

Mathematics Curriculum Guide

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Preface

This guide will assist the mathematics teacher in preparing students for the challenges of the twenty-first century. As established by the National Council of Teachers of Mathematics Principles and Standards for School Mathematics, educational goals for students are changing. A comprehensive and coherent set of mathematics standards for each and every student from prekindergarten through grade 12, Principles and Standards is the first set of rigorous, college and career readiness standards for the 21st century. Students should have many and varied experiences in their mathematical training to help them learn to value mathematics, become confident in their ability to do mathematics, become problem solvers, and learn to communicate and reason mathematically. This guide, along with the available division resources, VDOE resources, professional literature, alternative assessment methods, and in-service activities will assist the mathematics teacher in continuing to integrate these student goals into the curriculum.

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Introduction/General Comments

This curriculum guide follows the 2016 Virginia Math 4 SOLs as adopted by the Virginia Department of Education. It is extremely important and required that the Sequence of Instruction and Pacing be followed as presented in this guide. The textbook adopted as a resource for this course is *Go Math!* (Grade 4) by Houghton, Mifflin, Harcourt, 2017 edition.

Students will take two formative assessments during the year (dates to be determined annually). Each teacher-designed test will assess skill levels of the SOLs as presented in the Sequence of Instruction and Pacing and the formative assessment blueprint. The data collected from the formative assessments will help teachers determine students' strengths and weaknesses, and inform instructional decisions.

The Mapping for Instruction is based on specified SOLs, which are to be taught in the predetermined order. Note, some SOLs (or their parts) may be taught over multiple 9-week periods.

Refer to the Mathematics 2016 Standards of Learning Grade 4 Curriculum Framework during every lesson. It is located at the back of this guide. This will provide valuable information for the teacher (Understanding the Standard) and desired goals for instruction (Essential Knowledge and Skills). Examples of teaching techniques and strategies, definitions, and recommended manipulatives are included in the Curriculum Framework and on the VDOE website under Mathematics Instructional Plans (MIPs) http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/mip/index.shtml.

Resources Overview

Resources for all SOLs

IXL

PowerSchool

BrainPop VDOF

Promethean Planet

Number Talks

Performance Tasks

RCPS Common Assessments

get2math

DESMOS CALCULATOR

The Desmos <u>Virginia Four-Function Calculator</u> will be used for instruction and assessment.

Sequence of Instruction and Pacing

	First Nine Weeks		S	Second Nine Weeks		Т	hird Nine Weeks		Fo	ourth Nine Weeks	
SOL	Instructional Focus	Days	SOL	Instructional Focus	Days	SOL	Instructional Focus	Days	SOL	Instructional Focus	Days
4.1a,b,c	Whole numbers: place value comparing and ordering rounding	9	4.4a	Multiplication facts through 12 x 12	3	4.10a,b	Geometry: Points, lines, line segments, rays, and angles Intersecting, parallel, and perpendicular lines	4	4.8a,b,c,d	Measurement: Estimate and measure: length (US Customary and Metric) Estimate and measure: weight/mass (US Customary and Metric) Estimate and measure: liquid volume (US Customary) Identify equivalent measures: length, weight/mass, liquid volume (US Customary) Solve practical problems: length, weight/mass, and liquid volume (US Customary)	10
4.4b,d	Add and subtract whole numbers including estimating sums and differences Single step and multistep word problems	5	4.4b,d	Multiplication: Estimate and determine products of whole numbers Solve single-step and multi-step word problems	15	4.12	Classify quadrilaterals: Parallelograms Rectangles Squares Rhombi Trapezoids	3	4.13a,b,c	Probability	5

4.3a,b,c,d	Decimals: place value comparing and ordering rounding modeling	12	4.4c,d 4.2c	Division: Estimate and determine quotients of whole numbers with and without remainders Solve single-step practical problems involving division Identify the division statement that represents a fraction	15	4.11	Plane and solid figures: Identify, describe, compare, and contrast plane and solid figures	4	4.14a,b,c	Graphing: Bar graphs Line graphs Collect data, interpret, represent, and compare	5
4.6a,b	Decimals: Add and subtract Single-step and multi- step word problems	7	4.16	Equality in an equation	3	4.7	Determine area and perimeter (US Customary and Metric)	3		SOL Review & SOL Testing	15
4.9	Elapsed Time	5	4.15	Patterns: Identify, describe, create, and extend patterns found in objects, pictures, numbers, and tables Input/output tables	4	4.2a,b,c 4.3d	Fractions and mixed numbers: Compare and order Equivalent fractions Models Decimal and fraction equivalents	13		Hands-On Equations Level 1	5
						4.5a,b,c	Common multiplies and factors (LCM and GCF) Add and subtract fractions and mixed numbers with like and unlike denominators Solve single-step word problems with fractions and mixed numbers	13			
	Remediation, Review, Assessment	7		Remediation, Review, Assessment	5		Remediation, Review, Assessment	5		Remediation, Review, Assessment	5
	Total Days	45		Total Days	45		Total Days	45		Total Days	45

Mapping for Instruction - First Nine Weeks

SOLs

4.1 The student will

- a) read, write, and identify the place and value of each digit in a nine-digit whole number;
- b) compare and order whole numbers expressed through millions; and
- c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

4.3 The student will

- a) read, write, represent, and identify decimals expressed through thousandths;
- b) round decimals to the nearest whole number;
- c) compare and order decimals; and
- d) given a model, write the decimal and fraction equivalents.*

4.4 The student will

- b) estimate and determine sums, differences, and products of whole numbers;*
- d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

4.6 The student will

- a) add and subtract decimals;* and
- b) solve single-step and multistep practical problems involving addition and subtraction with decimals.
- 4.9 The student will solve practical problems related to elapsed time in hours and minutes within a 12-hour period.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

SOL	Instructional Focus	Vocabulary	Comments	Time Frame
	Read, write, and identify the place and value digits of whole numbers (up to nine digit numbers)	Digit, value, place value, ones period, thousands period, millions period, standard form, word form, expanded form, compare,	Teachers may go beyond nine digit numbers for enrichment.	9 days
4.1 a, b, c	Compare (limited to two) and order (limited to 4) whole numbers	greater than, less than, equal to, not equal to, order, greatest, least, round, number line		
	Round whole numbers	Symbols: >, <, =, ≠		
	(through millions)			
	Add and subtract:	Addends, add, sum, subtract, difference,		5 days
	Add and subtract whole numbers up to 999,999	estimate, <i>closer to</i> , <i>between</i> , <i>a little more than</i> , total, altogether, combined, in all, how	should be placed on thinking and reasoning rather than on	
4464	Estimate sums and differences	many more	key words.	
4.4 b, d	Create and solve single-step and multistep word problems		Note: SOL 4.4d allows the use of the calculator.	

4.3 a, b, c, d	Decimals: Read, write, represent, and identify decimals Round decimals Compare and order decimals Write the decimal and fraction equivalent	Decimal, "and", tenths, hundredths, thousandths, standard form, word form, expanded form, round, nearest whole number/dollar, compare, greater than, less than, equal to, not equal to, order, greatest, least Read the decimal point as "and"		12 days
4.6 a, b	Decimals: Add and subtract decimals (estimate and solve) Solve single-step and multistep word problems	Add, sum, subtract, difference, in all, altogether, how much more	In problem-solving, emphasis should be placed on thinking and reasoning rather than on key words. Note: SOL 4.6b allows the use of the calculator.	7 days
4.9	Elapsed time Includes 12-hour period across a.m. and p.m.	Elapsed time, a.m., p.m., digital clock, analog clock, timeline, ending time, beginning time, quarter after, half past, quarter till		5 days
			Remediation, Review, Assessment	7 days
			Total	45 days

	Resources – First Nine Weeks						
Common Assessment	Textbook	Links	Supplemental Materials				
Review/Pretest			Placement Test (Teacher Resource Materials) Power School: Math 4 Pretest (Benchmark)				
SOL 4.1 a, b, c	4.1 a - SE: 5–10, 11–16, 31-36 TE: 5A–5B, 11A–11B, 31A–31B Virginia SOL Success: VA16, VA17, VA20 4.1 b - SE: 17–22 TE: 17A–17B Virginia SOL Success: VA18 4.1 c - SE: 23–28 TE: 23A–23B Virginia SOL Success: VA19	www.funbrain.com/tens/index.html www.aaamath.com/index.html www.rainforestmaths.com www.berghuis.co.nz/ablator/maths/mcindex.html http://illuminations.nctm.org/ http://www.doe.virginia.gov/instruction/mathematics/resources/videos/index.shtml# http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess 4-1abc.pdf (Location, Location, Location) https://sites.google.com/site/rcpskahoots/ https://sites.google.com/a/solteacher.com/olteachercom/home/4th-grade/4th-grade-math	See MIPs: 4.1ab - Mystery Numbers: Number Sense (Word) / PDF Version 4.1c - Rounding: Identifying the Range (Word) / PDF Version VDOE Mathematics Vocabulary Cards – Grade 4 Literature • How Much is a Million by David Schwartz • Place Value by David Adler • Sir Cumference and all the King's Tens by Cindy Neuschwander • A Place for Zero by Angeline Lopresti Base-10 blocks Place value charts dice playing cards IXL Prodigy Common Assessments				
SOL 4.4 b, d	4.4 b - SE: 37–42, 43–48, 49-54, 63-68, 69-74, 75–80, 81–86, 87–92, 93–98, 99–104, 107–112, 113-118, 119–124, 131-136, 145–150, 151–156, 157–162, 163–168, 171–176, 177–182, 183-188 TE: 37A–B, 43A–B, 49A–B, 63A–B, 69A–B, 75A–B, 81A–B, 87A–B, 93A–B,99A–B,107A–B, 113A-B, 119A–B, 131A–B, 145A–B,151A–B, 157A–B, 163A–B, 171A–B, 177A–B,183A–B, Virginia SOL Success: 1A, 1–6, VA21, VA22, VA23, VA24, VA27, VA28, VA29, VA30, VA31, VA32, VA33, VA34, VA35, VA36, VA38, VA39, VA40, VA41, VA42, VA43, VA44, VA45 4.4 d - SE: 37–42, 43–48, 49–54, 63–68, 69–74, 75–80, 81–86, 87–92, 93–98, 99-104, 107–112, 113–118, 119-124, 131–136, 145–150, 151–156, 157-162, 163–168, 171–176, 177–182,	https://sites.google.com/site/rcpskahoots/ https://sites.google.com/a/solteacher.com/olteacher- com/home/4th-grade/4th-grade-math www.multiplication.com/interactive-games.html www.coolmath.com www.mathisfun.com www.mathisfun.com www.mathcats.com http://www.doe.virginia.gov/instruction/mathematics /resources/videos/index.shtml# http://www.doe.virginia.gov/testing/solsearch/sol/m ath/4/mess 4-4b 1.pdf (Modeling Addition and Subtraction) http://www.doe.virginia.gov/testing/solsearch/sol/m ath/4/mess 4-4d.pdf (Let's Do Lunch) http://www.doe.virginia.gov/testing/solsearch/sol/m ath/4/mess_4-4a.pdf (Estimation Game)	See MIPs: 4.4b - Addition and Subtraction with Whole Numbers (Word) / PDF Version 4.4b - Multiplying Two-Digit Numbers: Bridging from the Concrete to the Symbolic (Word) / PDF Version 4.4d - Shopping for Thanksgiving Lunch: Computation (Word) / PDF Version VDOE Mathematics Vocabulary Cards – Grade 4 Literature • 12 Ways to get 11 by Eve Merriam • Elevator Magic by Stuart J Murphy Basic-Facts Timed Tests (Teacher Resource Masters) calculators flash cards graph paper dice IXL Prodigy Common Assessments				

SOL 4.6 a, b 4.6 a - Virginia SOL Success: 127A, 127–132, 133A, 133–138, https://sites.google.com/a/solteacher.com/olteacher- https://sites.google.com/a/solteacher.com/olteacher- https://sites.google.com/a/solteacher.com/olteacher- https://sites.google.com/a/solteacher- https://sites.googl	127A, 127–132, 133A, 133–138, https://sites.google.com/a/solteacher.com/olteacher-	rd) / <u>PDF</u> az
	com/home/4th-grade/4th-grade-math	v 01 31011

	139A, 139–144, 145A, 145–150, VA111, VA112, VA113, VA114 4.6 b - Virginia SOL Success: 127A, 127–132, 133A, 133–138, 139A, 139–144, 145A, 145–150, VA111, VA112, VA113, VA114	http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess_4-4d.pdf (Let's Do Lunch) http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess_4-5c.pdf (Decimal Sums and Differences)	4.6b - Solving Contextual Problems: Adding and Subtracting Decimals (Word) / PDF Version VDOE Mathematics Vocabulary Cards – Grade 4 T charts money graph paper IXL Prodigy Common Assessments
SOL 4.9	SE: 691–696, 697-702 TE: 691A–691B, 697A–697B, Virginia SOL Success: VA139, VA140	https://sites.google.com/site/rcpskahoots/ https://sites.google.com/a/solteacher.com/olteacher- com/home/4th-grade/4th-grade-math www.mathisfun.com www.aaamath.com/index.html http://www.gamequarium.org/dir/Gamequarium/Mat h/ http://www.doe.virginia.gov/testing/solsearch/sol/m ath/4/mess 4-9.pdf (How Much Longer?)	See MIPs: 4.9 - How Much Longer? (Elapsed Time) (Word) / PDF Version VDOE Mathematics Vocabulary Cards – Grade 4 Performance Tasks

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Mapping for Instruction - Second Nine Weeks

SOLs

- 4.2 The student will
 - identify the division statement that represents a fraction, with models and in context.
- 4.4 The student will
 - demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
 - b) estimate and determine sums, differences, and products of whole numbers;*
 - c) estimate and determine quotients of whole numbers, with and without remainders;* and
 - d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.
- 4.15 The student will identify, describe, create, and extend patterns found in objects, pictures, numbers, and tables.
- 4.16 The student will recognize and demonstrate the meaning of equality in an equation.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

SOL	Instructional Focus	Vocabulary	Comments	Time Frame
4.4a	Multiplication facts 12 x 12 See Table of Problem Types in the Curriculum Framework.	Multiples, property of zero, factor, product, array, number line, hundreds chart, properties of one and zero	Student must understand and use the properties. They do not need to know the formal terms.	3 days
		Properties: commutative, associative, identity, distributive		
	Multiplication:	Multiples, factors, product, estimate, round, about,	Note: SOL 4.4d allows the	15 days
4.4 b, d	Estimate and determine products of whole numbers	in all, altogether, <i>closer to, between, a little more than</i> , array	use of the calculator.	
	Solve single-step and multi-step word problems			
	Division:	Dividend, divisor, quotient, remainder, about, closer	Note: SOL 4.4d allows the	15 days
	Estimate and determine quotients of whole numbers with and without remainders	to, between, a little more than, estimation, rounding	use of the calculator.	
4.4 c, d 4.2c	Solve single-step practical problems involving division			
4.20	Identify the division statement that represents a fraction			
	See Table of Problem Types in Curriculum Framework.			
4.16	Equality in an equation	Equivalent (=), not equivalent (≠), balanced, not balanced		3 days

	Patterns: Identify, describe, create, and extend patterns found in objects, pictures, numbers, fractions, and tables	Tables, graphs, symbols, numerical pattern, growing pattern, repeating pattern, input/output tables, geometric pattern, function, rule, extend		4 days
	Solve practical problems with patterns:			
4.15	 limited to add, subtract, multiply whole numbers 			
	 limited to add and subtract of fractions with denominators ≤ 12. 			
	Identify the rule:			
	- limited to add, subtract, multiply whole numbers			
			Remediation, Review, Assessment	5 days
			Total	45 days

	Resources – Second Nine Weeks					
Common Assessment	Textbook	Links	Supplemental Materials			
SOL 4.4 a	SE: 63–68, 69–74, 75–80, 279-284 TE: 63A–63B, 69A–69B, 75A–75B, 279A–279B Virginia SOL Success: 7A, 7–12, 13A, 13–18, VA25, VA26, VA27, VA28, VA29, VA59	www.coolmath.com www.mathisfun.com www.scottforesman.com www.aaamath.com/index.html www.internet4classrooms.com http://star.spsk12.net/math/math_04.htm https://classflow.com/ http://www.kidsnumbers.com/ http://www.kidsnumbers.com/ http://www.doe.virginia.gov/instruction/mathemat ics/resources/videos/index.shtml# http://www.multiplication.com/ http://www.abcya.com/ http://studyjams.scholastic.com/studyjams/jams/ma th/multiplication-division/multiplication.htm https://www.khanacademy.org/math/cc-fourth- grade-math/cc-4th-mult-div-topic http://gamequarium.com http://www.sheppardsoftware.com/math.htm http://www.doe.virginia.gov/testing/solsearch/sol/m ath/4/mess_4-4d.pdf http://www.doe.virginia.gov/testing/solsearch/sol/m ath/4/mess_4-4b_2.pdf	See MIPs: 4.4a - Multiplication: Fact Fluency (Word) / PDF Version 4.4a - Multiplication: Fluency with Facts (Word) / PDF Version VDOE Mathematics Vocabulary Cards – Grade 4 Performance Tasks • Decisions, Decisions • Holiday Inn Dilemma calculators multiplication flash cards multiplication Bingo multiplication fact sheets graph paper IXL Prodigy Common Assessments			
SOL 4.4 b, d	4.4 b - SE: 37–42, 43–48, 49-54, 63-68, 69-74, 75–80, 81–86, 87–92, 93–98, 99–104, 107–112, 113-118, 119–124, 131-136, 145–150, 151–156, 157–162, 163–168, 171–176, 177–182, 183-188 TE: 37A–B, 43A–B, 49A–B, 63A–B,69A–B,75A-B, 81A–B, 87A–B, 93A–B, 119A–B, 131A–B, 113A–B, 157A–B, 163A–B, 171A–B, 157A–B, 183A–B	www.coolmath.com www.mathisfun.com www.scottforesman.com www.aaamath.com/index.html www.internet4classrooms.com http://star.spsk12.net/math/math_04.htm http://www.kidsnumbers.com/ http://illuminations.nctm.org/ http://illuminations.nctm.org/ http://www.doe.virginia.gov/instruction/mathematics/resources/videos/index.shtml# http://studyjams.scholastic.com/studyjams/jams/math/index.htm https://www.khanacademy.org/math/cc-fourth-grade-math/cc-4th-mult-div-topic http://gamequarium.com	See MIPs: 4.4b - Addition and Subtraction with Whole Numbers (Word) / PDF Version 4.4b - Multiplying Two-Digit Numbers: Bridging from the Concrete to the Symbolic (Word) / PDF Version 4.4d - Shopping for Thanksgiving Lunch: Computation (Word) / PDF Version Literature • Spaghetti and Meatballs for All! by Marilyn Burns • Hersey Kisses Multiplication & Division by Jerry Pallotta			

> Virginia SOL Success: 1A, 1-6, VA21, VA22, VA23, VA24, VA27, VA28, VA29, VA30, VA31, VA32, VA33, VA34, VA35, VA36, VA38, VA39, VA40, VA41, VA42, VA43, VA44, VA45

4.4 d - SE: 37–42, 43–48, 49-54, 63-68, 69-74, 75-80, 81-86, 87-92, 93-98, 99-104, 107-112, 113-118, 119-124, 131-136, 145–150, 151–156, 157-162, 163–168, 171–176, 177–182, 183–188, 197– 202, 203-208, 209-214, 215-220, 221-226, 227-232, 235-240, 241-246, 247-252, 253-258, 259-264 TE: 37A-B, 43A-B, 49A-B, 63A-B, 69A-B, 75A-B, 81A-B, 87A-B, 93A-B, 99A-B, 107A-B, 113A-B, 119A-B, 131A-B, 145A-B, 151A-B, 157A-B, 163A-B, 171A-B, 177A-B, 183A-B, 197A-B, 203A-B, 209A-B, 215A-B, 221A-B, 227A-B,235A-B, 241A-B, 247A-B, 253A-B, 259A-B, Virginia SOL Success: 19A, 19-24, VA21, VA23, VA24, VA27, VA28, VA29, VA30, VA31, VA32, VA33, VA34, VA35, VA36, VA38, VA39, VA40, VA41, VA42, VA43, VA44, VA45, VA46, VA47, VA48, VA49, VA50, VA51, VA52, VA53, VA54, VA55,

VA56, VA58

https://classflow.com/

http://www.sheppardsoftware.com/math.htm http://www.doe.virginia.gov/testing/solsearch/sol/m

ath/4/mess 4-4d.pdf

http://www.doe.virginia.gov/testing/solsearch/sol/m

ath/4/mess 4-4b 2.pdf

Multiplying Menace: The Revenge of Rumpelstiltskin (A Math Adventure) by Pam Calvert

Calculators

dice

multiplication flash cards

multiplication Bingo

Game: Around the World

multiplication basic fact sheets

graph paper

ĬXL Prodigy

Common Assessments

SOL 4.4 c, d SOL 4.2 c

4.4 c - SE: 197–202, 203– 208, 209–214, 215–220, 221-226, 227-232, 235-240, 241–246, 247–252, 253-258, 259-264 TE: 197A-B. 203A-B. 209A-B, 215A-B, 221A-B, 227A-B. 235A-B. 241A-B. 247A-B, 253A-B, 259A-B Virginia SOL Success: VA46, VA47, VA48, VA49, VA50, VA51, VA52, VA53, VA54, VA55, VA56

4.4 d SE: 37–42, 43–48, 49-54, 63-68, 69-74, 75-80, 81-86, 87-92, 93-98, 99-104, 107-112, 113-118, 119-124, 131-136, 145-150, 151-156, 157-162, 163–168, 171–176, 177-182, 183-188, 197-202, 203-208, 209-214, 215-220, 221-226, 227-232, 235-240,241-246, 247-252, 253-258, 259-264 TE: 37A-B, 43A-B, 49A-B, 63A-B, 69A-B, 75A-B, 81A-B, 87A-B, 93A-B, 99A-B, 107A-B, 113A-B, 119A-B, 131A-B, 145A-B, 151A-B, 157A-B, 163A-B, 171A-B. 177A-B.183A-B. 197A-B. 203A-B. 209A-B.

215A-B, 221A-B, 227A-B.235A-B. 241A-B. 247A-B, 253A-B, 259A-B, Virginia SOL Success: 19A. 19-24, VA21, VA23, VA24, VA27, VA28, VA29, VA30, VA31, VA32, VA33, VA34, VA35, VA36, VA38, VA39,

www.aaamath.com/index.html,

www.coolmath.com

http://illuminations.nctm.org/

www.mathisfun.com

http://nlvm.usu.edu/en/nav/vlibrarv.html.

www.themathpage.com/ARITH/arithmetic.htm www.teachnet.com/lesson/math/fractioncitv.html

http://mathforum.org/paths/fractions/e.fraclessons. html

https://classflow.com/

http://www.kidsnumbers.com/

http://www.sheppardsoftware.com/math.htm

http://www.doe.virginia.gov/instruction/mathematic s/resources/videos/index.shtml#

http://studyjams.scholastic.com/studyjams/jams/ma th/index.htm

https://www.khanacademy.org/math/cc-fourthgrade-math/cc-4th-mult-div-topic

http://gameguarium.com

http://www.doe.virginia.gov/testing/solsearch/sol/m

ath/4/mess 4-4a.pdf

http://www.doe.virginia.gov/testing/solsearch/sol/m

ath/4/mess 4-4c.pdf

http://www.doe.virginia.gov/testing/solsearch/sol/m ath/4/mess 4-2abc.pdf

See MIPs:

4.2c - Sharing: Division Statements and Fractions (Word) / PDF Version

4.4c - Pears in a Basket: Dividing with Whole

Numbers (Word) / PDF Version

4.4d - Shopping for Thanksgiving Lunch: Computation (Word) / PDF Version

VDOE Mathematics Vocabulary Cards - Grade 4 Performance Tasks

- Poems all in a Row: Fun with Division Literature
 - Hersey Kisses Multiplication & Division by Jerry Pallotta
 - The Doorbell Rang by Pat Hutchins
 - The Great Divide by Dayle Dodds

Fraction strips

fraction circle Unifix cubes

pattern blocks

aeoboards

Rainbow Fraction Tiles

Cuisenaire rods

two-sided counters

number lines

10 by 10 grids

vard/meter sticks

decimal squares

money

IXL

Prodigy

Common Assessments

	VA40, VA41, VA42, VA43, VA44, VA45, VA46, VA47, VA48, VA49, VA50, VA51, VA52, VA53, VA54, VA55, VA56, VA58 4.2 c - Virginia SOL Success: 61A, 61–66		
SOL 4.16	SE: 31-36 TE: 31A, 31–36 Virginia SOL Success: VA66	http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess_4-16.pdf www.coolmath4kids.com www.mathisfun.com www.aaamath.com/index.html http://gamequarium.com http://illuminations.nctm.org/ https://classflow.com/ http://www.doe.virginia.gov/instruction/mathematics/resources/videos/index.shtml# http://studyjams.scholastic.com/studyjams/jams/math/index.htm http://www.sheppardsoftware.com/math.htm	See MIPs: 4.16 - Reasoning About Equations Using Equality (Word) / PDF Version VDOE Mathematics Vocabulary Cards – Grade 4 Vocabulary Cards IXL Prodigy Common Assessments
SOL 4.15	SE: 311–316, 587-592, 703-708 TE: 311A–B, 587A–B, 703A–B Virginia SOL Success: 91A, 91–96, 157A, 157–162, VA65, VA90, VA122, VA123	www.aaamath.com/index.html www.scottforesman.com www.mathisfun.com http://illuminations.nctm.org/ https://classflow.com/ http://www.doe.virginia.gov/instruction/mathematic s/resources/videos/index.shtml# http://studyjams.scholastic.com/studyjams/jams/ma th/index.htm http://www.doe.virginia.gov/testing/solsearch/sol/m ath/4/mess_4-15.pdf https://www.internet4classrooms.com/grade_level_ help/create_pattern_rule_words_math_fourth_4th_g rade.htm http://gamequarium.com http://gamequarium.com http://www.sheppardsoftware.com/math.htm	See MIPs: 4.15 - How Does the Pattern Grow? (Word) / PDF Version VDOE Mathematics Vocabulary Cards - Grade 4 Performance Tasks • Marathon Training Literature: • Two of Everything by Lily Toy Hong Number lines Graph paper In/out tables Unifix cubes • http://mathjourneys.com/slideshows/picture_pat terns_grade4.pptx IXL Prodigy Topic 6 VA SOL Prep Topic 6 Performance Assessment Practice Workbook Common Assessments

Mapping for Instruction - Third Nine Weeks

SOLs

- 4.2 The student will
 - a) compare and order fractions and mixed numbers, with and without models;*
 - b) represent equivalent fractions;* and
 - c) identify the division statement that represents a fraction, with models and in context.
- 4.3 The student will
 - d) given a model, write the decimal and fraction equivalents.*
- 4.5 The student will
 - a) determine common multiples and factors, including least common multiple and greatest common factor;
 - b) add and subtract fractions and mixed numbers having like and unlike denominators;* and
 - c) solve single-step practical problems involving addition and subtraction with fractions and mixed numbers.
- 4.7 The student will solve practical problems that involve determining perimeter and area in U.S. Customary and metric units.
- 4.10 The student will
 - a) identify and describe points, lines, line segments, rays, and angles, including endpoints and vertices; and
 - b) identify and describe intersecting, parallel, and perpendicular lines.
- 4.11 The student will identify, describe, compare, and contrast plane and solid figures according to their characteristics (number of angles, vertices, edges, and the number and shape of faces) using concrete models and pictorial representations.
- 4.12 The student will classify quadrilaterals as parallelograms, rectangles, squares, rhombi, and/or trapezoids.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

SOL	Instructional Focus	Vocabulary	Comments	Time Frame
4.10 a, b	Points, lines, line segments, rays, angles, endpoints, vertices Parallel and perpendicular lines	Point, line segment, line, endpoints, infinitely, ray, angle, right angle, vertex, vertices, intersecting, parallel, perpendicular, plane, congruence, sides	Students must know the symbolic notation for these concepts.	4 days
4.12	Classify quadrilaterals Identify properties of quadrilaterals Compare and contrast quadrilaterals Identify parallel sides, congruent sides, and right angles using correct markings	Quadrilateral, polygon, parallelogram, parallel, congruent, rectangle, right angles, square, trapezoid, rhombus, perpendicular, hash marks	Classify quadrilaterals in Math 4. All other polygons are moved to Math 3.	3 days
4.11	Plane and solid figures: Identify concrete models of solid figures Identify and describe solid figures according to their characteristics Compare and contrast solid figures	Plane figures, solid figures, face, angle, endpoint, vertex, vertices, lines, line segments, edge, intersection, point, ray, cube, congruent, square, rectangular prism, rectangle, sphere, square pyramid, triangle, cone, cylinder, closed shape, 2-D, 3-D		4 days

4.7	Area and perimeter: - Rectangles, given two adjacent sides - Square, given one side Perimeter only: Polygons with ≤ 8 sides	Perimeter, path, distance, plane figure, side, polygon, area, square units, square, rectangle, units, adjacent sides		3 days
4.2 a, b, c 4.3 d	Fractions: Compare and order (limited to four fractions) Use benchmarks (0, ½, 1) to compare fractions Equivalent fractions Decimal fraction equivalents	Fraction, equivalent fraction, whole, set, proper fraction, improper fraction, mixed number, numerator, denominator, whole number, common denominator, least common multiple, region/area model, set model, measurement model, division, compare, order, <i>less than</i> , <i>equal to, greater than, not equal</i> , decimal Symbols: <, >, =, ≠	The 4.2 a, b objectives are assessed without the use of a calculator on the state assessment.	13 days
4.5 a, b, c	Multiples and factors Add and subtract fractions and mixed numbers (like and unlike denominators) Practical problems with fractions and mixed numbers (single-step)	Factor, divisor, common factor, greatest common factor, product, multiple, common multiples, simplify, least common multiple, denominator, common denominator, proper fraction, improper fraction, mixed number, addend, minuend, subtrahend, sum, difference, simplest form, estimate	Add and subtract fractions (proper or improper) and/or mixed numbers, having like and unlike denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions. Word problems are limited to single-step practical problems. Note: SOL 4.5c allows the use of the calculator.	13 days
			Remediation, Review, Assessment	5 days
			Total	45 days

		Resources – Third Nine Weeks	
Common Assessment	Textbook	Links	Supplemental Materials
SOL 4.10 a, b	4.10 a - SE: 549–554 TE: 549A–549B Virginia SOL Success: 151A, 151–156, VA115, VA119 4.10 b - SE: 561–566 TE: 561A–561B Virginia SOL Success:151A, 151–156	https://www.brainpop.com/math/geomet ryandmeasurement/parallelandperpendic ularlines/ (Parallel and Perpendicular) https://jr.brainpop.com/math/geometry/pointslinessegmentsrays/ (Lines, line segments, points, and rays) http://www.math.com/school/subject3/lessons/S3U1L3GL.html (Types of Lines) https://illuminations.nctm.org http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/index.shtml https://sites.google.com/site/rcpskahoots//https://sites.google.com/a/solteacher.com/olteacher-com/home/4th-grade/4th-grade-math	See MIPs: 4.10ab - Geometry in Real-world Situations (Word) / PDF Version VDOE Mathematics Vocabulary Cards – Grade 4 Performance Tasks
SOL 4.12	SE: 567–572 TE: 567A–567B Virginia SOL Success: VA118	https://www.brainpop.com/math/geometryandmeasurement/polygons/ (polygons to introduce types of quadrilaterals) https://www.youtube.com/watch?v=WMkY_ulku9O (Quadrilaterals Song for Kids) https://illuminations.nctm.org http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/index.shtml https://sites.google.com/site/rcpskahoots/ https://sites.google.com/a/solteacher.com/olteachercom/home/4th-grade/4th-grade-math	See MIPs: 4.12 - Classifying Quadrilaterals (Word) / PDF Version VDOE Mathematics Vocabulary Cards – Grade 4 Performance Tasks

SOL 4.11	SE: 567-572 TE: 567A-B Virginia SOL Success: 151A, 151-156, VA118, VA119	https://www.youtube.com/watch?v=uZ8Jy1xgqPU (3D Shapes Song For Kids: Cubes, Spheres, Cylinders, and Pyramids) https://illuminations.nctm.org http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/index.shtml https://sites.google.com/a/solteacher.com/olteachercom/home/4th-grade/4th-grade-math	See MIPs: 4.11 - Characteristics of Solids (Word) / PDF Version VDOE Mathematics Vocabulary Cards – Grade 4 Literature • Cubes, Cones, Cylinders and Spheres by Tana Hoban Geometric Solids Newspaper to create geometric solids IXL Prodigy Common Assessments
SOL 4.7	SE: 175-180 TE: 175A-B Virginia SOL Success: VA142, VA143, VA144, VA145, VA146, VA147	https://www.brainpop.com/math/geometryandmeasurement/areaofpolygons/ (Area of Polygons) https://www.flocabulary.com/unit/area-and-perimeter/ (Area and Perimeter) https://jr.brainpop.com/math/measurement/perimeter/ (Perimeter) https://www.funbrain.com/games/shape-surveyor (Game measuring Perimeter and Area) https://illuminations.nctm.org http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/index.shtml https://sites.google.com/site/rcpskahoots/ https://sites.google.com/a/solteacher.com/olteacher-com/home/4th-grade/4th-grade-math	See MIPs: 4.7 - Will the Tablecloth Fit? (Word) / PDF Version VDOE Mathematics Vocabulary Cards – Grade 4 Performance Tasks
SOL 4.2 a, b, c SOL 4.3 d	4.2 a - SE: 359–364, 365–370, 371–376 TE: 359A–B, 365A–B, 371A–B, Virginia SOL Success: 73A, 73–78, VA77, VA78, VA79, VA80 4.2 b – SE: 327–332, 333–338, 339–344, 345– 350, 351–356 TE: 327A–B, 333A–B, 339A–B, 345A–B, 351A–B	https://www.brainpop.com/math/numbersandoperations/fractions/ (Fractions) https://www.brainpop.com/math/numbersandoperations/convertingfractionstodecimals/ (Converting Fractions to Decimals) https://www.brainpop.com/math/numbersandoperations/mixednumbers/ (Mixed Numbers) https://www.mathplayground.com/ASB_TugTeamFractions.html (Comparing Fractions Game)	See MIPs: 4.2ab - Fraction Strips: Comparing and Ordering Fractions (Word) / PDF Version 4.2a - Circle Fractions: Comparing and Ordering with Benchmarks (Word) / PDF Version 4.2a - Comparing Fractions: Developing Strategies (Word) / PDF Version 4.2c - Sharing: Division Statements and Fractions (Word) / PDF Version 4.3d - Fraction-Decimal Relationships with Grids (Word) / PDF Version

	Virginia SOL Success: 67A, 67–72, VA71, VA72, VA73, VA74, VA75, VA76 4.2 c – Virginia SOL Success: 61A, 61–66 4.3 d - SE: 495–500, 501–506, 507–512, 513–518 TE: 495A–B, 501A–B, 507A–B, 513A–B Virginia SOL Success: VA102, VA103, VA105, VA106	https://www.mathplayground.com/Triplets/index.html (Equivalent Fraction Game) https://www.mathplayground.com/number_climb.htm [(Ordering Fraction Game) https://www.mathplayground.com/puzzle_pics_fractions.html (Representing Fractions) https://www.sheppardsoftware.com/mathgames/menus/fractions.htm (Multiple Fraction Concepts) http://illuminations.nctm.org/Activity.aspx?id=4148 (Equivalent Fractions) https://illuminations.nctm.org http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/index.shtml https://sites.google.com/site/rcpskahoots/ https://sites.google.com/a/solteacher.com/olteacher-com/home/4th-grade/4th-grade-math	VDOE Mathematics Vocabulary Cards – Grade 4 Performance Tasks
SOL 4.5 a, b, c	4.5 a -SE: 279-284, 285-290, 291-296, 299-304 TE: 279A-B, 285A-B, 291A-B, 299A-B Virginia SOL Success: 25A, 25-30, VA59, VA60, VA61, VA62, VA63 4.5 b - SE: 385-390, 391-396, 397-402, 403-408, 409-414, 423-428, 435-440 TE: 385A-B, 391A-B, 397A-B, 403A-B, 409A-B, 423A-B, 435A-B Virginia SOL Success: 97A, 97-102, 103A, 103-108, 109A, 109-114, VA83, VA84, VA85, VA86, VA87, VA89, VA92, VA94, VA95, VA96	https://www.brainpop.com/math/numbersandoperations/addingandsubtractingfractions/ (Add and Subtract Fractions) https://www.brainpop.com/math/numbersandoperations/reducingfractions/ (Simplifying Fractions) https://www.quia.com/cb/62195.html (Adding and Subtracting Fractions Practice) https://illuminations.nctm.org http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/index.shtml https://sites.google.com/site/rcpskahoots/ https://sites.google.com/a/solteacher.com/olteacher-com/home/4th-grade/4th-grade-math	See MIPs: 4.5a - Factor Frenzy: Common Factors and Greatest Common Factor (Word) / PDF Version 4.5a - Multiple Madness: Common Multiples and Least Common Multiple (Word) / PDF Version 4.5a - Number Ray Investigators: Factors and Multiples (Word) / PDF Version 4.5b - Four in a Row: Fraction Addition and Subtraction (Word) / PDF Version 4.5b - Fraction Strips: What is the Meaning of Addition? (Word) / PDF Version 4.5b - Fraction Riddles: Adding and Subtracting Fractions (Word) / PDF Version 4.5b - Fraction Strips to Number Sentences: Adding Fractions (Word) / PDF Version 4.5b - Fraction Strips: Subtracting Fractions (Word) / PDF Version 4.5b - Fraction Strips: Subtracting Fractions (Word) / PDF Version

4.5 c - SE: 385–390, 391–396, 397–402, 403– 408, 409–414, 423–428, 435–440 TE: 385A–B, 391A–B, 397A–B, 403A–B, 409A–B, 423A– B, 435A–B Virginia SOL Success: 97A, 97–102, 103A, 103– 108, 109A, 109–114, VA83, VA84, VA85, VA86,	4.5b - Which is Closer? Estimating and Finding the Sum of Fractions (Word) / PDF Version 4.5c - Adding and Subtracting Fractions: Understanding the Context (Word) / PDF Version VDOE Mathematics Vocabulary Cards – Grade 4 Performance Tasks • The Name Game Literature • Lines, Segments, Rays, and Angles (My Path to Math) by Claire Piddock • Straight Lines, Parallel Lines,
	 <u>Lines, Segments, Rays, and Angles (My</u>

Mapping for Instruction - Fourth Nine Weeks

SOLs

4.8 The student will

- a) estimate and measure length and describe the result in U.S. Customary and metric units;
- b) estimate and measure weight/mass and describe the result in U.S. Customary and metric units;
- c) given the equivalent measure of one unit, identify equivalent measures of length, weight/mass, and liquid volume between units within the U.S. Customary system; and
- d) solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units.

4.13 The student will

- a) determine the likelihood of an outcome of a simple event;
- b) represent probability as a number between 0 and 1, inclusive; and
- c) create a model or practical problem to represent a given probability.

4.14 The student will

- a) collect, organize, and represent data in bar graphs and line graphs;
- b) interpret data represented in bar graphs and line graphs; and
- c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph, a chart and a line graph, or a pictograph and a bar graph).

SOL	Instructional Focus	Vocabulary	Comments	Time Frame
4.8 a, b, c, d	Measurement: Estimate and measure length and weight/mass in U.S Customary and metric units Identify equivalent measures of length, weight/mass, and liquid volume between units within the U.S. Customary system Compare estimates with actual measures Solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units	Length, ruler, yard stick, meter stick, tape measure, inch, foot, yard, mile, meter, centimeter, millimeter, kilometer, weight, mass, balance scale, scale, ounce, pound, gram, kilogram, liquid volume, cup, pint, quart, gallon		10 days
4.13 a, b, c	Probability: Determine the likelihood of an outcome of a simple event Represent probability as a number between 0 and 1 Create a model or practical problem to represent a given probability	Probability, outcome, certain, likely, equally likely, unlikely, impossible	Emphasis: Create a model or practical problem.	5 days
4.14 a, b, c	Graphing: Create and interpret bar and line graphs Compare different representations of the same data	Bar graph, line graph, data, survey, title, axis, trend, increase, decrease		5 days
			SOL Review and Testing	15 days
			Hands-On Equations Level 1	5 days
			Remediation, Review, Assessment	5 days
			Total	45 days

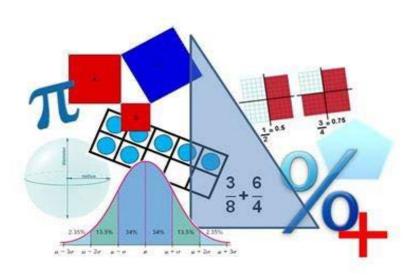
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	Resources – Fourth Nine Weeks			
Common Assessment	Textbook	Links	Supplemental Materials	
SOL 4.8 a, b, c, d	4.8 a – SE: 641–646 TE: 641A–641B Virginia SOL Success: 163A, 163–168, VA129, VA130 4.8 b – Virginia SOL Success: 169A, 169–174, VA132 4.8 c – SE: 647–652, 653–658, 659–664 TE: 647A–B, 653A–B, 659A–B Virginia SOL Success: VA131, VA133, VA134, VA141 4.8 d - SE: 647-652, 653-658, 659-664, 697-702 TE: 647A–B, 653A–B, 659A–B, 697A–B Virginia SOL Success: 163A, 163–168, 169A, 169–174, VA130, VA131, VA132, VA133, VA134, VA140	http://www.doe.virginia.gov/instruction/mathematic s/resources/vocab_cards/math_vocab_cards_4.pdf (Vocabulary Cards) http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess_4-7ab.pdf (Stick-Figure Measurements) http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess_4-6ab.pdf (This Fruit is a Mass!) http://studyjams.scholastic.com/studyjams/jams/math/measurement/units-of-measurement.htm http://studyjams.scholastic.com/studyjams/jams/math/measurement/tools-measurement.htm http://studyjams.scholastic.com/studyjams/jams/math/measurement/units-of-length.htm http://studyjams.scholastic.com/studyjams/jams/math/measurement/measure-length.htm	See MIPs: 4.8ac - Body Part Measurements (Word) / PDF Version 4.8bc - Fruit Basket Measurement (Word) / PDF Version 4.8b - Measuring Weight/Mass (Word) / PDF Version 4.8d - Liquid Volume: It's My Party (Word) / VDOE Mathematics Vocabulary Cards – Grade 4 Performance Task Camping out with Capacity Literature How Big Is It? By Ben Hillman Twelve Snails to One Lizard by Susan Hightower Me and the Measure of Things by Joan Sweeney Mow Long or How Wide – A Measuring Guide by Brian P. Cleary On the Scale a Weighty Tale by Brian P. Cleary Rulers Yard sticks and meter sticks Tape measures Scales IXL Prodigy Common Assessments	
SOL 4.13 a, b, c	4.13 a – SE: 79-84 TE: 79A–79B Virginia SOL Success: VA81 4.13 b – SE: 85-90 TE: 85A–85B Virginia SOL Success: VA82 4.13 c - SE: 85-90 TE: 85A–85B Virginia SOL Success: VA82	http://www.doe.virginia.gov/instruction/mathematics/resources/vocab_cards/math_vocab_cards_4.pdf (Vocabulary Cards) http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess_4-13ab_1.pdf (How Certain Are You?) http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess_4-13ab_2.pdf (Lucky Sums) http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess_4-13ab_3.pdf (Spinning Color)	See MIPs: 4.13ab - Probability: How Certain Are You? (Word) / PDF Version 4.13ab - Probability: Spinning Color (Word) / PDF Version 4.13abc - Probability: Sweet as Candy (Word) / PDF Version	

		http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess 4-13ab 4.pdf (Looking for a Pet) http://studyjams.scholastic.com/studyjams/jams/math/probability/find-probability.htm http://studyjams.scholastic.com/studyjams/jams/m	VDOE Mathematics Vocabulary Cards – Grade 4 Performance Task
		ath/probability/probability-fraction.htm http://studyjams.scholastic.com/studyjams/jams/m ath/probability/pidentify-outcomes.htm	Playing cards IXL Prodigy Common Assessments
SOL 4.14 a, b, c	4.14 a – Virginia SOL Success: 37A–37B, 37–42, 43A–43B, 43–48, 49A– 49B, 49–54, 55A–55B, 55–60, VA67, VA68, VA69, VA70 4.14 b- Virginia SOL Success: 37A–37B, 37–42, 43A–43B, 43–48, 49A– 49B, 49–54, 55A–55B, 55–60, VA67, VA68, VA69, VA70 4.14 c- Virginia SOL Success: 43–48, 55A, 55–60, VA68, VA70	http://www.doe.virginia.gov/instruction/mathematics/resources/vocab_cards/math_vocab_cards_4.pdf (Vocabulary Cards) http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess_4-14_2.pdf (Hot or Cold?) http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess_4-14_1.pdf (Sandwich Data) http://studyjams.scholastic.com/studyjams/jams/math/data-analysis/bar-graphs.htm	See MIPs: 4.14abc - Analyzing Temperature Data (Word) / PDF Version 4.14abc - Statistics: Sandwich Data (Word) / PDF Version VDOE Mathematics Vocabulary Cards – Grade 4 Performance Task • Soda Survey Literature • Guinness World Records: Wild Lives by Dina Anastasio and Ryan Herndon • Tiger Math by Ann Whitehead Nagda and Cindy Bickell • Lemonade for Sale by Stuart J. Murphy Graph paper IXL Prodigy Common Assessments
SOL Review and Testing		http://www.doe.virginia.gov/testing/sol/standards docs/mathematics/index.shtml	VDOE Website – Math SOL page
Hands-On Equations			Hands-On Equations Level 1 teacher booklet and reproducible classwork sheets Teacher demonstration kit Student kit (one per student)

Mathematics 2016 Standards of Learning

Grade 4 Curriculum Framework



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Virginia 2016 Mathematics Standards of Learning Curriculum Framework Introduction

The 2016 Mathematics Standards of Learning Curriculum Framework, a companion document to the 2016 Mathematics Standards of Learning, amplifies the Mathematics Standards of Learning and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 Mathematics Standards of Learning Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills

This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "...the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient, and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand, and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

"Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."

- National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

investigate and solve a variety of problem types.

Mathematics instruction in grades three through five should continue to foster the development of number sense, with greater emphasis on decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction in grades three through five allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names), and making connections among mathematics concepts as well as to other content areas. Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to

4.1 The student will

- a) read, write, and identify the place and value of each digit in a nine-digit whole number;
- b) compare and order whole numbers expressed through millions; and
- c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and
 Place value refers to the value of each digit and depends upon the position of the digit in the number. For example, in the number 7,864,352, the 8 is in the hundred thousand place, and the value of the 8 is eight hundred thousand or 800,000. Whole numbers may be written in a variety of forms: Standard: 1,234,567 Written: one million, two hundred thirty-four thousand, five hundred sixty-seven Expanded: (1,000,000 + 200,000 + 30,000 + 4,000 + 500 + 60 + 7) Numbers are arranged into groups of three places called <i>periods</i> (ones, thousands, millions). The value of the places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the value of the place and period of a number helps students determine values of digits in any number as well as read and write numbers. Students at this level will work with numbers through the millions period (nine-digit numbers). 	 Read nine-digit whole numbers, presented in standard form and represent the same number in written form. (a) Write nine-digit whole numbers in standard form when the numbers are presented orally or in written form. (a) Identify and communicate, orally and in written form, the place and value for each digit in a nine-digit whole number. (a) Compare two whole numbers expressed through millions, using the words greater than, less than, equal to, and not equal to or using the symbols >, <, =, or ≠. (b) Order up to four whole numbers expressed through millions. (b) Round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand place. (c) Identify the range of numbers that round to a given thousand, ten thousand, and hundred thousand. (c)

4.2 The student will

- a) compare and order fractions and mixed numbers, with and without models;*
- b) represent equivalent fractions;* and
- c) identify the division statement that represents a fraction, with models and in context.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard Essential Knowledge and Skills The student will use problem solving, mathematical A fraction is a way of representing part of a whole region (i.e., an area model), part of a group (i.e., a set model), or part of a length (i.e., a measurement model). communication, mathematical reasoning, connections, and representations to In the area and length/measurement fraction models, the parts must be equivalent. Compare and order no more than four fractions having like In a set model, each member of the set is an equivalent part of the set. In set models, the whole and unlike denominators of 12 or less, using concrete and needs to be defined, but members of the set may have different sizes and shapes. For instance, if pictorial models. (a) a whole is defined as a set of 10 animals, the animals within the set may be different. For • Use benchmarks (e.g., 0, $\frac{1}{2}$ or 1) to compare and order no example, students should be able to identify monkeys as representing $\frac{1}{2}$ of the animals in the more than four fractions having unlike denominators of 12 or following set. less. (a) Compare and order no more than four fractions with like denominators of 12 or less by comparing number of parts (numerators) (e.g., $\frac{1}{5} < \frac{3}{5}$). (a) Proper fractions, improper fractions, and mixed numbers are terms often used to describe Compare and order no more than four fractions with like fractions. A proper fraction is a fraction whose numerator is less than the denominator. An numerators and unlike denominators of 12 or less by improper fraction is a fraction whose numerator is equal to or greater than the denominator. An comparing the size of the parts (e.g., $\frac{3}{9} < \frac{3}{5}$). (a) improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{9}$). Compare and order no more than four fractions (proper or improper), and/or mixed numbers, having denominators of 12 The value of a fraction is dependent on both the number of equivalent parts in a whole or less. (a) (denominator) and the number of those parts being considered (numerator). Use the symbols >, <, =, and ≠ to compare fractions (proper or The more parts the whole is divided into, the smaller the parts (e.g., $\frac{1}{5} < \frac{1}{3}$). improper) and/or mixed numbers having denominators of 12 or less. (a) When fractions have the same denominator, they are said to have "common denominators" or Represent equivalent fractions through twelfths, using "like denominators." Comparing fractions with like denominators involves comparing only the region/area models, set models, and measurement/length numerators. models. (b)

- a) compare and order fractions and mixed numbers, with and without models;*
- b) represent equivalent fractions;* and
- c) identify the division statement that represents a fraction, with models and in context.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

On the state assessment, items measuring this objective are assessed without the use of a calculator.			
Understanding the Standard	Essential Knowledge and Skills		
 Strategies for comparing fractions having unlike denominators may include: comparing fractions to familiar benchmarks (e.g., 0, 1/2, 1); determining equivalent fractions, using models such as fraction strips, number lines, fraction circles, rods, pattern blocks, cubes, base-ten blocks, tangrams, graph paper, or patterns in multiplication chart; and determining a common denominator by determining the least common multiple (LCM) of both denominators and then rewriting each fraction as an equivalent fraction, using the LC as the denominator. 	a will receive when sharing 3 muffins equally). (c)		
 A variety of fraction models should be used to expand students' understanding of fractions and mixed numbers: 	l e e e e e e e e e e e e e e e e e e e		
 Region/area models: a surface or area is subdivided into smaller equal parts, and each part compared with the whole (e.g., fraction circles, pattern blocks, geoboards, grid paper, colo tiles). Set models: the whole is understood to be a set of objects, and subsets of the whole make fractional parts (e.g., counters, chips). Measurement models: similar to area models but lengths instead of areas are compared (e fraction strips, rods, cubes, number lines, rulers). 	up		
• Equivalent fractions name the same amount. Students should use a variety of representations a models to identify different names for equivalent fractions.	and		
• When presented with a fraction $\frac{3}{5}$ representing division, the division expression representing the fraction is written as $3 \div 5$.	ie		
• The fraction $\frac{3}{4}$ may be interpreted as the amount of cake each person will receive when 3 cake are divided equally among 4 people.	S		

- a) read, write, represent, and identify decimals expressed through thousandths;
- b) round decimals to the nearest whole number;
- c) compare and order decimals; and
- d) given a model, write the decimal and fraction equivalents.*

^{*}On the state assessment, items measuring this objective are assessed without the use of a calculator.

	Understanding the Standard	Essential Knowledge and Skills		
•	Decimal numbers expand the set of whole numbers and, like fractions, are a way of representing part of a whole. The structure of the base-ten number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This is known as a ten-to-one place value	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to Read and write decimals expressed through thousandths, using		
	relationship (e.g., in 2.35, 3 is in the tenths place since it takes ten one-tenths to make one whole). Use base-ten proportional manipulatives, such as place value mats/charts, decimal squares, base-ten blocks, meter sticks, as well as the ten-to-one non-proportional model, money, to investigate this relationship.	 Represent and identify decimals expressed through thousandths, using base-ten manipulatives, drawings, and numerical symbols. (a) Represent and identify decimals expressed through thousandths, using base-ten manipulatives, pictorial representations, and numerical symbols (e.g., relate the 		
•	A decimal point separates the whole number places from the places that are less than one. A number containing a decimal point is called a <i>decimal number</i> or simply a <i>decimal</i> . To read decimals,	 appropriate drawing to 0.05). (a) Investigate the ten-to-one place value relationship for decimals through thousandths, using base-ten manipulatives (e.g., place 		
	 read the whole number to the left of the decimal point; read the decimal point as "and"; read the digits to the right of the decimal point just as you would read a whole number; and say the name of the place value of the digit in the smallest place. 	 value mats/charts, decimal squares, and base-ten blocks). (a) Identify and communicate, both orally and in written form, the position and value of a decimal through thousandths (e.g., given 0.385, the 8 is in the hundredths place and has a value of 0.08. 		
•	Any decimal less than 1 will include a leading zero. For example 0.125 which can be read as "zero and one hundred twenty-five thousandths" or as "one hundred twenty-five thousandths."	 Round decimals expressed through thousandths to the nearest whole number. (b) 		
•	 Decimals may be written in a variety of forms: Standard: 26.537 Written: twenty-six and five hundred thirty-seven thousandths Expanded: 20 + 6 + 0.5 + 0.03 + 0.007. Strategies for rounding whole numbers can be applied to rounding decimals. 	 Compare two decimals expressed through thousandths, using symbols (>, <, =, and ≠) and/or words (greater than, less than, equal to, and not equal to). (c) Order a set of up to four decimals, expressed through thousandths, from least to greatest or greatest to least. (c) 		

- a) read, write, represent, and identify decimals expressed through thousandths;
- b) round decimals to the nearest whole number;
- c) compare and order decimals; and
- d) given a model, write the decimal and fraction equivalents.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

	Understanding the Standard	Essential Knowledge and Skills
decimals. We number line closer. Base-ten modelines, decimals and	es are useful tools when developing a conceptual understanding of rounding with When given a decimal to round to the nearest whole or ones place, locate it on the Property Next, determine the two whole numbers it is between. Then, identify to which it is podels concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number all squares, decimal circles, money). In the decimal circles of the same relationships; however, they are presented in two the same relationships; however, they are presented in two the same relationships; however, they are another way of writing the same relationships.	 Represent fractions for halves, fourths, fifths, and tenths as decimals through hundredths, using concrete objects. (d) Relate fractions to decimals, using concrete objects (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles, money). (d) Write the decimal and fraction equivalent for a given model (e.g., ¹/₄ = 0.25 or 0.25 = ¹/₄; 1.25 = ⁵/₄ or 1¹/₄). (d)

Computation and estimation in grades three through five should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to work with decimals. Instruction should focus on computation activities that enable students to model, explain, and develop proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades. Multiplication and division with decimals will be explored in grade five.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability to identify and use relationships among operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use properties of operations to solve problems (e.g., 7×28 is equivalent to $(7 \times 20) + (7 \times 8)$).

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., $\frac{2}{5} + \frac{1}{3}$ must be less than 1 because both fractions are less than $\frac{1}{2}$). Using estimation, students should develop strategies to recognize the reasonableness of their solutions.

Additionally, students should enhance their ability to select an appropriate problem-solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

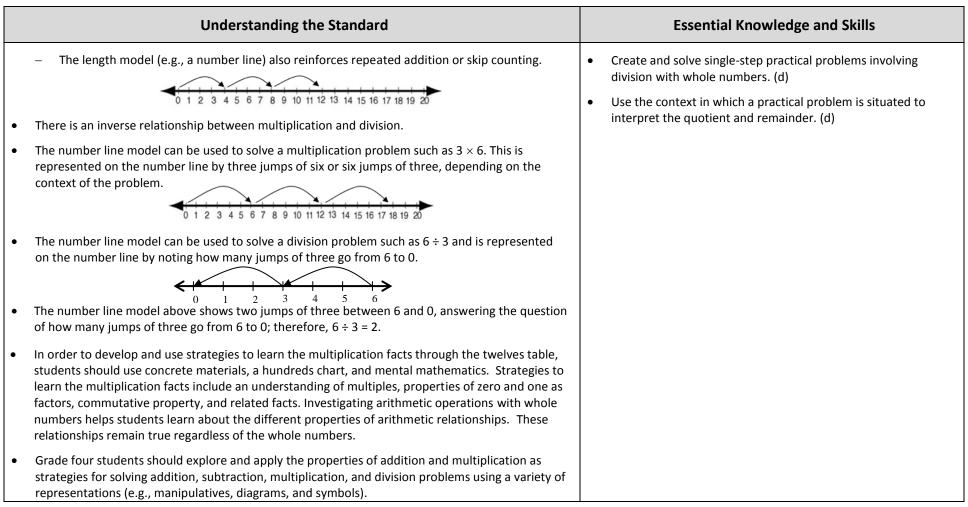
- a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
- b) estimate and determine sums, differences, and products of whole numbers;*
- c) estimate and determine quotients of whole numbers, with and without remainders;* and
- d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

Understanding the Standard Essential Knowledge and Skills Computational fluency is the ability to think flexibly in order to choose appropriate strategies to The student will use problem solving, mathematical communication, mathematical reasoning, connections, and solve problems accurately and efficiently. representations to The development of computational fluency relies on quick access to number facts. There are patterns and relationships that exist in the facts. These relationships can be used to learn and Demonstrate fluency with multiplication through 12×12 , and the corresponding division facts. (a) retain the facts. Estimate whole number sums, differences, products, and A certain amount of practice is necessary to develop fluency with computational strategies; however, the practice must be motivating and systematic if students are to develop fluency in quotients, with and without context. (b, c) computation, whether mental, with manipulative materials, or with paper and pencil. Apply strategies, including place value and the properties of In grade three, students developed an understanding of the meanings of multiplication and addition to determine the sum or difference of two whole division of whole numbers through activities and practical problems involving equal-sized groups, numbers, each 999,999 or less. (b) arrays, and length models. In addition, grade three students have worked on fluency of facts for Apply strategies, including place value and the properties of 0, 1, 2, 5, and 10. multiplication and/or addition, to determine the product of two whole numbers when both factors have two digits or Three models used to develop an understanding of multiplication include: fewer. (b) The equal-sets or equal-groups model lends itself to sorting a variety of concrete objects into equal groups and reinforces the concept of multiplication as a way to find the total number of Apply strategies, including place value and the properties of items in a collection of groups, with the same amount in each group, and the total number of multiplication and/or addition, to determine the quotient of items can be found by repeated addition or skip counting. two whole numbers, given a one-digit divisor and a two- or three-digit dividend, with and without remainders. (c) Refine estimates by adjusting the final amount, using terms The array model, consisting of rows and columns (e.g., three rows of four columns for a 3-bysuch as closer to, between, and a little more than. (b, c) 4 array), helps build an understanding of the commutative property. Create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication with whole numbers. (d)

^{*}On the state assessment, items measuring this objective are assessed without the use of a calculator.

- a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
- b) estimate and determine sums, differences, and products of whole numbers;*
- c) estimate and determine quotients of whole numbers, with and without remainders;* and
- d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.



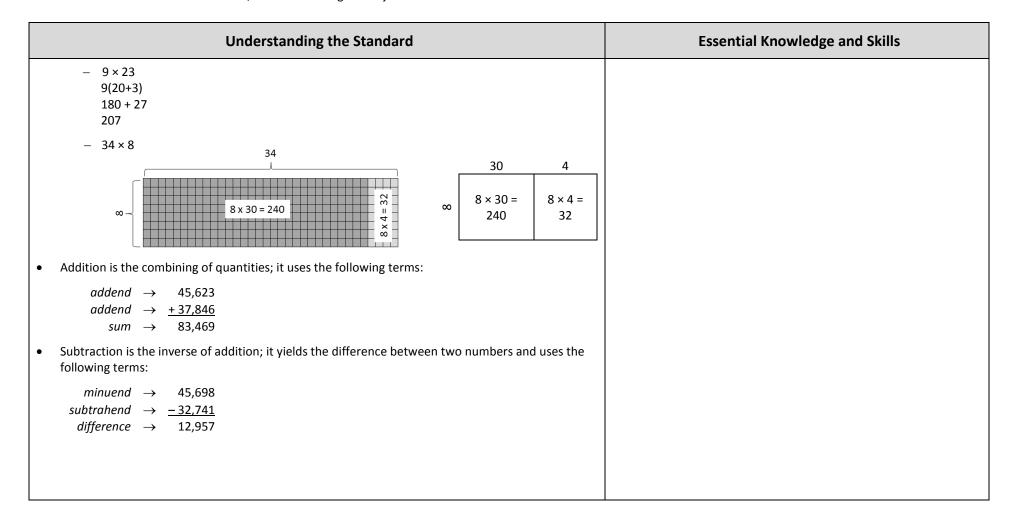
- a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
- b) estimate and determine sums, differences, and products of whole numbers;*
- c) estimate and determine quotients of whole numbers, with and without remainders;* and
- d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

^{*}On the state assessment, items measuring this objective are assessed without the use of a calculator.

	Understanding the Standard	Essential Knowledge and Skills
•	The properties of the operations are "rules" about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties are most appropriate for exploration at this level:	
	 The identity property of addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number. 	
	The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., $24 + 136 = 136 + 24$). Similarly, the commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., $12 \times 43 = 43 \times 12$).	
	The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., $15 + (35 + 16) = (15 + 35) + 16$). The associative property of multiplication states that the product stays the same when the grouping of factors is changed [e.g., $16 \times (40 \times 5) = (16 \times 40) \times 5$].	
	 The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products. Several examples are shown below: 	
	$-3(9) = 3(5+4)$ $3(5+4) = (3 \times 5) + (3 \times 4)$	
	$- 5 \times (3+7) = (5 \times 3) + (5 \times 7)$	
	$- (2 \times 3) + (2 \times 5) = 2 \times (3 + 5)$	

- a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
- b) estimate and determine sums, differences, and products of whole numbers;*
- c) estimate and determine quotients of whole numbers, with and without remainders;* and
- d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

^{*}On the state assessment, items measuring this objective are assessed without the use of a calculator.



- a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
- b) estimate and determine sums, differences, and products of whole numbers;*
- c) estimate and determine quotients of whole numbers, with and without remainders;* and
- d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

^{*}On the state assessment, items measuring this objective are assessed without the use of a calculator.

	Understanding the Standard	Essential Knowledge and Skills
•	The terms associated with multiplication are listed below:	
	$\begin{array}{ll} \textit{factor} & \rightarrow & 76 \\ \textit{factor} & \rightarrow & \times 23 \\ \textit{product} & \rightarrow & 1,748 \end{array}$	
•	In multiplication, one factor represents the number of equal groups and the other factor represents the number in or size of each group. The product is the total number in all of the groups.	
•	Multiplication can also refer to a multiplicative comparison, such as: "Gwen has six times as many stickers as Phillip". Both situations should be modeled with manipulatives.	
•	Models of multiplication may include repeated addition and collections of like sets, partial products, and area or array models.	
•	Division is the operation of making equal groups or shares. When the original amount and the number of shares are known, divide to determine the size of each share. When the original amount and the size of each share are known, divide to determine the number of shares. Both situations may be modeled with base-ten manipulatives.	
•	Division is the inverse of multiplication. Terms used in division are dividend, divisor, and quotient. $\frac{quotient}{dividend} \pm \frac{dividend}{divisor} = quotient$ $\frac{dividend}{divisor} = quotient$	
•	Students benefit from experiences with various methods of division, such as repeated subtraction and partial quotients.	

- a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
- b) estimate and determine sums, differences, and products of whole numbers;*
- c) estimate and determine quotients of whole numbers, with and without remainders;* and
- d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

^{*}On the state assessment, items measuring this objective are assessed without the use of a calculator.

	Understanding the Standard	Essential Knowledge and Skills
•	Estimation can be used to determine the approximation for and then to verify the reasonableness of sums, differences, products, and quotients of whole numbers. An estimate is a number that lies within a range of the exact solution, and the estimation strategy used in a particular problem determines how close the number is to the exact solution. An estimate tells about how much or about how many.	
•	Strategies such as rounding up or down, front-end, and compatible numbers may be used to estimate sums, differences, products, and quotients of whole numbers.	
•	The least number of steps necessary to solve a single-step problem is one.	
•	The problem-solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings.	
•	In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as <i>in all, altogether, difference,</i> etc., encourages students to perform a particular operation rather than make sense of the context of the problem. A key-word focus prepares students to solve a limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses.	
•	Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types.	
•	Students should experience a variety of problem types related to multiplication and division. Some examples are included in the following chart:	

- a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
- b) estimate and determine sums, differences, and products of whole numbers;*
- c) estimate and determine quotients of whole numbers, with and without remainders;* and
- d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

^{*}On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard				Essential Knowledge and Skills
GRADE 4: COMI	MON MULTIPLICA	ATION AND DIVI	SION PROBLEM TYPES	
		roup Problems		
Whole Unknown	Size of Grou	ps Unknown	Number of Groups Unknown	
(Multiplication)	(Partitive	Division)	(Measurement Division)	
There are six boxes of crayons.	If 144 crayons a	re shared	If 144 crayons are placed into school	
Each box contains 24 crayons.	equally among s	ix friends, how	boxes with each box containing 24	
How many crayons are there	many crayons w	ill each friend	crayons, how many school boxes can	
in all?	get?		be filled?	
	Multiplicative (Comparison Prob	lems	
Result Unknown	Start Ur	nknown	Comparison Factor Unknown	
Tyrone ran 30 miles last	Jasmine ran 120	miles. She ran	Jasmine ran 120 miles. Tyrone ran	
month. Jasmine ran four	four times as ma	any miles as	30 miles. How many times more	
times as many miles as Tyrone	Tyrone. How m	any miles did	miles did Jasmine run than Tyrone?	
during the same month. How	Tyrone run?			
many miles did Jasmine run?				
Array or Area Probler		Area Problems		
Whole Unknown	ı	(One Dimension Unknown	
There are 12 baseball teams competing in the		There are 108 k	paseball players competing in the	
tournament. Each team has 9 baseball players.		tournament. T	he players are divided equally among	
How many baseball players are there all		12 teams. How	many players are on each team?	
together?		Th 400 l	b - II - l t t t	
Mr. Myers's dog pen measures 15 feet by 22 feet. How many square feet are in the dog pen?			baseball players competing in the	
			here are exactly 9 players on each	
		team. How ma	ny teams are there?	
		The dog pen co	overs 60 square feet. The length of the	
		dog non is 15 fe	eet. What is the width of the dog pen?	

- a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
- b) estimate and determine sums, differences, and products of whole numbers;*
- c) estimate and determine quotients of whole numbers, with and without remainders;* and
- d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

^{*}On the state assessment, items measuring this objective are assessed without the use of a calculator.

Unde	Essential Knowledge and Skills	
Students need exposure to various types of practical problems in which they must interpret the quotient and remainder based on the context. The chart below includes one example of each type of problem.		
MAKING SEN	SE OF THE REMAINDER IN DIVISION	
TYPE OF PROBLEM	EXAMPLE	
Remainder is not needed and can be left over (or discarded).	Bill has 29 pencils to share fairly with 6 friends. How many pencils will each friend receive? 4 pencils with 5 pencils left over	
Remainder is partitioned and represented as a fraction or decimal.	Six friends will share 29 ounces of juice. How many ounces will each person get if all of the juice is shared equally? $4\frac{5}{6}$ ounces	
Remainder forces the answer to be ncreased to the next whole number.	There are 29 people going to the party by car. How many cars will be needed if each car holds 6 people? 5 cars	
Remainder forces the answer to be counded (giving an approximate answer).	Six children will share a bag of candy containing 29 pieces. About how many pieces of candy will each child get? about 5 pieces of candy	

- a) determine common multiples and factors, including least common multiple and greatest common factor;
- b) add and subtract fractions and mixed numbers having like and unlike denominators;* and
- c) solve single-step practical problems involving addition and subtraction with fractions and mixed numbers.

^{*}On the state assessment, items measuring this objective are assessed without the use of a calculator.

	Understanding the Standard	Essential Knowledge and Skills
•	A factor of a whole number is a whole number that divides evenly into that number with no remainder. A factor of a number is a divisor of the number.	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and
•	A common factor of two or more numbers is a divisor that all of the numbers share.	 Petermine common multiples and common factors of
•	The greatest common factor of two or more numbers is the largest of the common factors that all of the numbers share.	numbers. (a)
•	The product of the number and any natural number is a multiple of the number.	 Determine the least common multiple and greatest common factor of no more than three numbers. (a)
•	Common multiples and common factors can be useful when simplifying fractions.	Determine a common denominator for fractions, using
•	The least common multiple of two or more numbers is the lowest number that is a multiple of all of the given numbers.	common multiples. Common denominators should not exceed 60. (b)
•	Estimation keeps the focus on the meaning of the numbers and operations, encourages reflective thinking, and helps build informal number sense with fractions. Students can reason with benchmarks to get an estimate without using an algorithm.	Estimate the sum or difference of two fractions. (b, c) Add and subtract fractions (proper or improper) and/or mixed numbers, having like and unlike denominators limited
•	Reasonable answers to problems involving addition and subtraction of fractions can be established by using benchmarks such as $0, \frac{1}{2}$, and 1 . For example, $\frac{3}{5}$ and $\frac{4}{5}$ are each greater than $\frac{1}{2}$, so their sum is greater than 1 .	to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction. (Subtraction with fractions will be limited to problems that do not require regrouping). (b)
•	Students should investigate addition and subtraction with fractions, using a variety of models (e.g., fraction circles, fraction strips, lines, pattern blocks).	 Solve single-step practical problems that involve addition and subtraction with fractions (proper or improper) and/or mixed numbers, having like and unlike denominators limited to 2, 3,
•	While this standard requires instruction in solving problems with denominators of 2, 3, 4, 5, 6, 8, 10, and 12, students would benefit from experiences with other denominators.	4, 5, 6, 8, 10, and 12, and simplify the resulting fraction. (Subtraction with fractions will be limited to problems that do not require regrouping). (c)
•	When students use the least common multiple to determine common denominators to add or subtract fractions with unlike denominators, the least common multiple may be greater than 12, but will not exceed 60.	as

- a) determine common multiples and factors, including least common multiple and greatest common factor;
- b) add and subtract fractions and mixed numbers having like and unlike denominators;* and
- c) solve single-step practical problems involving addition and subtraction with fractions and mixed numbers.

^{*}On the state assessment, items measuring this objective are assessed without the use of a calculator.

	Understanding the Standard	Essential Knowledge and Skills
•	Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$).	
•	Instruction involving addition and subtraction of fractions should include experiences with proper fractions, improper fractions, and mixed numbers as addends, minuends, subtrahends, sums, and differences.	
•	A fraction is in simplest form when its numerator and denominator have no common factors other than one. The numerator can be greater than the denominator.	
•	The problem-solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings.	
•	In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as <i>in all, altogether, difference,</i> etc. encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions.	
•	At this level, denominators of fractions resulting from simplification will be limited to 12 or less.	

- a) add and subtract decimals;* and
- b) solve single-step and multistep practical problems involving addition and subtraction with decimals.

^{*}On the state assessment, items measuring this objective are assessed without the use of a calculator.

	Understanding the Standard	Essential Knowledge and Skills
•	Addition and subtraction of decimals may be explored, using a variety of models (e.g., 10-by-10 grids, number lines, money).	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
•	The problem-solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings.	Estimate sums and differences of decimals. (a)
•	In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as <i>in all, altogether, difference,</i> etc. encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions. The least number of steps necessary to solve a single-step problem is one.	 Add and subtract decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil. (a) Solve single-step and multistep practical problems that involve adding and subtracting with decimals through thousandths. (b)

Students in grades three through five should be actively involved in measurement activities that require a dynamic interaction between students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, perimeter, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure length, weight/mass, liquid volume/capacity, area, perimeter, temperature, and time. Students' understanding of measurement continues to be enhanced through experiences using appropriate tools such as rulers, balances, clocks, and thermometers.

The study of geometry helps students represent and make sense of the world. In grades three through five, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, draw representations of, and describe the relationships among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence; parallel, intersecting, and perpendicular lines; and classification of triangles.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

• Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

- Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of the parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three.)
- Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

4.7 The student will solve practical problems that involve determining perimeter and area in U.S. Customary and metric units.

Understanding the Standard	Essential Knowledge and Skills
 Perimeter is the path or distance around any plane figure. To determine the perimeter of any polygon, determine the sum of the lengths of the sides. 	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
 Area is the surface included within a plane figure. Area is measured by the number of square units needed to cover a surface or plane figure. Students should have opportunities to investigate and discover, using manipulatives, the formulas for the area of a square and the area of a rectangle. 	Determine the perimeter of a polygon with no more than eight sides, when the lengths of the sides are given, with diagrams.
 Area of a square = side length × side length Area of rectangle = length × width 	 Determine the perimeter and area of a rectangle when given the measure of two adjacent sides, with and without diagrams.
Perimeter and area should always be labeled with the appropriate unit of measure.	 Determine the perimeter and area of a square when the measure of one side is given, with and without diagrams. Solve practical problems that involve determining perimeter and area in U.S. Customary and metric units.

- a) estimate and measure length and describe the result in U.S. Customary and metric units;
- b) estimate and measure weight/mass and describe the result in U.S. Customary and metric units;
- c) given the equivalent measure of one unit, identify equivalent measures of length, weight/mass, and liquid volume between units within the U.S. Customary system; and
- d) solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units.

	Understanding the Standard	Essential Knowledge and Skills
•	The measurement of an object must include the unit of measure along with the number of iterations.	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and
•	Length is the distance between two points along a line. U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter rulers, meter sticks, and tape measures. When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch $(\frac{1}{2}, \frac{1}{4}, \frac{1}{8})$, foot, or yard. Weight and mass are different. Mass is the amount of matter in an object. Weight is determined	 Petermine an appropriate unit of measure (inch, foot, yard, mile, millimeter, centimeter, and meter) to use when measuring length in both U.S. Customary and metric units. (a) Estimate and measure length in U.S. Customary and metric units, measuring to the nearest part of an inch (¹/₂, ¹/₄, ¹/₈), and to the nearest foot, yard, millimeter, centimeter, or meter, and record the length including the unit of measure (e.g., 24 inches). (a)
•	by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term weight (e.g., "How much does it weigh?" versus "What is its mass?"). Balances are appropriate measuring devices to measure weight in U.S. Customary units (ounces, pounds) and mass in metric units (grams, kilograms).	 Compare estimates of the length with the actual measurement of the length. (a) Determine an appropriate unit of measure (ounce, pound, gram, and kilogram) to use when measuring the weight/mass of everyday objects in both U.S. Customary and metric units. (b)
•	Practical experience measuring the weight/mass of familiar objects (e.g., foods, pencils, book bags, shoes) helps to establish benchmarks and facilitates the student's ability to estimate weight/mass. Students should measure the liquid volume of everyday objects in U.S. Customary units, including cups, pints, quarts, gallons, and record the volume including the appropriate unit of measure (e.g., 24 gallons).	 Estimate and measure the weight/mass of objects in both U.S. Customary and metric units (ounce, pound, gram, or kilogram) to the nearest appropriate measure, using a variety of measuring instruments. (b) Record the weight/mass of an object with the unit of measure (e.g., 24 grams). (b)

- a) estimate and measure length and describe the result in U.S. Customary and metric units;
- b) estimate and measure weight/mass and describe the result in U.S. Customary and metric units;
- c) given the equivalent measure of one unit, identify equivalent measures of length, weight/mass, and liquid volume between units within the U.S. Customary system; and
- d) solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units.

Understanding the Standard	Essential Knowledge and Skills
 Students at this level will be given the equivalent measure of one unit when asked to determine equivalencies between units in the U.S. Customary system. For example, students will be told one gallon is equivalent to four quarts and then will be asked to apply that relationship to determine: the number of quarts in five gallons; the number of gallons equal to 20 quarts; When empty, Tim's 10-gallon container can hold how many quarts?; or Maria has 20 quarts of lemonade. How many empty one-gallon containers will she be able to fill? 	 Given the equivalent measure of one unit, identify equivalent measures between units within the U.S. Customary system for: length (inches and feet, feet and yards, inches and yards); yards and miles; weight/mass (ounces and pounds); and liquid volume (cups, pints, quarts, and gallons). (c) Solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units. (d)

4.9 The student will solve practical problems related to elapsed time in hours and minutes within a 12-hour period.

Understanding the Standard	Essential Knowledge and Skills
 Elapsed time is the amount of time that has passed between two given times. Elapsed time should be modeled and demonstrated using analog clocks and timelines. Elapsed time can be found by counting on from the beginning time or counting back from the ending time. 	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to • Solve practical problems related to elapsed time in hours and minutes, within a 12-hour period (within a.m., within p.m., and across a.m. and p.m.): - when given the beginning time and the ending time, determine the time that has elapsed; - when given the beginning time and amount of elapsed time in hours and minutes, determine the ending time; or - when given the ending time and the elapsed time in hours and minutes, determine the beginning time.

- a) identify and describe points, lines, line segments, rays, and angles, including endpoints and vertices; and
- b) identify and describe intersecting, parallel, and perpendicular lines.

	Understanding the Standard	Essential Knowledge and Skills
•	Points, lines, line segments, rays, and angles, including endpoints and vertices are fundamental components of noncircular geometric figures.	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and
•	Intersecting lines have one point in common.	

- a) identify and describe points, lines, line segments, rays, and angles, including endpoints and vertices; and
- b) identify and describe intersecting, parallel, and perpendicular lines.

Understanding the Standard	Essential Knowledge and Skills
• Perpendicular lines intersect at right angles. The symbol \bot is used to indicate that two lines are perpendicular. For example, the notation $\overrightarrow{AB} \bot \overrightarrow{CD}$ is read as "line AB is perpendicular to line CD."	
Students need experiences using geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides.	
• Parallel lines lie in the same plane and never intersect. Parallel lines are always the same distance apart and do not share any points. The symbol \parallel indicates that two or more lines are parallel. For example, the notation $\overrightarrow{BC} \parallel \overrightarrow{FG}$ is read as "line BC is parallel to line FG".	

4.11 The student will identify, describe, compare, and contrast plane and solid figures according to their characteristics (number of angles, vertices, edges, and the number and shape of faces) using concrete models and pictorial representations.

Undo	erstanding the Standard		Essential Knowledge and Skills
 The study of geometric figures must such as graph paper, pattern blocks, Opportunity must be provided for busolid figures. A plane figure is any closed, two-dim A solid figure is three-dimensional, he A face is any flat surface of a solid figure is formed by two rays with wherever lines and/or line segments. An edge is the line segment where two and the point at which two or solid figures, a vertex is the point at at a cube is a solid figure with six congrates eight vertices and 12 edges. A rectangular prism is a solid figure is eight vertices and 12 edges. A cube a sphere is a solid figure with all of it common vertex. A square pyramid he Characteristics of solid figures included solid figure in the point is a solid figure with all of it common vertex. A square pyramid he Characteristics of solid figures included solid figure in figure in the point at a solid figure with all of it common vertex. A square pyramid he Solid Figure # of Faces Cube 6 Rectangular Prism 6 Square Pyramid 5 Sphere 0 	nensional shape. having length, width, and height gure. a common endpoint called the sintersect. wo faces of a solid figure intersect more lines, line segments, or rewhich three or more faces meet gruent, square faces. All edges a in which all six faces are rectangulates a special case of a rectangulatits points the same distance from the square base and four faces has five vertices and eight edges ded at this grade level are defined.	and computer software tools). cabulary to describe plane and nt. e vertex. Angles are found sect. rays meet to form an angle. In et. are the same length. A cube agles. A rectangular prism has ar prism. om its center. s that are triangles with a es.	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to Identify concrete models and pictorial representations of solid figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder). Identify and describe solid figures (cube, rectangular prism, square pyramid, and sphere) according to their characteristics (number of angles, vertices, edges, and by the number and shape of faces). Compare and contrast plane and solid figures (circle/sphere, square/cube, triangle/square pyramid, and rectangle/rectangular prism) according to their characteristics (number of sides, angles, vertices, edges, and the number and shape of faces).

4.12 The student will classify quadrilaterals as parallelograms, rectangles, squares, rhombi, and/or trapezoids.

Understanding the Standard	Essential Knowledge and Skills
 A quadrilateral is a polygon with four sides. A parallelogram is a quadrilateral with both pairs of opposite sides parallel and congruent. Congruent figures have the same size and shape. Congruent sides are the same length. A rectangle is a quadrilateral with four right angles, and, opposite sides that are parallel and congruent. The geometric markings shown on the rectangle below indicate parallel sides with an equal number of arrows and congruent sides indicated with an equal number of hatch (hash) marks. A square is a rectangle with four congruent sides and four right angles. A trapezoid is a quadrilateral with exactly one pair of parallel sides. A rhombus is a quadrilateral with four congruent sides. Properties of a rhombus include the following: opposite sides are congruent opposite sides are congruent opposite angles are congruent 	 The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to Develop definitions for parallelograms, rectangles, squares, rhombi, and trapezoids. Identify properties of quadrilaterals including parallel, perpendicular, and congruent sides. Classify quadrilaterals as parallelograms, rectangles, squares, rhombi, and/or trapezoids. Compare and contrast the properties of quadrilaterals. Identify parallel sides, congruent sides, and right angles using geometric markings to denote properties of quadrilaterals.

Students entering grades three through five have begun to explore the concept of the measurement of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students have had initial experiences in predicting whether a game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction in grades three through five is to deepen their understanding of the concepts of probability by

- offering opportunities to set up models simulating practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities that instill a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on:

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions ("How many?" "How much?") from the data displays;
- identifying and justifying comparisons ("Which is the most? Which is the least?" "Which is the same? Which is different?") about the information;
- · comparing their initial predictions to the actual results; and
- communicating to others their interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.

- a) determine the likelihood of an outcome of a simple event;
- b) represent probability as a number between 0 and 1, inclusive; and
- c) create a model or practical problem to represent a given probability.

Understanding the Standard Essential Knowledge and Skills A spirit of investigation and experimentation should permeate probability instruction, where students The student will use problem solving, mathematical communication, mathematical reasoning, connections, and are actively engaged in explorations and have opportunities to use manipulatives. representations to Probability is the measure of likelihood that an event will occur. An event is a collection of outcomes Model and determine all possible outcomes of a given from an investigation or experiment. simple event where there are no more than 24 possible The terms certain, likely, equally likely, unlikely, and impossible can be used to describe the likelihood outcomes, using a variety of manipulatives (e.g., coins, of an event. If all outcomes of an event are equally likely, the probability of an event can be expressed number cubes, and spinners). (a) as a fraction, where the numerator represents the number of favorable outcomes and the Determine the outcome of an event that is least likely to denominator represents the total number of possible outcomes. If all the outcomes of an event are occur or most likely to occur where there are no more equally likely to occur, the probability of the event is equal to: than 24 possible outcomes. (a) number of favorable outcomes total number of possible outcomes. Write the probability of a given simple event as a fraction, where there are no more than 24 possible outcomes. (b) Probability is quantified as a number between 0 and 1. An event is "impossible" if it has a probability of 0 (e.g., if eight balls are in a bag, four yellow and four blue, there is zero probability that a red ball Determine the likelihood of an event occurring and relate could be selected). An event is "certain" if it has a probability of one (e.g., the probability that if 10 it to its whole number or fractional representation (e.g., coins, all pennies, are in a bag that it is certain a penny could be selected). impossible or zero; equally likely; certain or one). (a, b) For an event such as flipping a coin, the things that can happen are called *outcomes*. For example, Create a model or practical problem to represent a given there are two possible outcomes when flipping a coin: the coin can land heads up, or the coin can land probability. (c) tails up. The two possible outcomes, heads up or tails up, are equally likely. For another event such as spinning a spinner that is one-third red and two-thirds blue, the two outcomes, red and blue, are not equally likely. Equally likely events can be represented with fractions of equivalent value. For example, on a spinner with eight sections of equal size, where three of the sections are labeled G (green) and three are labeled B (blue), the chances of landing on green or on blue are equally likely; the probability of each of these events is the same, or $\frac{3}{8}$.

- a) determine the likelihood of an outcome of a simple event;
- b) represent probability as a number between 0 and 1, inclusive; and
- c) create a model or practical problem to represent a given probability.

Understanding the Standard	Essential Knowledge and Skills
 Students need opportunities to create a model or practical problem that represents a given probability. For example, if asked to create a box of marbles where the probability of selecting a black marble is \$\frac{4}{8}\$, sample responses might include: When a probability experiment has very few trials, the results can be misleading. The more times an 	
experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time).	

- a) collect, organize, and represent data in bar graphs and line graphs;
- b) interpret data represented in bar graphs and line graphs; and
- c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph, a chart and a line graph, or a pictograph and a bar graph).

Understanding the Standard

- Data analysis helps describe data, recognize patterns or trends, and make predictions.
- Investigations involving practical data should occur frequently; data can be collected through brief class surveys or through more extended projects taking many days.
- Students formulate questions, predict answers to questions under investigation, collect and represent initial data, and consider whether the data answer the questions.
- There are two types of data: categorical (e.g., qualitative) and numerical (e.g., quantitative). Categorical data are observations about characteristics that can be sorted into groups or categories, while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. While students need to be aware of the differences, they do not have to know the terms for each type of data.
- Bar graphs display grouped data such as categories using rectangular bars whose length represents
 the quantity the bar represents. Bar graphs should be used to compare counts of different
 categories (categorical or qualitative data). Grid paper can assist students in creating graphs with
 greater accuracy.
 - A bar graph uses horizontal or vertical bars to represent counts for several categories.
 One bar is used for each category, with the length of the bar representing the count for that category.
 - There is space before, between, and after the bars.
 - The axis that displays the scale representing the count for the categories should begin at zero and extend one increment above the greatest recorded piece of data. Grade four students should collect and represent data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, 10, or 100.
 - Each axis should be labeled, and the graph should be given a title.
- Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect data, using, for example, observations, measurement, surveys, scientific experiments, polls, or questionnaires. (a)
- Organize data into a chart or table. (a)
- Represent data in bar graphs, labeling one axis with equal whole number increments of one or more (numerical data) (e.g., 2, 5, 10, or 100) and the other axis with categories related to the title of the graph (categorical data) (e.g., swimming, fishing, boating, and water skiing as the categories of "Favorite Summer Sports"). (a)
- Represent data in line graphs, labeling the vertical axis with equal whole number increments of one or more and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years. Line graphs will have no more than 10 identified points along a continuum for continuous data. (a)
- Title the graph or identify an appropriate title. Label the axes or identify the appropriate labels. (a)
- Interpret data by making observations from bar graphs and line graphs by describing the characteristics of the data and the data as a whole (e.g., the time period when the temperature increased the most, the category with the greatest/least, categories with the same number of responses, similarities and differences, the total number).
 One set of data will be represented on a graph. (b)

- a) collect, organize, and represent data in bar graphs and line graphs;
- b) interpret data represented in bar graphs and line graphs; and
- c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph, a chart and a line graph, or a pictograph and a bar graph).

Understanding the Standard Essential Knowledge and Skills Line graphs are used to show how two data sets (numerical or quantitative data) are related. Line Interpret data by making inferences from bar graphs and line graphs may be used to show how one variable changes over time (numerical or quantitative data). graphs. (b) By looking at a line graph, it can be determined whether the change in the data set is increasing, Interpret the data to answer the question posed, and decreasing, or staying the same over time. compare the answer to the prediction (e.g., "The summer The values along the horizontal axis represent continuous data, usually some measure of time sport preferred by most is swimming, which is what I (e.g., time in years, months, or days). The data presented on a line graph is referred to as predicted before collecting the data."). (b) "continuous data," as it represents data collected over a continuous period of time. Write at least one sentence to describe the analysis and The values along the vertical axis represent the range of values in the collected data set at the interpretation of the data, identifying parts of the data that given time interval on the horizontal axis. The scale values on the vertical axis should represent have special characteristics, including categories with the equal increments of multiples of whole numbers, fractions, or decimals, depending upon the greatest, the least, or the same. (b) data being collected. The scale should extend one increment above the greatest recorded piece of data. Compare two different representations of the same data Plot a point to represent the data collected for each time increment. Use line segments to (e.g., a set of data displayed on a chart and a bar graph; a connect the points in order moving left to right. chart and a line graph; a pictograph and a bar graph). (c) Each axis should be labeled, and the graph should be given a title. Statements representing an analysis and interpretation of the characteristics of the data in the graph should be included (e.g., trends of increase and/or decrease, and least and greatest). For example, a line graph documenting data gathered during a planting cycle might show length of time and the height of a plant at any given interval. Different situations call for different types of graphs. The way data are displayed is often dependent upon what someone is trying to communicate. Comparing different types of representations (charts and graphs) provide students an opportunity to learn how different graphs can show different aspects of the same data. Following construction of graphs, students benefit from discussions around what information each graph provides. Tables or charts organize the exact data and display numerical information. They do not show visual comparisons, which generally means it takes longer to understand or to examine trends.

- a) collect, organize, and represent data in bar graphs and line graphs;
- b) interpret data represented in bar graphs and line graphs; and
- c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph, a chart and a line graph, or a pictograph and a bar graph).

	Understanding the Standard			Essential Knowledge and Skills
	graphs display da eases to be seen	_	tinuously over time. This allows overall increases or	
com	Bar graphs can be used to compare data easily and see relationships. They provide a visual display comparing the numerical values of different categories. The scale of a bar graph may affect how one perceives the data.			
whic	Examples of some questions that could be explored in comparing a chart to a line graph include: In which representation do you readily see the increase or decrease of temperature over time? In which representation is it easiest to determine when the greatest rise in temperature occurred? Temperature Over Time			
	Time	Temperature	50	
	9 a.m.	12	£ 40	
	10 a.m.	26	€ 30	
	11 a.m.	33		
	12 p.m.	39	E 20 /	
			Temperature (°F)	

Time

Students entering grades three through five have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships within a pattern.

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write "rules" for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

4.15 The student will identify, describe, create, and extend patterns found in objects, pictures, numbers, and tables.

Understanding the Standard

- Patterns and functions can be represented in many ways and described using words, tables, graphs, and symbols.
- Patterning activities should involve making connections between concrete materials and numerical representations (e.g., number sequence, table, description). Numeric patterns, at this level, will include both growing and repeating patterns (limited to addition, subtraction, and multiplication of whole numbers and addition and subtraction of fractions with like denominators of 12 or less).
- Students need experiences with growing patterns using concrete materials and calculators.
- Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing the relationship symbolically or algebraically.
- Sample growing patterns that are, or can be, represented as numerical (arithmetic) growing patterns include:
 - 2, 4, 8, 16, ...;
 - 8, 10, 13, 17, ...;

$$-\frac{1}{4}$$
, $\frac{3}{4}$, $1\frac{1}{4}$, $1\frac{3}{4}$...; and









Students in grade three had experiences working with input/output tables. At this level, input/output tables should be analyzed for a pattern to determine an unknown value or describe the rule that explains how to find the output when given the input. Determining and applying rules builds the foundation for functional thinking. Sample input/output tables that require determination of the rule or missing terms can be found below:

Rule: ?	
Input	Output
4	11
5	12
6	13
10	17

Rule: ?		
Input	Output	
145	130	
100	85	
75	60	
50	?	

Rule: ?	
Input	Output
2	8
4	16
?	20
8	32

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify and describe patterns, using words, objects, pictures, numbers, and tables.
- Create patterns using objects, pictures, numbers, and tables.
- Extend patterns, using objects, pictures, numbers, and tables.
- Solve practical problems that involve identifying, describing, and extending single-operation input and output rules, limited to addition, subtraction, and multiplication of whole numbers and addition and subtraction of fractions with like denominators of 12 or less.
- Identify the rule in a single-operation numerical pattern found in a list or table, limited to addition, subtraction, and multiplication of whole numbers.

4.16 The student will recognize and demonstrate the meaning of equality in an equation.

	Understanding the Standard	Essential Knowledge and Skills
•	Mathematical relationships can be expressed using equations. An expression is a representation of a quantity. It is made up of numbers, variables, and/or computational symbols. It does not have an equal symbol (e.g., 8 , 15×12).	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
•	An equation represents the relationship between two expressions of equal value (e.g., $12 \times 3 = 72 \div 2$).	 Write an equation to represent the relationship between equivalent mathematical expressions (e.g., 4 × 3 = 2 × 6; 10 + 8 = 36 ÷ 2; 12 × 4 = 60 − 12). Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal, using addition, subtraction, multiplication, and division (e.g., 4 × 12 = 8 × 6 and 64 ÷ 8 ≠ 8 × 8).
•	The equal-symbol (=) means that the values on either side are equivalent (balanced). The not equal symbol (\neq) means that the values on either side are not equivalent (not balanced).	