lesson 2-2 6-1 6-2 6-3 6-4 6-5 6-6	https://www.mathplayground.com/Decention/index.html (FDP) https://www.mathplayground.com/Triplets/index.html (FDP w/models) https://www.youtube.com/watch?v=5vv--qrUJXo (Comparing and Ordering FDP) https://www.youtube.com/watch?v=WV5VY76Pf5U (Converting fractions to decimals)	MIPs: 6.2a - Rational Speed Match (Word) / PDF Version 6.2b - Compare and Order Rational Numbers / PDF Version Fraction bars, base 10 blocks, fraction circles, number lines, calculators, colored counters, cubes, decimal squares, shaded figures, shaded grids, area model, set model, measurement model
6.12 a, b, c, d	Lesson 5-2 to 5-10 VA-6 VA-7	https://www.youtube.com/watch?v=USmit5zUGas (Math Antics- Intro to Proportions) https://www.youtube.com/watch?v=liW_ALj4Qj8&t=129s (Unit Rates) https://www.youtube.com/watch?v=JwRYcBUGz5Q (Ratio tables)	MIPs: 6.12ab - Ratio Tables and Unit Rates (Word) / PDF Version 6.12cd - Identifying and Representing Proportional Relationships / PDF Version
6.7 a, b, c	Lesson 2-6 7-1 to 7-4 VA-8 VA-9	https://www.youtube.com/watch?v=8-cazxAL_tU (Intro to Pi)	MIPs: 6.7ab - Going the Distance (Word) / PDF Version 6.7c - Practical Problems Involving Area and Perimeter / PDF Version Tiles, one-inch cubes, graph paper, geoboards, tracing paper, concrete materials, computer models

Mapping for Instruction - Fourth Nine Weeks

SOLs

3.15 (Review) The student will
- **a) collect, organize, and represent data in pictographs or bar graphs; and**
- **b) read and interpret data represented in pictographs and bar graphs.**

4.14 (Review) The student will
- **a) collect, organize, and represent data in bar graphs and line graphs;**
- **b) interpret data represented in bar graphs and line graphs; and**
- **c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph, a chart and a line graph, or a pictograph and a bar graph).**

5. 16 (Review) The student, given a practical problem, will
- **a) represent data in line plots and stem-and-leaf plots;**
- **b) interpret data represented in line plots and stem-and-leaf plots; and**
- **c) compare data represented in a line plot with the same data represented in a stem-and-leaf plot.**

6.9 The student will determine congruence of segments, angles, and polygons.

6.10 The student, given a practical situation, will
- **a) represent data in a circle graph;**
- **b) make observations and inferences about data represented in a circle graph; and**
- **c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.**

6.11 The student will
- **a) represent the mean of a data set graphically as the balance point; and**
- **b) determine the effect on measures of center when a single value of a data set is added, removed, or changed.**

SOL	Instructional Focus	Vocabulary	Comments	Blocks
3.15 a, b **4.14 a, b, c** **5.16 a, b, c** **6.10 a, b, c**	**Circle graphs and related graphs:** **Review: Line plots, pictographs, bar graphs, and stem & leaf plots** **Collect, organize, and represent data in a circle graph** **Collect, organize, and represent data in a circle graph** **Make observations and inferences about data in circle graphs** **Compare data in circle graphs with same data in bar graphs, pictographs, line plots**	Data, relationship, part to whole, percent, frequency, category, construct, analyze, survey, categorical data, numerical data, line plot, bar graph, pictograph, scale, compare, predict, inference, collect, organize, represent, observations, table, title, scale, key, data categories, labels		10-15
6.11 a, b	**Mean as the balance point in a line plot** **Calculate mean, median, mode** **Determine effect on measures of center when a single value is added, subtracted, or changed**	Measures of center, data set, mean, median, mode, balance point, average, values, outlier, descriptor	*Standard 6.11b; emphasize effects on measures of center when a single value is added, subtracted, or changed.*	10-15

6.9	**Determine congruence** **Identify regular polygons** **Draw lines of symmetry in regular polygons**	Segment, angle, polygon, congruent, non-congruent, regular polygon, line of symmetry, parallel, interior angles Symbols for congruence: hash marks, angle curves Symbol for parallelism: arrows	*Emphasize identifying regular polygons and drawing lines of symmetry in regular polygons.*	10-15
			Remediation, Review, Assessment	10-15
			Total Blocks	**45**

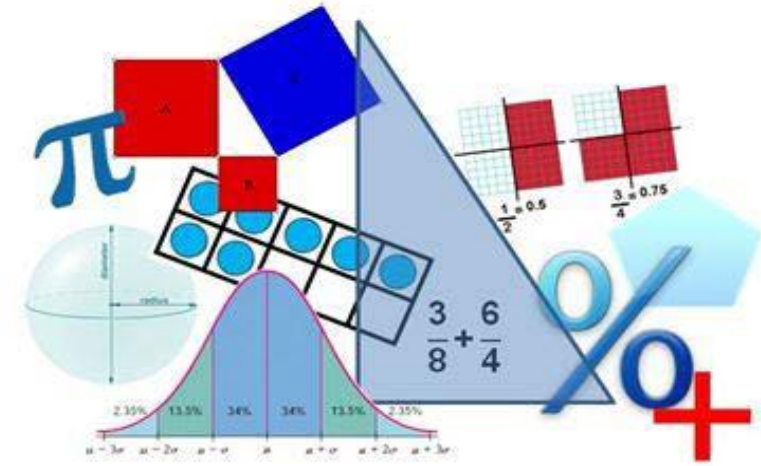
Resources – Fourth Nine Weeks			
SOL	**Textbook**	**Links**	**Supplemental Materials**
6.10 a, b, c	Lesson 8-1 VA-11	https://www.youtube.com/watch?v=pzasYiTzyqs (interpreting circle graphs) https://www.youtube.com/watch?v=KzXZfv9anpU (circle graphs) https://www.youtube.com/watch?v=x5GxZ2Ezsmc (interpreting circle graphs) https://www.youtube.com/watch?v=9ldcKQhUCG0 (interpreting circle graphs)	MIPs: 6.10abc - May I have Fries with That? (Word) / PDF Version Ruler, coloring supplies
6.11 a, b	Lesson 8-2 VA-12 8-5 8-6 8-7		MIPs: 6.11a - Balancing Act (Word) / PDF Version 6.11b - Effects on Measures of Center (Word) / PDF Version Ruler, Number line, centimeter cubes
6.9	VA-10		MIPs: 6.9 - Side to Side (Word) / PDF Version Concrete materials

Mathematics

2016 Standards of Learning

Grade 6

Curriculum Framework



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Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills

This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and to see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "… the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Mathematics instruction in grades six through eight continues to focus on the development of number sense, with emphasis on rational and real numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to many middle school mathematics topics.

Students develop an understanding of integers and rational numbers using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.

Students develop an understanding of real numbers and the properties of operations on real numbers through experiences with rational and irrational numbers and apply the order of operations.

Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

Understanding the Standard	Essential Knowledge and Skills
• A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities. Ratios are used in practical situations when there is a need to compare quantities. • In the elementary grades, students are taught that fractions represent a part-to-whole relationship. However, fractions may also express a measurement, an operator (multiplication), a quotient, or a ratio. Examples of fraction interpretations include: – Fractions as parts of wholes: $\frac{3}{4}$ represents three parts of a whole, where the whole is separated into four equal parts. – Fractions as measurement: the notation $\frac{3}{4}$ can be interpreted as three one-fourths of a unit. – Fractions as an operator: $\frac{3}{4}$ represents a multiplier of three-fourths of the original magnitude. – Fractions as a quotient: $\frac{3}{4}$ represents the result obtained when three is divided by four. – Fractions as a ratio: $\frac{3}{4}$ is a comparison of 3 of a quantity to the whole quantity of 4. • A ratio may be written using a colon (*a*:*b*), the word *to* (*a* to *b*), or fraction notation $\left(\frac{a}{b}\right)$. • The order of the values in a ratio is directly related to the order in which the quantities are compared. – Example: In a certain class, there is a ratio of 3 girls to 4 boys (3:4). Another comparison that could represent the relationship between these quantities is the ratio of 4 boys to 3 girls (4:3). Both ratios give the same information about the number of girls and boys in the class, but they are distinct ratios. When you switch the order of comparison (girls to boys vs. boys to girls), there are different ratios being expressed. • Fractions may be used when determining equivalent ratios. – Example: The ratio of girls to boys in a class is 3:4, this can be interpreted as: number of girls = $\frac{3}{4}$ · number of boys. In a class with 16 boys, number of girls = $\frac{3}{4}$ · (16) = 12 girls.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent a relationship between two quantities using ratios. • Represent a relationship in words that makes a comparison by using the notations $\frac{a}{b}$, *a*:*b*, and *a to b*. • Create a relationship in words for a given ratio expressed symbolically.

6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

Understanding the Standard	Essential Knowledge and Skills

- Example: A similar comparison could compare the ratio of boys to girls in the class as being 4:3, which can be interpreted as:

  number of boys = $\frac{4}{3}$ · number of girls.

  In a class with 12 girls, number of boys = $\frac{4}{3}$ · (12) = 16 boys.

- A ratio can compare two real-world quantities (e.g., miles per gallon, unit rate, and circumference to diameter of a circle).
- Ratios may or may not be written in simplest form.
- A ratio can represent different comparisons within the same quantity or between different quantities.

Ratio	Comparison
part-to-whole (within the same quantity)	compare part of a whole to the entire whole
part-to-part (within the same quantity)	compare part of a whole to another part of the same whole
whole-to-whole (different quantities)	compare all of one whole to all of another whole
part-to-part (different quantities)	compare part of one whole to part of another whole

6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

Understanding the Standard	Essential Knowledge and Skills

- Examples: Given Quantity A and Quantity B, the following comparisons could be expressed.

Ratio	Example	Ratio Notation(s)
part-to-whole (within the same quantity)	compare the number of unfilled stars to the total number of stars in Quantity A	3:8; 3 to 8; or $\frac{3}{8}$
part-to-part [1] (within the same quantity)	compare the number of unfilled stars to the number of filled stars in Quantity A	3:5 or 3 to 5
whole-to-whole [1] (different quantities)	compare the number of stars in Quantity A to the number of stars in Quantity B	8:5 or 8 to 5
part-to-part [1] (different quantities)	compare the number of unfilled stars in Quantity A to the number of unfilled stars in Quantity B	3:2 or 3 to 2

[1]Part-to-part comparisons and whole-to-whole comparisons are ratios that are not typically represented in fraction notation except in certain contexts, such as determining whether two different ratios are equivalent.

6.2 The student will

a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and

b) compare and order positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Fractions, decimals and percents can be used to represent part-to-whole ratios. – Example: The ratio of dogs to the total number of pets at a grooming salon is 5:8. This implies that 5 out of every 8 pets being groomed is a dog. This part-to-whole ratio could be represented as the fraction $\frac{5}{8}$ ($\frac{5}{8}$ of all pets are dogs), the decimal 0.625 (0.625 of the number of pets are dogs), or as the percent 62.5% (62.5% of the pets are dogs). • Fractions, decimals, and percents are three different ways to express the same number. Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent. • Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, base ten blocks, fraction circles, number lines, colored counters, cubes, decimal squares, shaded figures, shaded grids, or calculators). • *Percent* means "per 100" or how many "out of 100"; *percent* is another name for *hundredths*. • A number followed by a percent symbol (%) is equivalent to a fraction with that number as the numerator and with 100 as the denominator (e.g., 30% = $\frac{30}{100} = \frac{3}{10}$; 139% = $\frac{139}{100}$). • Percents can be expressed as decimals (e.g., 38% = $\frac{38}{100}$ = 0.38; 139% = $\frac{139}{100}$ = 1.39). • Some fractions can be rewritten as equivalent fractions with denominators of powers of 10, and can be represented as decimals or percents (e.g., $\frac{3}{5} = \frac{6}{10} = \frac{60}{100}$ = 0.60 = 60%). Fractions, decimals, and percents can be represented by using an area model, a set model, or a measurement model. For example, the fraction $\frac{1}{3}$ is shown below using each of the three models. 	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent ratios as fractions (proper or improper), mixed numbers, decimals, and/or percents. (a) • Determine the decimal and percent equivalents for numbers written in fraction form (proper or improper) or as a mixed number, including repeating decimals. (a) • Represent and determine equivalencies among decimals, percents, fractions (proper or improper), and mixed numbers that have denominators that are 12 or less or factors of 100. (a) • Compare two percents using pictorial representations and symbols (<, ≤, ≥, >, =). (b) • Order no more than four positive rational numbers expressed as fractions (proper or improper), mixed numbers, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less or factors of 100). Ordering may be in ascending or descending order. (b)

6.2 The student will

a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and

b) compare and order positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Percents are used to solve practical problems including sales, data description, and data comparison. • The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of positive rational numbers are: $\sqrt{25}$, 0.275, $\frac{1}{4}$, 82, 75%, $\frac{22}{5}$, $4.\overline{59}$. • Students are not expected to know the names of the subsets of the real numbers until grade eight. • Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). • Strategies using 0, $\frac{1}{2}$ and 1 as benchmarks can be used to compare fractions. – Example: Which is greater, $\frac{4}{7}$ or $\frac{3}{9}$? $\frac{4}{7}$ is greater than $\frac{1}{2}$ because 4, the numerator, represents more than half of 7, the denominator. The denominator tells the number of parts that make the whole. $\frac{3}{9}$ is less than $\frac{1}{2}$ because 3, the numerator, is less than half of 9, the denominator, which tells the number of parts that make the whole. Therefore, $\frac{4}{7} > \frac{3}{9}$. • When comparing two fractions close to 1, use the distance from 1 as your benchmark. – Example: Which is greater, $\frac{6}{7}$ or $\frac{8}{9}$? $\frac{6}{7}$ is $\frac{1}{7}$ away from 1 whole. $\frac{8}{9}$ is $\frac{1}{9}$ away from 1 whole. Since, $\frac{1}{9} < \frac{1}{7}$, then $\frac{6}{7}$ is a greater distance away from 1 whole than $\frac{8}{9}$. Therefore, $\frac{6}{7} < \frac{8}{9}$.	

6.2 The student will

a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and

b) compare and order positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Some fractions such as $\frac{1}{8}$, have a decimal representation that is a terminating decimal (e. g., $\frac{1}{8} = 0.125$) and some fractions such as $\frac{2}{9}$, have a decimal representation that does not terminate but continues to repeat (e. g., $\frac{2}{9}$ = 0.222…). The repeating decimal can be written with ellipses (three dots) as in 0.222… or denoted with a bar above the digits that repeat as in $0.\overline{2}$.	

6.3 The student will

a) identify and represent integers;

b) compare and order integers; and

c) identify and describe absolute value of integers.

Understanding the Standard	Essential Knowledge and Skills
• The set of integers includes the set of whole numbers and their opposites {…-2, -1, 0, 1, 2, …}. Zero has no opposite and is an integer that is neither positive nor negative. • Integers are used in practical situations, such as temperature (above/below zero), deposits/withdrawals in a checking account, golf (above/below par), time lines, football yardage, positive and negative electrical charges, and altitude (above/below sea level). • Integers should be explored by modeling on a number line and using manipulatives, such as two-color counters, drawings, or algebra tiles. • The opposite of a positive number is negative and the opposite of a negative number is positive. • Positive integers are greater than zero. • Negative integers are less than zero. • A negative integer is always less than a positive integer. • When comparing two negative integers, the negative integer that is closer to zero is greater. • An integer and its opposite are the same distance from zero on a number line. − Example: the opposite of 3 is −3 and the opposite of −10 is 10. • On a conventional number line, a smaller number is always located to the left of a larger number (e.g.,–7 lies to the left of –3, thus –7 < –3; 5 lies to the left of 8 thus 5 is less than 8) • The absolute value of a number is the distance of a number from zero on the number line regardless of direction. Absolute value is represented using the symbol | |(e.g., $\|-6\| = 6$ and $\|6\| = 6$). • The absolute value of zero is zero.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model integers, including models derived from practical situations. (a) • Identify an integer represented by a point on a number line. (a) • Compare and order integers using a number line. (b) • Compare integers, using mathematical symbols ($<, \leq, >, \geq, =$). (b) • Identify and describe the absolute value of an integer. (c)

6.4 The student will recognize and represent patterns with whole number exponents and perfect squares.

Understanding the Standard	Essential Knowledge and Skills
• The symbol • can be used in grade six in place of "x" to indicate multiplication. • In exponential notation, the base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In 8^3, 8 is the base and 3 is the exponent (e.g., $8^3 = 8 \cdot 8 \cdot 8$). • Any real number other than zero raised to the zero power is 1. Zero to the zero power (0^0) is undefined. • A perfect square is a whole number whose square root is an integer (e.g., $36 = 6 \cdot 6 = 6^2$). Zero (a whole number) is a perfect square. • Perfect squares may be represented geometrically as the areas of squares the length of whose sides are whole numbers (e.g., $1 \cdot 1$, $2 \cdot 2$, $3 \cdot 3$, etc.). This can be modeled with grid paper, tiles, geoboards and virtual manipulatives. • The examination of patterns in place value of the powers of 10 in grade six leads to the development of scientific notation in grade seven.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize and represent patterns with bases and exponents that are whole numbers. • Recognize and represent patterns of perfect squares not to exceed 20^2, by using grid paper, square tiles, tables, and calculators. • Recognize powers of 10 with whole number exponents by examining patterns in place value.

The computation and estimation strand in grades six through eight focuses on developing conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring an understanding as to why procedures work and make sense.

Students develop and refine estimation strategies based on an understanding of number concepts, properties and relationships. The development of problem solving, using operations with integers and rational numbers, builds upon the strategies developed in the elementary grades. Students will reinforce these skills and build on the development of proportional reasoning and more advanced mathematical skills.

Students learn to make sense of the mathematical tools available by making valid judgments about the reasonableness of answers. Students will balance the ability to make precise calculations through the application of the order of operations with knowing when calculations may require estimation to obtain appropriate solutions to practical problems.

6.5 The student will

a) multiply and divide fractions and mixed numbers;*

b) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and

c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor. • When the numerator and denominator have no common factors other than 1, then the fraction is in simplest form. • Addition and subtraction are inverse operations as are multiplication and division. • Models for representing multiplication and division of fractions may include arrays, paper folding, repeated addition, repeated subtraction, fraction strips, fraction rods, pattern blocks, and area models. • It is helpful to use estimation to develop computational strategies. – Example: $2\frac{7}{8} \cdot \frac{3}{4}$ is about $\frac{3}{4}$ of 3, so the answer is between 2 and 3. • When multiplying a whole number by a fraction such as $3 \cdot \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. • When multiplying a fraction by a fraction such as $\frac{2}{3} \cdot \frac{3}{4}$, we are asking for part of a part. • When multiplying a fraction by a whole number such as $\frac{1}{2} \cdot 6$, we are trying to determine a part of the whole. • A multistep problem is a problem that requires two or more steps to solve. • Different strategies can be used to estimate the result of computations and judge the reasonableness of the result. – Example: What is an approximate answer for 2.19 ÷ 0.8? The answer is around 2 because 2.19 ÷ 0.8 is about 2 ÷ 1 = 2.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate/model multiplication and division of fractions (proper or improper) and mixed numbers using multiple representations. (a) • Multiply and divide fractions (proper or improper) and mixed numbers. Answers are expressed in simplest form. (a) • Solve single-step and multistep practical problems that involve addition and subtraction with fractions (proper or improper) and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form. (b) • Solve single-step and multistep practical problems that involve multiplication and division with fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form. (b) • Solve multistep practical problems involving addition, subtraction, multiplication and division with decimals. Divisors are limited to a three-digit number, with decimal divisors limited to hundredths. (c)

6.5 The student will

a) multiply and divide fractions and mixed numbers;*

b) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and

c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Understanding the placement of the decimal point is important when determining quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, and by 0.06. • Solving multistep problems in the context of practical situations enhances interconnectedness and proficiency with estimation strategies. • Examples of practical situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, and sharing the cost of a pizza or the prize money from a contest.	

6.6 The student will

a) add, subtract, multiply, and divide integers;*

b) solve practical problems involving operations with integers; and

c) simplify numerical expressions involving integers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• The set of integers is the set of whole numbers and their opposites (e.g., …-3, -2, -1, 0, 1, 2, 3…). Zero has no opposite and is neither positive nor negative. • Integers are used in practical situations, such as temperature changes (above/below zero), balance in a checking account (deposits/withdrawals), golf, time lines, football yardage, and changes in altitude (above/below sea level). • Concrete experiences in formulating rules for adding, subtracting, multiplying, and dividing integers should be explored by examining patterns using calculators, using a number line, and using manipulatives, such as two-color counters, drawings, or by using algebra tiles. • Sums, differences, products and quotients of integers are either positive, negative, undefined or zero. This may be demonstrated through the use of patterns and models. • The order of operations is a convention that defines the computation order to follow in simplifying an expression. Having an established convention ensures that there is only one correct result when simplifying an expression. • The order of operations is as follows: – First, complete all operations within grouping symbols.[1] If there are grouping symbols within other grouping symbols, do the innermost operation first. – Second, evaluate all exponential expressions. – Third, multiply and/or divide in order from left to right. – Fourth, add and/or subtract in order from left to right. [1]Parentheses ( ), absolute value | | (e.g., $\|3(-5+2)\|$), and the division bar (e.g., $\frac{3+4}{5+6}$) should be treated as grouping symbols. • Expressions are simplified using the order of operations and applying the properties of real numbers. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard):	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model addition, subtraction, multiplication and division of integers using pictorial representations or concrete manipulatives. (a) • Add, subtract, multiply, and divide two integers. (a) • Solve practical problems involving addition, subtraction, multiplication, and division with integers. (b) • Use the order of operations and apply the properties of real numbers to simplify numerical expressions involving more than two integers. Expressions should not include braces { } or brackets [ ], but may contain absolute value bars | |. Simplification will be limited to three operations, which may include simplifying a whole number raised to an exponent of 1, 2 or 3. (c)

6.6 The student will

a) add, subtract, multiply, and divide integers;*

b) solve practical problems involving operations with integers; and

c) simplify numerical expressions involving integers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
– Commutative property of addition: $a + b = b + a$. – Commutative property of multiplication: $a \cdot b = b \cdot a$. – Associative property of addition: $(a + b) + c = a + (b + c)$. – Associative property of multiplication: $(ab)c = a(bc)$. – Subtraction and division are neither commutative nor associative. – Distributive property (over addition/subtraction): $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$. – Identity property of addition (additive identity property): $a + 0 = a \; and \; 0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \; and \; 1 \cdot a = a$. – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division. – Inverse property of addition (additive inverse property): $a + (-a) = 0 \; and \; (-a) + a = 0$. – Multiplicative property of zero: $a \cdot 0 = 0 \; and \; 0 \cdot a = 0$ – Substitution property: If $a = b$ then *b* can be substituted for *a* in any expression, equation or inequality. • The power of a number represents repeated multiplication of the number (e.g., $8^3 = 8 \cdot 8 \cdot 8$). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In the example, 8 is the base, and 3 is the exponent. • Any number, except zero, raised to the zero power is 1. Zero to the zero power (0^0) is undefined.	

Measurement and geometry in the middle grades provide a natural context and connection among many mathematical concepts. Students expand informal experiences with geometry and measurement in the elementary grades and develop a solid foundation for further exploration of these concepts in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

Students develop measurement skills through exploration and estimation. Physical exploration to determine length, weight/mass, liquid volume/capacity, and angle measure are essential to develop a conceptual understanding of measurement. Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, and geometry software provide experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, square-based pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

Students explore and develop an understanding of the Pythagorean Theorem. Understanding how the Pythagorean Theorem can be applied in practical situations has a far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one.)

Level 2: Analysis. Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three.)

Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

Level 4: Deduction. Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

6.7 The student will

a) derive π (pi);

b) solve problems, including practical problems, involving circumference and area of a circle; and

c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles.

Understanding the Standard	Essential Knowledge and Skills
• The value of pi (π) is the ratio of the circumference of a circle to its diameter. Thus, the circumference of a circle is proportional to its diameter. • The calculation of determining area and circumference may vary depending upon the approximation for pi. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi (π) button on a calculator. • Experiences in deriving the formulas for area, perimeter, and volume using manipulatives such as tiles, one-inch cubes, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and their use. • Perimeter is the path or distance around any plane figure. The perimeter of a circle is called the circumference. • The circumference of a circle is about three times the measure of its diameter. • The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle. • The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve. • The area of a circle is computed using the formula $A = \pi r^2$, where r is the radius of the circle. • The perimeter of a square whose side measures s can be determined by multiplying 4 by s ($P = 4s$), and its area can be determined by squaring the length of one side ($A = s^2$). • The perimeter of a rectangle can be determined by computing the sum of twice the length and twice the width ($P = 2l + 2w$, or $P = 2(l + w)$), and its area can be determined by computing the product of the length and the width ($A = lw$). • The perimeter of a triangle can be determined by computing the sum of the side lengths ($P = a + b + c$), and its area can be determined by computing $\frac{1}{2}$ the product of base and the height ($A = \frac{1}{2}bh$).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. (a) • Solve problems, including practical problems, involving circumference and area of a circle when given the length of the diameter or radius. (b) • Solve problems, including practical problems, involving area and perimeter of triangles and rectangles.(c)

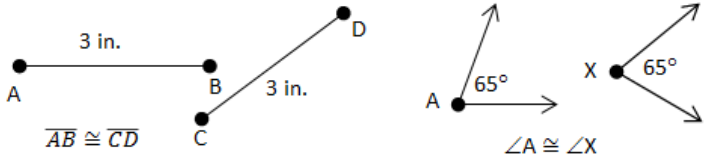
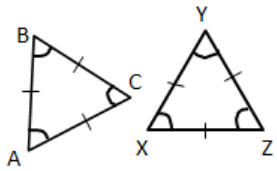
6.8 The student will

a) identify the components of the coordinate plane; and

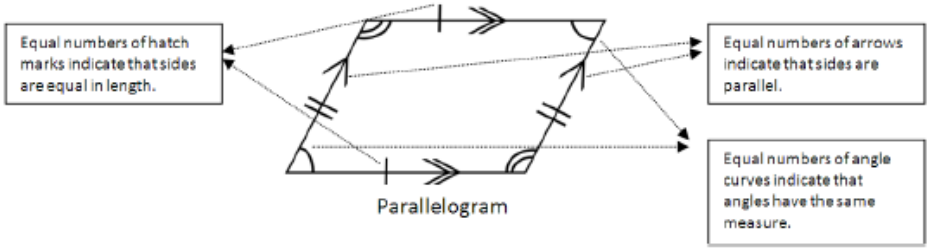
b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.

Understanding the Standard	Essential Knowledge and Skills
• In a coordinate plane, the coordinates of a point are typically represented by the ordered pair (*x*, *y*), where *x* is the first coordinate and *y* is the second coordinate. • Any given point is defined by only one ordered pair in the coordinate plane. • The grid lines on a coordinate plane are perpendicular. • The axes of the coordinate plane are the two intersecting perpendicular lines that divide it into its four quadrants. The *x*-axis is the horizontal axis and the *y*-axis is the vertical axis. • The quadrants of a coordinate plane are the four regions created by the two intersecting perpendicular lines (*x*- and *y*-axes). Quadrants are named in counterclockwise order. The signs on the ordered pairs for quadrant I are (+,+); for quadrant II, (–,+); for quadrant III, (–, –); and for quadrant IV, (+,–). • In a coordinate plane, the origin is the point at the intersection of the *x*-axis and *y*-axis; the coordinates of this point are (0, 0). • For all points on the *x*-axis, the *y*-coordinate is 0. For all points on the *y*-axis, the *x*-coordinate is 0. • The coordinates may be used to name the point. (e.g., the point (2, 7)). It is not necessary to say "the point whose coordinates are (2, 7)." The first coordinate tells the location or distance of the point to the left or right of the *y*-axis and the second coordinate tells the location or distance of the point above or below the *x*-axis. For example, (2, 7) is two units to the right of the *y*-axis and seven units above the *x*-axis. • Coordinates of points having the same *x*-coordinate are located on the same vertical line. For example, (2, 4) and (2, -3) are both two units to the right of the *y*-axis and are vertically seven units from each other. • Coordinates of points having the same *y*-coordinate are located on the same horizontal line. For example, (-4, -2) and (2, -2) are both two units below the *x*-axis and are horizontally six units from each other.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and label the axes, origin, and quadrants of a coordinate plane. (a) • Identify the quadrant or the axis on which a point is positioned by examining the coordinates (ordered pair) of the point. Ordered pairs will be limited to coordinates expressed as integers. (a) • Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. (b) • Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. (b) • Relate the coordinates of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line. Ordered pairs will be limited to coordinates expressed as integers. (b) • Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate. Ordered pairs will be limited to coordinates expressed as integers. Apply these techniques in the context of solving practical and mathematical problems. (b)

6.9 The student will determine congruence of segments, angles, and polygons.

Understanding the Standard	Essential Knowledge and Skills
• The symbol for congruency is $\cong$. • Congruent figures have exactly the same size and the same shape. Line segments are congruent if they have the same length. Angles are congruent if they have the same measure. Congruent polygons have an equal number of sides, and all the corresponding sides and angles are congruent. – Examples:   $\Delta ABC \cong \Delta XYZ$ • A polygon is a closed plane figure composed of at least three line segments that do not cross. • A regular polygon has congruent sides and congruent interior angles. • The number of lines of symmetry of a regular polygon is equal to the number of sides of the polygon. • A line of symmetry divides a figure into two congruent parts, each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines. • Noncongruent figures may have the same shape but not the same size. • Students should be familiar with geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides. An equal number of hatch (hash) marks indicate that those sides are equal in length. An equal number of arrows indicate that those sides are parallel. An equal number of angle curves indicate that those angles have the same measure. See the diagram below.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify regular polygons. • Draw lines of symmetry to divide regular polygons into two congruent parts. • Determine the congruence of segments, angles, and polygons given their properties. • Determine whether polygons are congruent or noncongruent according to the measures of their sides and angles.

6.9 The student will determine congruence of segments, angles, and polygons.

Understanding the Standard	Essential Knowledge and Skills
 • The determination of the congruence or noncongruence of two figures can be accomplished by placing one figure on top of the other or by comparing the measurements of all corresponding sides and angles. • Construction of congruent line segments, angles, and polygons helps students understand congruency.	

In the middle grades, students develop an awareness of the power of data analysis and the application of probability through fostering their natural curiosity about data and making predictions.

The exploration of various methods of data collection and representation allows students to become effective at using different types of graphs to represent different types of data. Students use measures of center and dispersion to analyze and interpret data.

Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability. Through experiments and simulations, students build on their understanding of the Fundamental Counting Principle from elementary mathematics to learn more about probability in the middle grades.

6.10 The student, given a practical situation, will

a) represent data in a circle graph;

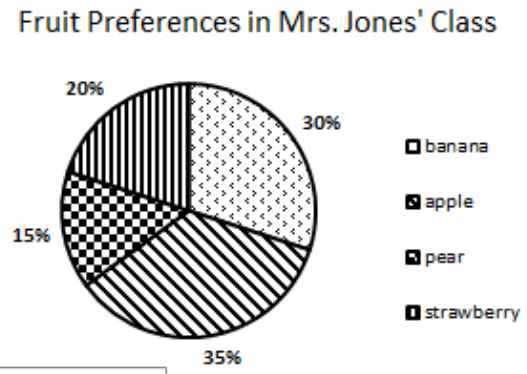
b) make observations and inferences about data represented in a circle graph; and

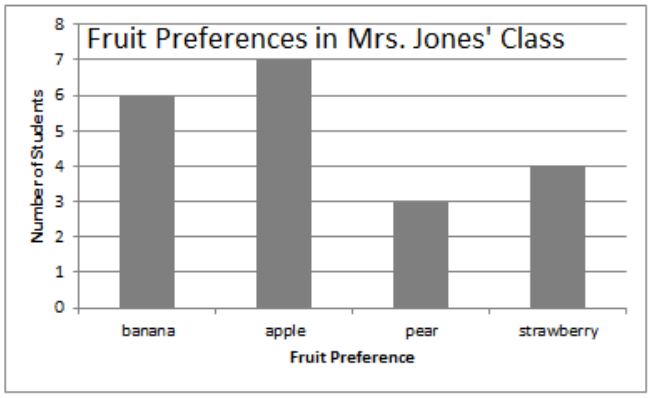
c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.

Understanding the Standard

- Circle graphs are used for data showing a relationship of the parts to the whole.
  - Example: the favorite fruit of 20 students in Mrs. Jones class was recorded in the table. Compare the same data displayed in both a circle graph and a bar graph.

Fruit Preference	# of students
banana	6
apple	7
pear	3
strawberry	4





- Circle graphs can represent percent or frequency.
- Circle graphs are not useful for representing data with large numbers of categories.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect, organize and represent data in a circle graph. The number of data values should be limited to allow for comparisons that have denominators of 12 or less or those that are factors of 100 (e.g., in a class of 20 students, 7 choose apples as a favorite fruit, so the comparison is 7 out of 20, $\frac{7}{20}$, or 35%). (a)
- Make observations and inferences about data represented in a circle graph. (b)
- Compare data represented in a circle graph with the same data represented in bar graphs, pictographs, and line plots. (c)

6.10 The student, given a practical situation, will

a) represent data in a circle graph;

b) make observations and inferences about data represented in a circle graph; and

c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.

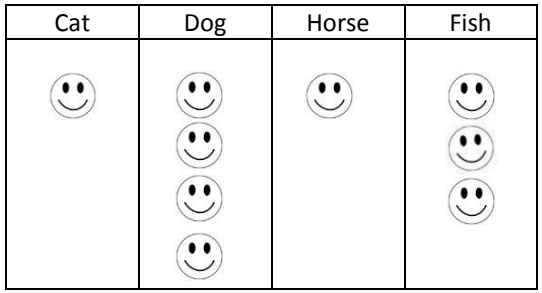
Understanding the Standard	Essential Knowledge and Skills
• Teachers should be reasonable about the selection of data values. The number of data values can affect how a circle graph is constructed (e.g., 10 out of 25 would be 40%, but 7 out of 9 would be $77.\overline{7}\%$, making the construction of a circle graph more complex). Students should have experience constructing circle graphs, but a focus should be placed on the analysis of circle graphs. • Students are not expected to construct circle graphs by multiplying the percentage of data in a category by 360° in order to determine the central angle measure. Limiting comparisons to fraction parameters noted will assist students in constructing circle graphs. • To collect data for any problem situation, an experiment can be designed, a survey can be conducted, or other data-gathering strategies can be used. The data can be organized, displayed, analyzed, and interpreted to solve the problem. • Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. • Different types of graphs can be used to display categorical data. The way data are displayed often depends on what someone is trying to communicate. – A line plot is used for categorical and discrete numerical data and is used to show frequency of data on a number line. It is a simple way to organize data. Example:  	

6.10 The student, given a practical situation, will

a) represent data in a circle graph;

b) make observations and inferences about data represented in a circle graph; and

c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.

Understanding the Standard	Essential Knowledge and Skills
– A bar graph is used for categorical and discrete numerical data (e.g., number of months or number of people with a particular eye color) and is used to show comparisons. – A pictograph is mainly used to show categorical data. Pictographs are used to show frequency and compare items. However, the use of partial pictures can give misleading information. o Example: The Types of Pets We Have   = 1 student • A circle graph is used for categorical and discrete numerical data. Circle graphs are used to show a relationship of the parts to a whole. • All graphs must include a title, percent or number labels for data categories, and a key. A key is essential to explain how to read the graph. A title is essential to explain what the graph represents. • A scale should be chosen that is appropriate for the data values being represented. • Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. • The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inferences).	

6.11 The student will

a) represent the mean of a data set graphically as the balance point; and

b) determine the effect on measures of center when a single value of a data set is added, removed, or changed.

Understanding the Standard	Essential Knowledge and Skills
• Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. • Measures of center are types of averages for a data set. They represent numbers that describe a data set. Mean, median, and mode are measures of center that are useful for describing the average for different situations. – Mean may be appropriate for sets of data where there are no values much higher or lower than those in the rest of the data set. – Median is a good choice when data sets have a couple of values much higher or lower than most of the others. – Mode is a good descriptor to use when the set of data has some identical values, when data is non-numeric (categorical) or when data reflects the most popular item. • Mean can be defined as the point on a number line where the data distribution is balanced. This requires that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances from the mean of all the data points below the mean. This is the concept of mean as the balance point. – Example: Given the data set: 2, 3, 4, 7 The mean value of 4 can be represented on a number line as the balance point:  • The mean can also be found by calculating the numerical average of the data set. • In grade five mathematics, mean is defined as fair share. • Defining mean as the balance point is a prerequisite for understanding standard deviation, which is addressed in high school level mathematics.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent the mean of a set of data graphically as the balance point represented in a line plot. (a) • Determine the effect on measures of center when a single value of a data set is added, removed, or changed. (b)

6.11 The student will

a) represent the mean of a data set graphically as the balance point; and

b) determine the effect on measures of center when a single value of a data set is added, removed, or changed.

Understanding the Standard	Essential Knowledge and Skills
• The median is the middle value of a data set in ranked order. If there are an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the numerical average of the two middle values. • The mode is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there is more than one value that occurs most often, all these most-frequently-occurring values are modes. When there are exactly two modes, the data set is bimodal.	

Patterns, functions and algebra become a larger mathematical focus in the middle grades as students extend their knowledge of patterns developed in the elementary grades.

Students make connections between the numeric concepts of ratio and proportion and the algebraic relationships that exist within a set of equivalent ratios. Students use variable expressions to represent proportional relationships between two quantities and begin to connect the concept of a constant of proportionality to rate of change and slope. Representation of relationships between two quantities using tables, graphs, equations, or verbal descriptions allow students to connect their knowledge of patterns to the concept of functional relationships. Graphing linear equations in two variables in the coordinate plane is a focus of the study of functions which continues in high school mathematics.

Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality, domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades. Students learn to solve equations by using concrete materials. They expand their skills from one-step to multistep equations and inequalities through their application in practical situations.

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine whether a proportional relationship exists between two quantities; and
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills
• A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities. • Equivalent ratios arise by multiplying each value in a ratio by the same constant value. For example, the ratio of 4:2 would be equivalent to the ratio 8:4, since each value in the first ratio could be multiplied by 2 to obtain the second ratio. • A proportional relationship consists of two quantities where there exists a constant number (constant of proportionality) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity. • Proportional thinking requires students to thinking multiplicatively, versus additively. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e.., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, because context can help students to see the relationship. Students will explore algebraic representations of additive relationships in grade seven. – Example: Additive relationship: Multiplicative relationship: x y (+8: 2→10, 3→11, 4→12, 5→13) x y (·5: 2→10, 3→15, 4→20, 5→25)	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio. (a) • Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a practical situation. (a) • Identify the unit rate of a proportional relationship represented by a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (b) • Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate. Unit rates are limited to positive values. (b) • Determine whether a proportional relationship exists between two quantities, when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (c) • Determine whether a proportional relationship exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. (c)

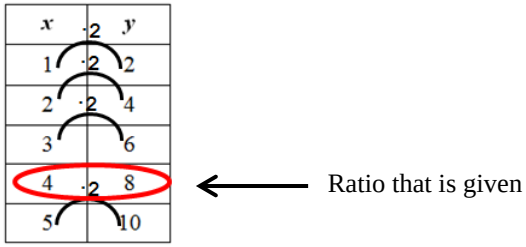
6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills
○ In the additive relationship, *y* is the result of adding 8 to *x*. ○ In the multiplicative relationship, *y* is the result of multiplying 5 times *x*. ○ The ordered pair (2, 10) is a quantity in both relationships; however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative. • Students have had experiences with tables of values (input/output tables that are additive and multiplicative) in elementary grades. • A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios. A constant exists that can be multiplied by the measure of one quantity to get the measure of the other quantity for every ratio pair. The same proportional relationship exists between each pair of quantities in a ratio table. – Example: Given that the ratio of *y* to *x* in a proportional relationship is 8:4, create a ratio table that includes three additional equivalent ratios.  Students have had experience with tables of values (input/output tables) in elementary grades and the concept of a ratio table should be connected to their prior knowledge of representing number patterns in tables.	• Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values. (d)

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine whether a proportional relationship exists between two quantities; and
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills

- A rate is a ratio that involves two different units and how they relate to each other. Relationships between two units of measure are also rates (e.g., inches per foot).
- A unit rate describes how many units of the first quantity of a ratio correspond to one unit of the second quantity.
  - Example: If it costs \$10 for 5 items at a store (a ratio of 10:5 comparing cost to the number of items), then the unit rate would be \$2.00/per item (a ratio of 2:1 comparing cost to number of items).

Unit Rate Given ratio

# of items (*x*)	1	2	5	10
Cost in \$ (*y*)	\$2.00	\$4.00	\$10.00	\$20.00

- Any ratio can be converted into a unit rate by writing the ratio as a fraction and then dividing the numerator and denominator each by the value of the denominator. Example: It costs \$8 for 16 gourmet cookies at a bake sale. What is the price per cookie (unit rate) represented by this situation?

$$\frac{8}{16} = \frac{8 \div 16}{16 \div 16} = \frac{0.5}{1}$$

So, it would cost \$0.50 per cookie, which would be the unit rate.

  - Example: $\frac{8}{16}$ and 40 to 10 are ratios, but are not unit rates. However, $\frac{0.5}{1}$ and 4 to 1 are unit rates.
- Students in grade six should build a conceptual understanding of proportional relationships and unit rates before moving to more abstract representations and complex computations in higher grade levels. Students are not expected to use formal calculations for slope and unit rates (e.g., slope formula) in grade six.

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine whether a proportional relationship exists between two quantities; and
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills

- Example of a proportional relationship:

  Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza. This ratio table represents the cost (*y*) per number of pizzas ordered (*x*).

x number of pizzas	1	2	3	4
y total cost	8	16	24	32

x8 x8

  In this relationship, the ratio of *y* (cost in $) to *x* (number of pizzas) in each ordered pair is the same:

$$\frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4}$$

- Example of a non-proportional relationship:

  Uptown Pizza sells medium pizzas for $7 each but charges a $3 delivery fee per order. This table represents the cost per number of pizzas ordered.

x number of pizzas	1	2	3	4
y total cost	10	17	24	31

  The ratios represented in the table above are not equivalent.

  In this relationship, the ratio of *y* to *x* in each ordered pair is not the same:

$$\frac{10}{1} \neq \frac{17}{2} \neq \frac{24}{3} \neq \frac{31}{4}$$

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills

Other non-proportional relationships will be studied in later mathematics courses.

- Proportional relationships can be described verbally using the phrases "for each," "for every," and "per."
- Proportional relationships involve collections of pairs of equivalent ratios that may be graphed in the coordinate plane. The graph of a proportional relationship includes ordered pairs (x, y) that represent pairs of values that may be represented in a ratio table.
- Proportional relationships can be expressed using verbal descriptions, tables, and graphs.
  - Example: (verbal description) To make a drink, mix 1 liter of syrup with 3 liters of water. If x represents how many liters of syrup are in the mixture and y represents how many liters of water are in the mixture, this proportional relationship can be represented using a ratio table:

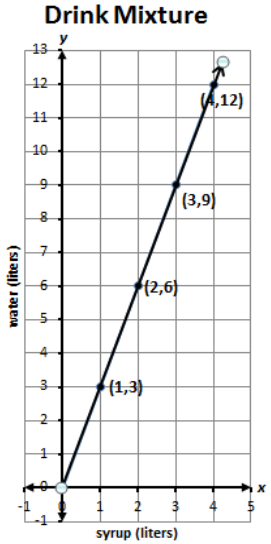
Syrup (liters) x	1	2	3	4
Water (liters) y	3	6	9	12

The ratio of the amount of water (y) to the amount of syrup (x) is 3:1. Additionally, the proportional relationship may be graphed using the ordered pairs in the table.

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine whether a proportional relationship exists between two quantities; and
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills



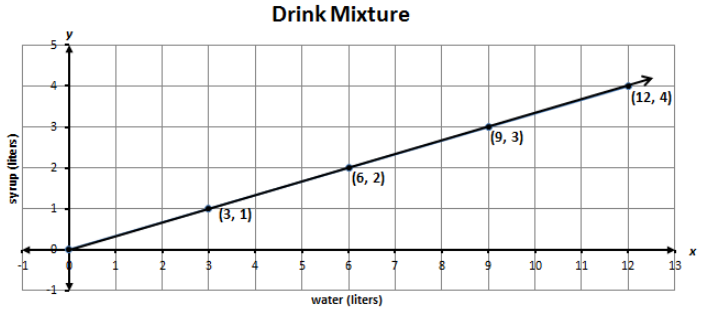
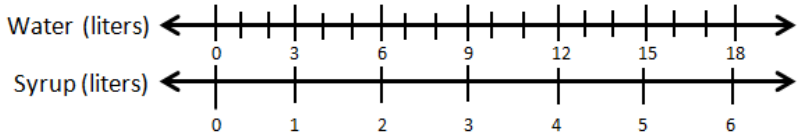
- The representation of the ratio between two quantities may depend upon the order in which the quantities are being compared.
  - Example: In the mixture example above, we could also compare the ratio of the liters of syrup per liters of water, as shown:

Water (liters) x	3	6	9	12
Syrup (liters) y	1	2	3	4

In this comparison, the ratio of the amount of syrup (y) to the amount of water (x) would be 1:3.

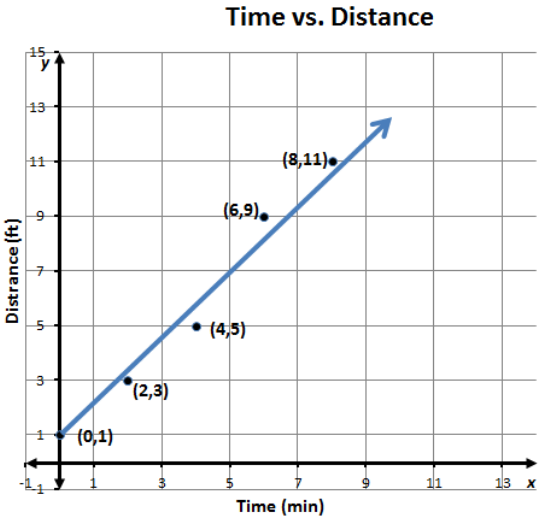
6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine whether a proportional relationship exists between two quantities; and
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills
The graph of this relationship could be represented by:  Students should be aware of how the order in which quantities are compared affects the way in which the relationship is represented as a table of equivalent ratios or as a graph. • Double number line diagrams can also be used to represent proportional relationships and create collections of pairs of equivalent ratios. – Example:  In this proportional relationship, there are three liters of water for each liter of syrup represented on the number lines.	

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine whether a proportional relationship exists between two quantities; and
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills
• A graph representing a proportional relationship includes ordered pairs that lie in a straight line that, if extended, would pass through (0, 0), creating a pattern of horizontal and vertical increases. The context of the problem and the type of data being represented by the graph must be considered when determining whether the points are to be connected by a straight line on the graph. – Example of the graph of a non-proportional relationship:  The relationship of distance (y) to time (x) is non-proportional. The ratio of y to x for each ordered pair is not equivalent. That is, $\frac{11}{8} \neq \frac{9}{6} \neq \frac{5}{4} \neq \frac{3}{2} \neq \frac{1}{0}$	

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills

The points of the graph do not lie in a straight line. Additionally, the line does not pass through the point (0, 0), thus the relationship of *y* to *x* cannot be considered proportional.

- Practical situations that model proportional relationships can typically be represented by graphs in the first quadrant, since in most cases the values for *x* and *y* are not negative.
- Unit rates are not typically negative in practical situations involving proportional relationships.
- A unit rate could be used to find missing values in a ratio table.
  - Example: A store advertises a price of $25 for 5 DVDs. What would be the cost to purchase 2 DVDs? 3 DVDs? 4 DVDs?

# DVDs	1	2	3	4	5
Cost	$5	?	?	?	$25

The ratio of $25 per 5 DVDs is also equivalent to a ratio of $5 per 1 DVD, which would be the unit rate for this relationship. This unit rate could be used to find the missing value of the ratio table above. If we multiply the number of DVDs by a constant multiplier of 5 (the unit rate) we obtain the total cost. Thus, 2 DVDs would cost $10, 3 DVDs would cost $15, and 4 DVDs would cost $20.

6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
• A one-step linear equation may include, but not be limited to, equations such as the following: $2x = 5$; $y - 3 = -6$; $\frac{1}{5}x = -3$; $a - (-4) = 11$. • A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable. • An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an "equal sign (=)" (e.g., $\frac{3}{4}$, $5x$, $140 - 38.2$, $18 \cdot 21$, $5 + x$.) • An expression that contains a variable is a variable expression. A variable expression is like a phrase: As a phrase does not have a verb, so an expression does not have an "equal sign (=)". An expression cannot be solved. • A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. Example, the verbal expression "a number multiplied by 5" could be represented by the variable expression "$n \cdot 5$" or "$5n$." • An algebraic expression is a variable expression that contains at least one variable (e.g., $x - 3$). • A verbal sentence is a complete word statement (e.g., "The sum of a number and two is five" could be represented by "$n + 2 = 5$"). • An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., $2x = 7$). • A term is a number, variable, product, or quotient in an expression of sums and/or differences. In $7x^2 + 5x - 3$, there are three terms, $7x^2$, $5x$, and 3. • A coefficient is the numerical factor in a term. Example: in the term $3xy^2$, 3 is the coefficient; in the term z, 1 is the coefficient. • An equation is a mathematical sentence stating that two expressions are equal. • A variable is a symbol used to represent an unknown quantity.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Identify examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient. • Represent and solve one-step linear equations in one variable, using a variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale. • Apply properties of real numbers and properties of equality to solve a one-step equation in one variable. Coefficients are limited to integers and unit fractions. Numeric terms are limited to integers. • Confirm solutions to one-step linear equations in one variable. • Write verbal expressions and sentences as algebraic expressions and equations. • Write algebraic expressions and equations as verbal expressions and sentences. • Represent and solve a practical problem with a one-step linear equation in one variable.

6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
• The solution to an equation is a value that makes it a true statement. Many equations have one solution and are represented as a point on a number line. Solving an equation or inequality involves a process of determining which value(s) from a specified set, if any, make the equation or inequality a true statement. Substitution can be used to determine whether a given value(s) makes an equation or inequality true. • Properties of real numbers and properties of equality can be used to solve equations, justify equation solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard). – Commutative property of addition: $a + b = b + a$. – Commutative property of multiplication: $a \cdot b = b \cdot a$. – Subtraction and division are neither commutative nor associative. – Identity property of addition (additive identity property): $a + 0 = a \; and \; 0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \; and \; 1 \cdot a = a$. – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division. – Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$). – Inverse property of addition (additive inverse property): $a + (-a) = 0 \; and \; (-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \; and \; \frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0 \; and \; 0 \cdot a = 0$. – Division by zero is not a possible mathematical operation. It is undefined. – Addition property of equality: If $a = b$, then $a + c = b + c$.	

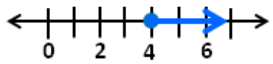
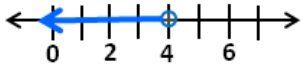
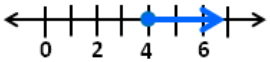
6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
– Subtraction property of equality: If $a = b$, then $a - c = b - c$. – Multiplication property of equality: If $a = b$, then $a \cdot c = b \cdot c$. – Division property of equality: If $a = b \; and \; c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. – Substitution property: If $a = b$ then *b* can be substituted for *a* in any expression, equation or inequality.	

6.14 The student will

a) represent a practical situation with a linear inequality in one variable; and

b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.

Understanding the Standard	Essential Knowledge and Skills
• The solution set to an inequality is the set of all numbers that make the inequality true. • Inequalities can represent practical situations. Example: Jaxon works at least 4 hours per week mowing lawns. Write an inequality representing this situation and graph the solution. $x \geq 4$ or $4 \leq x$  Students might then be asked: "Would Jaxon ever work 3 hours in a week? 6 hours? • The variable in an inequality may represent values that are limited by the context of the problem or situation. Example: if the variable represents all children in a classroom who are taller than 36 inches, the variable will be limited to have a minimum and maximum value based on the heights of the children. Students are not expected to represent these situations with a compound inequality (e.g., 36 < x < 70) but only recognize that the values satisfying the single inequality (x > 36) will be limited by the context of the situation. • Inequalities using the < or > symbols are represented on a number line with an open circle on the number and a shaded line over the solution set. Example: When graphing $x < 4$, use an open circle above the 4 to indicate that the 4 is not included.  • Inequalities using the ≤ or ≥ symbols are represented on a number line with a closed circle on the number and shaded line in the direction of the solution set. Example: When graphing $x \geq 4$ fill in the circle above the 4 to indicate that the 4 is included.  • It is important for students to see inequalities written with the variable before the inequality symbol and after. Example: $x > 5$ is not the same relationship as $5 > x$. However, $x > 5$ is the same relationship as $5 < x$.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Given a verbal description, represent a practical situation with a one-variable linear inequality. (a) • Apply properties of real numbers and the addition or subtraction property of inequality to solve a one-step linear inequality in one variable, and graph the solution on a number line. Numeric terms being added or subtracted from the variable are limited to integers. (b) • Given the graph of a linear inequality with integers, represent the inequality two different ways (e.g., $x < -5$ or $-5 > x$) using symbols. (b) • Identify a numerical value(s) that is part of the solution set of a given inequality. (a, b)

6.14 The student will

a) represent a practical situation with a linear inequality in one variable; and

b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.

Understanding the Standard	Essential Knowledge and Skills
• A one-step linear inequality may include, but not be limited to, inequalities such as the following: $2 + x > 5$; $y - 3 \leq -6$; $a - (-4) \geq 11$. • Solving an equation or inequality involves a process of determining which value(s) from a specified set, if any, make the equation or inequality a true statement. Substitution can be used to determine whether a given value(s) makes an equation or inequality true. • Properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard): – Commutative property of addition: $a + b = b + a$. – Commutative property of multiplication: $a \cdot b = b \cdot a$. – Subtraction and division are neither commutative nor associative. – Identity property of addition (additive identity property): $a + 0 = a \; and \; 0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \; and \; 1 \cdot a = a$. – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division. – Inverses are numbers that combine with other numbers and result in identity elements (e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$). – Inverse property of addition (additive inverse property): $a + (-a) = 0 \; and \; (-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \; and \; \frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0 \; and \; 0 \cdot a = 0$.	

6.14 The student will

a) represent a practical situation with a linear inequality in one variable; and

b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.

Understanding the Standard	Essential Knowledge and Skills
– Addition property of inequality: If $a < b$, then $a + c < b + c$; if $a > b$, then $a + c > b + c$ (this property also applies to $\leq and \geq$). – Subtraction property of inequality: If $a < b$, then $a - c < b - c$; if $a > b$, then $a - c > b - c$ (this property also applies to $\leq and \geq$). – Substitution property: If $a = b$ then *b* can be substituted for *a* in any expression, equation or inequality.	