WGSD Curriculum – Pre-Algebra
DRAFT
Mathematical Practices 1

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP1

Learning Goal	**Proficiency Scale**
Students will be able to make sense of problems and persevere in solving them.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: Student demonstrates mastery with the learning goal as evidenced by: • discussing, explaining, and solving a problem with multiple representations and in multiple ways. • struggling with various attempts over time. • learning from previous solution attempts. • checking answers using a different method or strategy. 2: Student demonstrates he/she is nearing proficiency by: • explaining his/her thought processes when solving a problem and representing it in several ways. • trying several approaches in find a solution and seeking hints only if stuck. 1: Student demonstrates a limited understanding or skill with the learning goal by: • explaining their thought processes when solving a problem one way. • staying with a challenging problem for more than one attempt.

Learning Targets

- Explain the meaning of a problem and look for efficient ways to solve it
- Use concrete objects or pictures to help conceptualize and solve problems
- Checks their thinking by asking themselves, “Does this make sense?”

- Listens to the strategies of others and tries different approaches
- Uses a different strategies to check answers
- Takes time to thoughtfully consider problems

Learning Design

- Provides time and facilitates discussion in problem solutions
- Facilitates discourse in the classroom so that students UNDERSTAND the approaches of others
- Provides opportunities for students to explain themselves, the meaning of a problem, etc.
- Provides opportunities for students to connect concepts to “their” world
- Provides students TIME to think and become “patient” problem solvers
- Facilitates and encourages students to check their answers using different methods (not calculators)
- Provides problems that focus on relationships and are “generalizable”

WGSD Curriculum – Pre-Algebra
DRAFT
Mathematical Practices 2

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP2

Learning Goal	**Proficiency Scale**
Students will be able to reason abstractly and quantitatively.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: Student demonstrates mastery with the learning goal as evidenced by: • converting situations into symbols to solve problems. • converting mathematical equations into meaningful situations. 2: Student demonstrates he/she is nearing proficiency by translating situations into symbols to solve problems. 1: Student demonstrates a limited understanding or skill with the learning goal by reasoning with models or pictorial representations to solve problems.

Learning Targets

- Recognize that a number represents a specific quantity
- Connect the quantity to written symbols and create a logical representation of the problem at hand
- Consider both the appropriate units involved and the meaning of quantities
- Write simple expressions that record calculations with numbers and symbols
- Represent or round numbers using place value concepts

Learning Design

- Provides a range of representations of math problem situations and encourages various solutions
- Provides opportunities for students to make sense of quantities and their relationships in problem situations
- Provides problems that require flexible use of properties of operations and objects
- Emphasizes quantitative reasoning which entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them and/or rules; and knowing and flexibly using different properties of operations and objects

WGSD Curriculum – Pre-Algebra
DRAFT
Mathematical Practices 3

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP3

Learning Goal	**Proficiency Scale**
Students will be able to construct viable arguments and critique the reasoning of others.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: Student demonstrates mastery with the learning goal as evidenced by: • justifying and explaining, with accurate language and vocabulary, why his/her solution is correct. • comparing his/her strategy to other students' strategies, asking questions, and making connections with his/her own thinking. • explaining the reasoning of others. 2: Student demonstrates he/she is nearing proficiency by: • explaining his/her thinking and the thinking of others with accurate vocabulary. • explaining other students' solutions and identifying strengths and weaknesses of the strategy. 1: Student demonstrates a limited understanding or skill with the learning goal by: • explaining his/her solution. • discussing other ideas, approaches, and strategies.

Learning Targets

- Construct arguments using concrete referents, such as objects, pictures, and drawings
- Refine their mathematical communication skills by answering questions like "How do you know?" and "Can you show me another way?"
- Refine their mathematical communication skills by asking others questions like "How do you know?" and "How did you get that?"
- Explain their thinking to others and respond to others' thinking

Learning Design

- Provides ALL students opportunities to understand and use stated assumptions, definitions, and previously established results in constructing arguments
- Provides ample time for students to make conjectures and build a logical progression of statements to explore the truth of their conjectures
- Provides opportunities for students to construct arguments and critique arguments of peers
- Facilitates and guides students in recognizing and using counterexamples
- Encourages and facilitates students justifying their conclusions, communicating, and responding to the arguments of others
- Asks useful questions to clarify and/or improve students' arguments

WGSD Curriculum – Pre-Algebra
DRAFT
Mathematical Practices 4

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP4
TILS 5.C.a: Recognize that there are a variety of ways to share information
TILS 5.C.c: Effectively share information

Learning Goal	**Proficiency Scale**
Students will be able to model with mathematics.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: Student demonstrates mastery with the learning goal as evidenced by: • recognizing math in everyday situations. • using a variety of models, symbolic representations, and technology tools to represent the solution to a problem and accurately explain the solution representation. 2: Student demonstrates he/she is nearing proficiency by: • recognizing math in everyday situations, when prompted. • using models and symbols to represent and solve a problem. 1: Student demonstrates a limited understanding or skill with the learning goal by using models to represent and solve a problem with teacher support.

Learning Targets

- Represents problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. and use all of these representations as needed
- Connect different representations and explain the connections
- Evaluate results in the context of the situation and reflect on whether the results make sense
- Evaluate the utility of models to determine which models are most useful and efficient to solve problems

Learning Design

- Provides problem situations that apply to everyday life
- Provides rich tasks that focus on conceptual understanding, relationships, etc.

WGSD Curriculum – Pre-Algebra
DRAFT
Mathematical Practices 5

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>
CCSS.Math.Practice.MP5</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to use appropriate tools strategically.</td><td><u>Proficiency Scale</u>
4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3: Student demonstrates mastery with the learning goal as evidenced by combining various tools to explore and solve a problem as well as justifying his/her tool selection and problem solution.

2: Student demonstrates he/she is nearing proficiency by selecting from a variety of provided tools the ones that can be used to solve a problem and explaining his/her reasoning for the selection.

1: Student demonstrates a limited understanding or skill with the learning goal by using the appropriate tool, when provided, to find a solution.</td></tr>
<tr><td colspan="2"><u>Learning Targets</u>
• Consider the available tools (including, but not limited to estimation, graph paper, manipulatives, table, list, etc.) when solving a mathematical problem and decide when certain tools might be helpful
• For example, they may use unit cubes to fill a rectangular prism and a ruler to measure the dimensions
• Use graph paper to accurately create graphs and solve problems or make predictions from real world data</td></tr>
<tr><td colspan="2"><u>Learning Design</u>
• Provides a variety of tools and technology for students to explore to deepen their understanding of math concepts
• Provides problem solving tasks that require students to consider a variety of tools for solving (Tools might include pencil/paper, concrete models, manipulatives, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software, etc.)</td></tr>
</table>

WGSD Curriculum – Pre-Algebra
DRAFT
Mathematical Practices 6

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>
CCSS.Math.Practice.MP6</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to attend to precision.</td><td><u>Proficiency Scale</u>
4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3: Student demonstrates mastery with the learning goal as evidenced by using appropriate symbols, vocabulary, and labeling to communicate effectively and exchange ideas.

2: Student demonstrates he/she is nearing proficiency by incorporating appropriate vocabulary and symbols in most mathematical communications.

1: Student demonstrates a limited understanding or skill with the learning goal by communicating his/her reasoning and solution to others, with support.</td></tr>
<tr><td colspan="2"><u>Learning Targets</u>
• Use clear and precise language in their discussions with others and in their own reasoning
• Specify units of measure and state the meaning of the symbols used
• Report answers that appropriately address the context of a problem</td></tr>
<tr><td colspan="2"><u>Learning Design</u>
• Facilitates, encourages and expects precision in communication
• Provides opportunities for students to explain and/or write their reasoning to others</td></tr>
</table>

WGSD Curriculum – Pre-Algebra
DRAFT
Mathematical Practices 7

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP7

Learning Goal	**Proficiency Scale**
Students will be able to look for and make use of structure.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: Student demonstrates mastery with the learning goal as evidenced by: • noticing mathematical expressions as component parts. • using mathematical generalizations to identify the most efficient solution to mathematical tasks. 2: Student demonstrates he/she is nearing proficiency by composing and decomposing number situations and relationships in order to simplify solutions. 1: Student demonstrates a limited understanding or skill with the learning goal by looking for structure or patterns within mathematics to help him/her solve problems efficiently.

Learning Targets

- Look closely to discover a pattern or structure
- For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals.
- Examine numerical patterns and relate them to a rule or a graphical representation

Learning Design

- Provides opportunities and time for students to explore patterns and relationships to solve problems
- Provides rich tasks and facilitates pattern seeking and understanding of relationships in numbers rather than following a set of steps and/or procedures

WGSD Curriculum – Pre-Algebra
DRAFT
Mathematical Practices 8

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP8

Learning Goal	**Proficiency Scale**
Students will be able to look for and express regularity in repeated reasoning.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: Student demonstrates mastery with the learning goal as evidenced by: • connecting prior knowledge to an unfamiliar mathematical situation. • creating a model or equation that unifies the various aspects of a problem. • noticing patterns, making generalizations, and predicting patterns. 2: Student demonstrates he/she is nearing proficiency by finding and explaining patterns. 1: Student demonstrates a limited understanding or skill with the learning goal by connecting prior knowledge to new situations and noticing patterns with prompting from a teacher or peer.

Learning Targets

- Notice repetitive actions in computation and look for more shortcut methods
- Use repeated reasoning to understand algorithms and make generalizations about patterns

Learning Design

- Provides problem situations that allow students to explore regularity and repeated reasoning
- Provides rich tasks that encourage students to use repeated reasoning to form generalizations and provides opportunities for students to communicate these generalizations

WGSD Curriculum – Pre-Algebra
DRAFT
The Number System

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>

CCSS.Math.Content.8.NS.A.1, NS.A.2, CCSS.Math.Content.6.RP.A.3</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to compare and order rational numbers and represent rational numbers in multiple ways.</td><td><u>Proficiency Scale</u>

4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3: The student demonstrates mastery of the learning goal as evidenced by
• using rational approximations of irrational numbers to locate them on a number line and to make numerical comparisons; converting between fractions and repeating decimals; and comparing rational numbers.
• using ratio reasoning to solve and understand the concept of unit rates in unfamiliar or multi-step problems, including instances of unit pricing and constant speed, and solving percent problems by finding the whole, given a part and the percent.
• describing a ratio relationship between any two number quantities (denominators less than or equal to 12).

2: The student demonstrates he/she is nearing proficiency by
• recognizing and recalling specific vocabulary, such as decimal, fraction, percent, ratio, rational number, irrational number, order.

• performing specific processes, such as
○ identifying approximate locations of familiar irrational numbers on a number line; identifying numbers as rational or irrational; and converting between fractions and terminating decimals.
○ understanding the concept of unit rate in straightforward, well-posed problems.
○ solving straightforward, well-posed, one-step problems requiring ratio reasoning.

1: The student demonstrates limited understanding or skill with the learning goal by
• identifying square roots of numbers less than 100.</td></tr>
</table>

	• identifying pi as not rational. • understanding that every rational number has a decimal expansion. • describing a ratio relationship between two whole number quantities. • finding missing values in tables that display a proportional relationship, and plotting the pairs of values from a table on the coordinate plane. • finding a percent as a rate per hundred and convert measurement units.

Learning Targets

- Know that there are numbers that are not rational, and approximate them by rational numbers.
    - Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
    - Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations*.
- Understand ratio concepts and use ratio reasoning to solve problems.
    - Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Learning Design

WGSD Curriculum – Pre-Algebra
DRAFT
Functions

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>

CCSS.Math.Content.8.F.B.4, 8.F.B.5,</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to use functions to model relationships between quantities.</td><td><u>Proficiency Scale</u>

4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3: The student demonstrates mastery of the learning goal as evidenced by
• constructing a function to represent a linear relationship between two quantities.
• constructing a graph to represent verbally described qualitative features.
• determining the rate of change and initial value of a function from a graph, a verbal description of a relationship, or from two sets of xy-values given as coordinate pairs or displayed in a table.
• analyzing a graph of a linear or nonlinear function to qualitatively describe it.

2: The student demonstrates he/she is nearing proficiency by

• recognizing or recalling specific vocabulary, such as graph, table, function, linear function, ordered pair, coordinate pairs.

• performing processes, such as
○ constructing a graphical or tabular model to represent a linear relationship between two quantities.
○ finding the rate of change of a linear relationship displayed in a graph or table.
○ analyzing a graph of a linear function to qualitatively describe it.

1: The student demonstrates limited understanding or skill with the learning goal by
• identifying a function that models a linear relationship between two quantities.</td></tr>
</table>

Learning Targets

- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Learning Design

WGSD Curriculum – Pre-Algebra
DRAFT
Graphing Functions

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSA-CED.A.1,CED.A.2
CCSS.Math.Content.HSF-IF.A.1, HSF-IF.A.2, HSF-IF.B.5, HSF-IF.B.6, HSF-IF.C.9

Learning Goal	**Proficiency Scale**
Students will be able to graph and analyze functions and represent data.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal as evidenced by • creating and using linear equations and inequalities to model an unfamiliar situation and to solve an unfamiliar problem. • graphing an equation in two variables and rearranging a linear multi-variable formula for a particular given quantity. • using function notation to evaluate a function given in function notation for a particular input. • identifying the domain and range for any given function presented in any form, e.g., as a graph, a verbal description, or a sequence. • graphing linear functions and interpreting and relating key features, including range and domain, in familiar or scaffolded contexts. • specifying the average rate of change of a function on a given domain from its equation or approximating the average rate of change of a function from its graph. • analyzing and comparing properties of two functions of different types represented in different ways and understanding equivalent forms of functions. 2: The student demonstrates he/she is nearing proficiency by • recognizing and recalling specific vocabulary, such as *solution, solving for a variable, domain, range, slope, rate of change, function.* • performing processes, such as • creating and using linear equations and linear inequalities in one and two variables to model

	a familiar situation and to solve a familiar problem. • graphing a linear equation in two variables and be able to rearrange a familiar formula or an unfamiliar linear formula in one or two variables for a particular given quantity. • understanding the concept of a function in order to distinguish a relation as a function or not a function. • identifying domain and range of a function given a graph of a linear function and they should understand that the graph of a function f(x) is the graph of the equation y = f(x). • specifying the average rate of change from an equation of a linear function and approximating it from a graph of a linear function. • graphing linear functions by hand and compare properties of two functions of the same type, i.e., linear to linear, represented in different ways and understanding equivalent forms of functions. 1: The student demonstrates limited understanding or skill with the learning goal by • creating and using one-step linear equations in one variable to model a familiar situation and to solve a familiar problem. • distinguishing between functions and nonfunctions. • stating the domain and range given a graph. • interpreting linear functions in context, and given the key features of a linear graph, identifying the appropriate graph. • graphing a linear function by hand and identifying equivalent forms of linear functions.

Learning Targets

- Create equations that describe numbers or relationships.
    - Create equations and inequalities in one variable and use them to solve problems.
    - Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Understand the concept of a function and use function notation.
    - Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the

domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).

- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- Interpret functions that arise in applications in terms of the context.
  - Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
  - Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

- Analyze functions using different representations.
  - Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Learning Design

WGSD Curriculum – Pre-Algebra
DRAFT
Expressions and Polynomials

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSA-SSE.A.1, SSE.A.1a,
CCSS.Math.Content.HSA-APR.A.1

Learning Goal	**Proficiency Scale**
Students will be able to simplify expressions and polynomials.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal as evidenced by • recognizing equivalent forms of expressions and use the structure of an expression to identify ways to rewrite it. • interpreting complicated expressions by viewing one or more of their parts as a single entity. • adding, subtracting, and multiplying multi-variable polynomials of any degree and understand that polynomials are closed under subtraction and multiplication. 2: The student demonstrates he/she is nearing proficiency by • recognizing or recalling specific vocabulary, such as *equivalent experessions, polynomial, monomial, binomial, distribute, FOIL.* • performing processes, such as o interpreting parts of an expression, such as terms, factors, coefficients, exponents, etc., and interpreting simple compound expressions by viewing one or more of their parts as a single entity. o recognizing equivalent forms of linear expressions. o adding, subtracting, and multiplying multi-variable polynomials made up of monomials of degree 2 or less. o understanding that polynomials are closed under addition.

	1: The student demonstrates limited understanding or skill with the learning goal by • identifying parts of an expression, such as terms, factors, coefficients, exponents, etc. • adding, subtracting, and multiplying single variable polynomials of degree 2 or less.

Learning Targets

- Interpret the structure of expressions.
    - Interpret expressions that represent a quantity in terms of its context.
    - Interpret parts of an expression, such as terms, factors, and coefficients.
- Perform arithmetic operations on polynomials.
    - Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Learning Design

WGSD Curriculum – Pre-Algebra
DRAFT
Equations and Inequalities

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.8.EE.C.7
CCSS.Math.Content.8.EE.C.8a
CCSS.Math.Content.HSA-CED.A.1
CCSS.Math.Content.HSA-REI.A.1, REI.B.3, REI.D.12

Learning Goal	**Proficiency Scale**
Students will be able to analyze and solve linear equations and inequalities.	4: In addition to score 3.0 performance, the student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal as evidenced by • solving and producing examples of linear equations in one variable, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. • analyzing and solve systems of linear equations graphically by understanding that the solution of a system of linear equations in two variables corresponds to the point of intersection on a plane. • graphing and estimating the solution of systems of equations and systems of linear inequalities. • understanding that the plotted line, curve, or region represents the solution set to an equation or inequality. • solving multi-step linear inequalities in one variable with real roots. 2: The student demonstrates he/she is nearing proficiency by • recognizing and recalling specific vocabulary, such as *system, intersection, substitution, inequality.* • performing specific processes, such as o analyzing and solving systems of linear equations graphically by understanding that the solution of a system of linear equations in two variables corresponds to the point of intersection on a plane. o representing linear equations and inequalities with integer coefficients in one and two

	variables graphically on a coordinate plane. o understanding that the plotted line or curve represents the solution set to an equation. o graphing and estimating the solution of systems of linear equations. o solving one-step linear inequalities and in one variable with integer roots. 1: The student demonstrates limited understanding or skill with the learning goal by • solving one-step linear equations in one variable. • representing a linear equation with an integer-valued slope in two variables graphically on a coordinate plane. • Solving a one-step linear inequality.

Learning Targets

- Solve linear equations in one variable.
  - Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
- Analyze and solve pairs of simultaneous linear equations.
  - Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- Create equations that describe numbers or relationships.
  - Create equations and inequalities in one variable and use them to solve problems.
- Understand solving equations as a process of reasoning and explain the reasoning.
  - Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.
- Solve equations and inequalities in one variable.
  - Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
  - Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and

graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Learning Design

WGSD Curriculum – Algebra 1
DRAFT

In Algebra 1, instructional time will focus on three critical areas: (1) deepen and extend understanding of linear and exponential relationships by contrasting them with each other, (2) applying linear models to data that exhibit a linear trend, and (3) analyzing, solving, and using quadratic functions.

While the content learning goals describe the mathematics students should be able to understand and do, the first eight learning goals (The Standards for Mathematical Practice) describe how students should engage with these mathematical concepts and skills as they grow in mathematical maturity and expertise. Teachers should connect the mathematical practices to mathematical content in all mathematics instruction. These learning goals merit the most time, resources, innovation, and focus necessary to qualitatively improve the instruction, assessment, and student achievement in mathematics.

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP1

Learning Goal	**Proficiency Scale**
Students will be able to make sense of problems and persevere in solving them.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: Student demonstrates mastery with the learning goal as evidenced by: • Discussing, explaining, and solving a problem with multiple representations and in multiple ways. • Struggling with various attempts over time. • Learning from previous solution attempts. • Checking answers using a different method or strategy. 2: Student demonstrates he/she is nearing proficiency by: • Explaining his/her thought processes when solving a problem and representing it in several ways. • Trying several approaches in find a solution and seeking hints only if stuck. 1: Student demonstrates a limited understanding or skill with the learning goal by: • Explaining his/her thought processes when solving a problem one way. • Staying with a challenging problem for more than one attempt.

Learning Targets

- Explain the meaning of a problem and look for efficient ways to solve it
- Use concrete objects or pictures to help conceptualize and solve problems
- Checks their thinking by asking themselves, “Does this make sense?”
- Listens to the strategies of others and tries different approaches
- Uses a different strategies to check answers
- Takes time to thoughtfully consider problems

Learning Design

- Provides time and facilitates discussion in problem solutions
- Facilitates discourse in the classroom so that students UNDERSTAND the approaches of others
- Provides opportunities for students to explain themselves, the meaning of a problem, etc.
- Provides opportunities for students to connect concepts to “their” world
- Provides students TIME to think and become “patient” problem solvers
- Facilitates and encourages students to check their answers using different methods (not calculators)
- Provides problems that focus on relationships and are “generalizable”

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP2

Learning Goal	**Proficiency Scale**
Students will be able to reason abstractly and quantitatively.	4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal. 3: Student demonstrates mastery with the learning goal as evidenced by • Converting situations into symbols to solve problems. • Converting mathematical equations into meaningful situations. 2: Student demonstrates he/she is nearing proficiency by translating situations into symbols to solve problems. 1: Student demonstrates a limited understanding or skill with the learning goal by reasoning with models or pictorial representations to solve problems.

Learning Targets

- Recognize that a number represents a specific quantity
- Connect the quantity to written symbols and create a logical representation of the problem at hand
- Consider both the appropriate units involved and the meaning of quantities
- Write simple expressions that record calculations with numbers and symbols
- Represent or round numbers using place value concepts

Learning Design

- Provides a range of representations of math problem situations and encourages various solutions
- Provides opportunities for students to make sense of quantities and their relationships in problem situations
- Provides problems that require flexible use of properties of operations and objects
- Emphasizes quantitative reasoning which entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them and/or rules; and knowing and flexibly using different properties of operations and objects

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>
CCSS.Math.Practice.MP3</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to construct viable arguments and critique the reasoning of others.</td><td><u>Proficiency Scale</u>
4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal.

3: Student demonstrates mastery with the learning goal as evidenced by:
• Justifying and explaining, with accurate language and vocabulary, why his/her solution is correct.
• Comparing his/her strategy to other students' strategies, asking questions, and making connections with his/her own thinking.
• Explaining the reasoning of others.

2: Student demonstrates he/she is nearing proficiency by:
• Explaining his/her thinking and the thinking of others with accurate vocabulary.
• Explaining other students' solutions and identifying strengths and weaknesses of the strategy.

1: Student demonstrates a limited understanding or skill with the learning goal by:
• Explaining his/her solution.
• Discussing other ideas, approaches, and strategies.</td></tr>
<tr><td colspan="2"><u>Learning Targets</u>
• Construct arguments using concrete referents, such as objects, pictures, and drawings
• Refine their mathematical communication skills by answering questions like "How do you know?" and "Can you show me another way?"
• Refine their mathematical communication skills by asking others questions like "How do you know?" and "How did you get that?"
• Explain their thinking to others and respond to others' thinking</td></tr>
</table>

Learning Design

- Provides ALL students opportunities to understand and use stated assumptions, definitions, and previously established results in constructing arguments
- Provides ample time for students to make conjectures and build a logical progression of statements to explore the truth of their conjectures
- Provides opportunities for students to construct arguments and critique arguments of peers
- Facilitates and guides students in recognizing and using counterexamples
- Encourages and facilitates students justifying their conclusions, communicating, and responding to the arguments of others
- Asks useful questions to clarify and/or improve students' arguments

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>
CCSS.Math.Practice.MP4</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to model with mathematics.</td><td><u>Proficiency Scale</u>
4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal.

3: Student demonstrates mastery with the learning goal as evidenced by:
• Recognizing math in everyday situations.
• Using a variety of models, symbolic representations, and technology tools to represent the solution to a problem and accurately explain the solution representation.

2: Student demonstrates he/she is nearing proficiency by:
• Recognize math in everyday situations, when prompted.
• Using models and symbols to represent and solve a problem.

1: Student demonstrates a limited understanding or skill with the learning goal by using models to represent and solve a problem with teacher support.</td></tr>
<tr><td colspan="2"><u>Learning Targets</u>
• Represents problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. and use all of these representations as needed
• Connect different representations and explain the connections
• Evaluate results in the context of the situation and reflect on whether the results make sense
• Evaluate the utility of models to determine which models are most useful and efficient to solve problems</td></tr>
<tr><td colspan="2"><u>Learning Design</u>
• Provides problem situations that apply to everyday life
• Provides rich tasks that focus on conceptual understanding, relationships, etc.</td></tr>
</table>

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>
CCSS.Math.Practice.MP5</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to use appropriate tools strategically.</td><td><u>Proficiency Scale</u>
4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal.

3: Student demonstrates mastery with the learning goal as evidenced by combining various tools to explore and solve a problem as well as justifying his/her tool selection and problem solution.

2: Student demonstrates he/she is nearing proficiency by selecting from a variety of provided tools the ones that can be used to solve a problem and explaining his/her reasoning for the selection.

1: Student demonstrates a limited understanding or skill with the learning goal by using the appropriate tool, when provided, to find a solution.</td></tr>
<tr><td colspan="2"><u>Learning Targets</u>
• Consider the available tools (including, but not limited to estimation, graph paper, manipulatives, table, list, etc.) when solving a mathematical problem and decide when certain tools might be helpful
• For example, they may use unit cubes to fill a rectangular prism and a ruler to measure the dimensions
• Use graph paper to accurately create graphs and solve problems or make predictions from real world data</td></tr>
<tr><td colspan="2"><u>Learning Design</u>
• Provides a variety of tools and technology for students to explore to deepen their understanding of math concepts
• Provides problem solving tasks that require students to consider a variety of tools for solving (Tools might include pencil/paper, concrete models, manipulatives, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software, etc.)</td></tr>
</table>

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP6

Learning Goal	**Proficiency Scale**
Students will be able to attend to precision.	4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal. 3: Student demonstrates mastery with the learning goal as evidenced by using appropriate symbols, vocabulary, and labeling to communicate effectively and exchange ideas. 2: Student demonstrates he/she is nearing proficiency by incorporating appropriate vocabulary and symbols in most mathematical communications. 1: Student demonstrates a limited understanding or skill with the learning goal by communicating his/her reasoning and solution to others, with support.

Learning Targets

- Use clear and precise language in their discussions with others and in their own reasoning
- Specify units of measure and state the meaning of the symbols used
- Report answers that appropriately address the context of a problem

Learning Design

- Facilitates, encourages and expects precision in communication
- Provides opportunities for students to explain and/or write their reasoning to others

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>
CCSS.Math.Practice.MP7</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to look for and make use of structure.</td><td><u>Proficiency Scale</u>
4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal.

3: Student demonstrates mastery with the learning goal as evidenced by:
• Noticing mathematical expressions as component parts.
• Using mathematical generalizations to identify the most efficient solution to mathematical tasks.

2: Student demonstrates he/she is nearing proficiency by composing and decomposing number situations and relationships in order to simplify solutions.

1: Student demonstrates a limited understanding or skill with the learning goal by looking for structure or patterns within mathematics to help him/her solve problems efficiently.</td></tr>
<tr><td colspan="2"><u>Learning Targets</u>
• Look closely to discover a pattern or structure
o For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals.
• Examine numerical patterns and relate them to a rule or a graphical representation</td></tr>
<tr><td colspan="2"><u>Learning Design</u>
• Provides opportunities and time for students to explore patterns and relationships to solve problems
• Provides rich tasks and facilitates pattern seeking and understanding of relationships in numbers rather than following a set of steps and/or procedures</td></tr>
</table>

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>
CCSS.Math.Practice.MP8</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to look for and express regularity in repeated reasoning.</td><td><u>Proficiency Scale</u>
4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal.

3: Student demonstrates mastery with the learning goal as evidenced by:
• Connecting prior knowledge to an unfamiliar mathematical situation.
• Creating a model or equation that unifies the various aspects of a problem.
• Noticing patterns, making generalizations, and predicting patterns.

2: Student demonstrates he/she is nearing proficiency by finding and explaining patterns.

1: Student demonstrates a limited understanding or skill with the learning goal by connecting prior knowledge to new situations and noticing patterns with prompting from a teacher or peer.</td></tr>
<tr><td colspan="2"><u>Learning Targets</u>
• Notice repetitive actions in computation and look for more shortcut methods
• Use repeated reasoning to understand algorithms and make generalizations about patterns</td></tr>
<tr><td colspan="2"><u>Learning Design</u>
• Provides problem situations that allow students to explore regularity and repeated reasoning
• Provides rich tasks that encourage students to use repeated reasoning to form generalizations and provides opportunities for students to communicate these generalizations</td></tr>
</table>

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.A.1
CCSS.Math.Content.HSF-IF.A.2
CCSS.Math.Content.HSF-IF.A.3
CCSS.Math.Content.HSF-IF.B.4
CCSS.Math.Content.HSF-IF.B.5
CCSS.Math.Content.HSF-IF.B.6
CCSS.Math.Content.HSF-IF.C.7
CCSS.Math.Content.HSF-IF.C.7a
CCSS.Math.Content.HSF-IF.C.7b

Learning Goal	**Proficiency Scale**

Students will be able to graph and analyze functions including transformations.	4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal.
	
	3: Student demonstrates mastery with the learning goal as evidenced by:
	• Identifying the domain and range of a function.
	• Finding the domain of a function given the range.
	• Graphing a function.
	• Writing linear functions.
	
	2: Student demonstrates he/she is nearing proficiency by:
	• Recognizing and recalling specific vocabulary such as: input, output, domain, range, function, linear, quadratic, exponential, functional notation f(x), intercepts, increasing, decreasing, maximum, minimum, behavior, zeros, inverse functions, square roots, cube roots, piece-wise function, arithmetic, geometric, recursive, explicit.
	• Performing processes such as:
	  ○ Creating a table of values.
	  ○ Determining if a function is linear, quadratic, exponential or absolute value.

	○ Finding the range of a function given the domain. ○ Determine if a relation is a function given a table of values. ○ Recognizing maximum, minimum, and y-intercepts from a graph. 1: Student demonstrates a limited understanding or skill with the learning goal by determining if a graph is a function using the vertical line test.

Learning Targets

- Understand the concept of a function and use function notation
- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range; if f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x; the graph of f is the graph of the equation y = f(x)
- Interpret functions that arise in applications in terms of a context
- Analyze functions using different representations
- Build a function that models a relationship between two quantities
- Build new functions from existing functions
- Interpret expressions for functions in terms of the situation they model

Learning Design

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>
CCSS.Math.Content.HSA-REI.A.1
CCSS.Math.Content.HSA-REI.A.2
CCSS.Math.Content.HSA-REI.B.3
CCSS.Math.Content.HSA-SSE.A.1a</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to solve, write, and graph equations and inequalities.</td><td><u>Proficiency Scale</u>
4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal.

3: Student demonstrates mastery with the learning goal as evidenced by:
• Creating and manipulating linear equations in multiple forms.
• Solving multi-step equations.
• Solving and graphing linear inequalities.

2: Student demonstrates he/she is nearing proficiency by:
• Recognizing and recalling specific vocabulary such as: linear equation, inequality, equations, slope-intercept form, y-intercepts, x-intercepts, coefficient, constant, variable, coordinate plane, quadrants, distributive property, collecting like terms, point-slope form, standard form, half-plane, undefined, parallel, perpendicular
• Performing processes such as:
o Writing and graphing equations when given slope and y-intercept.
o Finding the slope from two ordered pairs.
o Identifying the intercepts and slope from a graph of a line.
o Solving 1- and 2- step equations.
o Solving and graphing 1- and 2- step inequalities.

1: Student demonstrates a limited understanding or skill with the learning goal by finding the slope and y-intercept from an equation in slope-intercept form.</td></tr>
</table>

Learning Targets
• Represent and solve equations and inequalities graphically • Construct and compare linear, quadratic, and exponential models and solve problems • Interpret parts of an expression, such as terms, factors, and coefficients • Create equations and inequalities in one variable and use them to solve problems • Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales
Learning Design

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSA-REI.C.5
CCSS.Math.Content.HSA-REI.C.6
CCSS.Math.Content.HSA-REI.D.10
CCSS.Math.Content.HSA-REI.D.12

Learning Goal	**Proficiency Scale**
Students will be able to solve systems of equations and inequalities.	4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal. 3: Student demonstrates mastery with the learning goal as evidenced by: • Solving systems of linear equations by substitution and elimination. • Solving a system of linear inequalities. • Solving a system of linear and quadratic equations algebraically and graphically. • Writing and solving a system of linear equations. 2: Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary such as: system of equations, point of intersection, parallel, infinite solutions, substitution, elimination, solution set, half-planes, coefficient • Performing processes such as solving systems of linear equations by graphing. 1: Student demonstrates a limited understanding or skill with the learning goal.

Learning Targets

- Solve systems of equations and inequalities
- Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions
- Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables
- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane
- Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph

the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes
Learning Design

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>
CCSS.Math.Content.HSN-RN.A.1
CCSS.Math.Content.HSN-RN.A.2
CCSS.Math.Content.HSA-APR.A.1</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to simplify exponential and polynomial expressions.</td><td><u>Proficiency Scale</u>
4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal.

3: Student demonstrates mastery with the learning goal as evidenced by:
• Multiplying polynomials by polynomials.
• Simplifying expressions using multiple rules of exponents.
• Factoring complex polynomials.

2: Student demonstrates he/she is nearing proficiency by:
• Recognizing and recalling specific vocabulary such as: polynomials, monomials, binomials, trinomials, base, exponent, coefficient, constant, greatest common factor, terms, like terms, expression, degree of term, degree of polynomial, factor
• Performing processes such as:
o Adding and subtracting polynomials.
o Multiplying a polynomial by a monomial.
o Writing a polynomial in standard form and determine degree of polynomial.
o Simplifying expressions using a single rule of exponents.

1: Student demonstrates a limited understanding or skill with the learning goal.</td></tr>
</table>

Learning Targets
• Extend the properties of exponents to rational exponents • Perform arithmetic operations on polynomials
Learning Design

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>

CCSS.Math.Content.HSF-IF.C.8
CCSS.Math.Content.HSF-IF.C.8a
CCSS.Math.Content.HSA-REI.B.4
CCSS.Math.Content.HSA-REI.B.4a
CCSS.Math.Content.HSA-REI.B.4b</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to model, analyze and solve quadratic relationships.</td><td><u>Proficiency Scale</u>
4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal.

3: Student demonstrates mastery with the learning goal as evidenced by:
• Solving quadratic equations using factoring, quadratic formula, completing the square, and taking square roots.
• Creating and using quadratic equations to model a situation.
• Graphing quadratic equations and identify intercepts and maximum or minimum.
• Representing a quadratic relationship in equivalent forms (graph, table, and equation)

2: Student demonstrates he/she is nearing proficiency by:
• Recognizing and recalling specific vocabulary such as: maximum, minimum, zeros, intercepts, quadratic, square root, factor, equivalent, parabola, and symmetry.
• Performing processes such as:
o Solving quadratic equations using the quadratic formula.
o Graphing quadratic equations using a table.

1: Student demonstrates a limited understanding or skill with the learning goal.</td></tr>
</table>

Learning Targets

- Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context
- Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function
- Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions
- Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions

Learning Design

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>
CCSS.Math.Content.HSS-ID.A.1
CCSS.Math.Content.HSS-ID.A.2
CCSS.Math.Content.HSS-ID.A.3
CCSS.Math.Content.HSS-ID.B.6
CCSS.Math.Content.HSS-ID.B.6c
CCSS.Math.Content.HSS-ID.C.7</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to summarize, represent and interpret data.</td><td><u>Proficiency Scale</u>
4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal.

3: Student demonstrates mastery with the learning goal as evidenced by:
• Representing data on a scatterplot and determine line of best fit.
• Determining a set of data to fit specified measures of center.
• Interpreting the meaning of the slope and intercept of a linear model in the context of the data.

2: Student demonstrates he/she is nearing proficiency by:
• Recognizing and recalling specific vocabulary such as: positive correlation, negative correlation, no correlation, outlier, scatterplot, mean, median, mode, interquartile range, slope, intercept, line of best fit, range.
• Performing processes such as:
o Determining the type of correlation for a given data set.
o Calculating the measures of center for a given data set.

1: Student demonstrates a limited understanding or skill with the learning goal by creating a histogram, box plot, dot plot, and scatterplot given a set of data.</td></tr>
</table>

Learning Targets

- Represent data with plots on the real number line (dot plots, histograms, and box plots)
- Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range) of two or more different data sets
- Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers)
- Represent data on two quantitative variables on a scatter plot, and describe how the variables are related
- Fit a linear function for a scatter plot that suggests a linear association
- Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data

Learning Design

WGSD Curriculum – Algebra 1
DRAFT
Mathematical Practices

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>
CCSS.Math.Content.HSA-CED.A.4,
CCSS.Math.Content.HSA-SSE.B.3,
CCSS.Math.Content.HSF-IF.C.8</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to manipulate equations.</td><td><u>Proficiency Scale</u>
4: Student demonstrates an in-depth or advanced application, or innovates with the learning goal.

3: Student demonstrates mastery with the learning goal as evidenced by:
• Solving an equation for a single variable.
• Identifying which variable to solve for in a given situation.
• Rewriting equations in other forms (standard ax+by=c, slope-intercept y=mx+b, point-slope $y-y_1 = m(x-x_1)$, vertex $y=a(x-h)^2+k$).

2: Student demonstrates he/she is nearing proficiency by:
• Recognizing and recalling specific vocabulary such as: formula, variable, solve, function, simplify
• Performing a process such as isolating a variable in an equation, without completely simplifying.

1: Student demonstrates a limited understanding or skill with the learning goal.</td></tr>
<tr><td colspan="2"><u>Learning Targets</u>
• Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law V = IR to highlight resistance R
• Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales
• Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression
• Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function</td></tr>
</table>

Learning Design

Sources:
http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/11/Smarter-Balanced-Math-ALDs.pdf
http://www.corestandards.org/Math
http://katm.org/wp/wp-content/uploads/flipbooks/
http://itembank.marzanoresearch.com/search_details.aspx
http://www.ocf.berkeley.edu/~mimiyang/misc/rubric.pdf

WGSD Curriculum -- Geometry
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
CCSS.Math.Content.HSG-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
CCSS.Math.Content.HSG-CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
CCSS.Math.Content.HSG-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
CCSS.Math.Content.HSG-SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor:

Learning Goal	**Proficiency Scales**
Students will be able to identify and apply transformations of figures in the coordinate plane.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • drawing a reflection, translation, rotation, and dilation of objects in the coordinate plane. • analyzing figures in terms of their symmetries using concepts of reflection, rotation, and translation and combinations of these. • explaining the effects of each transformation. • 2. Student demonstrates he/she is nearing proficiency by • recognizing and recalling specific vocabulary, such as *angle of rotation, dilation, line of symmetry, reflection, rotation, rotational symmetry, translation.* • performing specific processes such as ▪ identifying the transformation applied to an object.

	▪ reflecting an object across an axis. ▪ rotating an object 180 degrees. ▪ translating an object horizontally, vertically, or diagonally. 1. Students will demonstrate limited understanding of the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Geometry
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.
CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.

Learning Goal	**Proficiency Scales**
Students will be able to apply laws of logic to analyze and write arguments.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • applying inductive and deductive reasoning to form conjectures. • verifying conjectures by applying geometric theorems or rejecting them using counterexamples. • analyzing and writing conditional statements and determining their validity. • applying mathematical methods of proof to develop justifications for basic theorems. 2. Student demonstrates he/she is nearing proficiency by: • recognizing and recalling specific vocabulary, such as *conclusion, conditional statement, conjecture, contrapositive, converse, counterexample, hypothesis, inverse, negation, postulate, proof, theorem, truth value.* • performing specific processes such as o identifying the converse, contrapositive, and inverses of a conditional statement. o translating a conditional statement into an if-then statement. o determining the validity of a conditional statement.

	• Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum -- Geometry
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
CCSS.Math.Content.HSG-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
CCSS.Math.Content.HSG-CO.C.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*.

Learning Goal	**Proficiency Scales**
Students will identify congruent angles and congruent triangles and will use congruency as a foundation for formal proofs.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • using theorems to prove triangle congruence. • explaining what criteria are needed to prove two or more figures are congruent. • applying the properties of congruence to solve problems. • identify congruent angles formed by parallel lines cut by a transversal. 2. Student demonstrates he/she is nearing proficiency by: • recognizing and recalling specific vocabulary, such as *midpoint, bisector, congruent, SAS, ASA, SSS, AAS, vertical angles, included angle, included side, alternate interior angles, corresponding angles, transversal, parallel lines* • performing specific processes such as ▪ writing congruency statements given congruent figures. ▪ identifying congruent sides and angles given a congruency statement. ▪ determining the appropriate congruence theorem for a given problem. 1. Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum -- Geometry
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
CCSS.Math.Content.HSG-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
CCSS.Math.Content.HSG-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
CCSS.Math.Content.HSG-C.A.1 Prove that all circles are similar.

Learning Goal	**Proficiency Scales**
The student will be able to identify similar figures and use similarity to solve problems.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • examining polygons and determining if they are similar. • creating proportions and using them to solve problems. 2. Student demonstrates he/she is nearing proficiency by: • recognizing and recalling specific vocabulary, such as *proportion, ratio, scale factor, similar polygons*. • performing specific processes such as • identifying criteria for similar polygons. • setting up proportions needed to solve problems. • identifying proportional segments and congruent angles given a similarity statement. 1. Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum -- Geometry
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
CCSS.Math.Content.HSG-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.
CCSS.Math.Content.HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Learning Goal	**Proficiency Scales**
Students will be able to define trigonometric ratios and solve problems involving right triangles.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • explaining and using the trigonometric ratios (sine, cosine, and tangent) to find missing sides and angles of right triangles. • using the Pythagorean Theorem to determine if a triangle is a right triangle. • applying the properties of special right triangles. 2. Student demonstrates he/she is nearing proficiency by: • recognizing and recalling specific vocabulary, such as *angle of depression, angle of elevation, cosine, Pythagorean triple, sine, tangent, 30-60-90 triangle, 45-45-90 triangle.* • performing specific processes such as ▪ identifying the appropriate trigonometric ratio to use given a figure. ▪ using the Pythagorean Theorem to find missing sides of a right triangle. 1. Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum -- Geometry
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
CCSS.Math.Content.HSG-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
CCSS.Math.Content.HSG-C.A.4 (+) Construct a tangent line from a point outside a given circle to the circle.
CCSS.Math.Content.HSG-C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
CCSS.Math.Content.HSG-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Learning Goal	**Proficiency Scales**
Students will understand and apply properties of circles.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • using the properties of tangent lines to solve problems. • applying the circumference and area formulas to find missing parts. • explaining that pi is the ratio of the circumference to the diameter of any circle. • finding arc length and sector area of a circle and providing the answer in exact and approximate measurements. • 2. Student demonstrates he/she is nearing proficiency by: • recognizing and recalling specific vocabulary, such as *arc length, center, chord, circumference, diameter, pi, radius, sector area, tangent.* • performing specific processes such as

	○ finding circumference and area of circles. ○ identifying the parts of a circle. • Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum -- Geometry
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone
CCSS.Math.Content.HSG-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects
CCSS.Math.Content.HSG-MG.A.2 Apply concepts of density based on area and volume in modeling situations

Learning Goal	**Proficiency Scales**
Students will be able to justify two and three dimensional measurement formulas and apply the formulas to solve problems.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • explaining how the formulas are derived and why they work. • applying the measurement formulas to find missing parts of two and three dimensional figures. • applying the measurement formulas to compare two or more shapes' areas and/or volumes. • finding the areas and/or volumes of composite figures. 2. Student demonstrates he/she is nearing proficiency by: • recognizing and recalling specific vocabulary, such as *area, base,* • *composite figure, cone, cylinder, height, lateral area, parallelogram,* • *rectangle, slant height, sphere, surface area, trapezoid, triangle*. • performing specific processes such as • finding perimeter and area of two dimensional shapes. • finding the surface area and volume of three dimensional solids. 1. Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum – Geometry
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
CCSS.Math.Content.HSG-CO.C.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*.

Learning Goal	**Proficiency Scales**
The students will be able to use coordinate geometry to identify relationships among lines and polygons.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • using slope and distance formulas to classify polygons. • writing equations of parallel and perpendicular lines. 2. Student demonstrates he/she is nearing proficiency by: • recognizing and recalling specific vocabulary, such as *parallel, perpendicular, slope, distance, quadrilateral, parallelogram, rectangle, square, trapezoid, opposite, reciprocal.* • performing specific processes such as • determining if lines are parallel or perpendicular. • calculating the slope between two points. 1. Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum – Honors Geometry (no change)
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
CCSS.Math.Content.HSG-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
CCSS.Math.Content.HSG-CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
CCSS.Math.Content.HSG-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
CCSS.Math.Content.HSG-SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor.

Learning Goal	**Proficiency Scales**
Students will be able to identify and apply transformations of figures in the coordinate plane.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • drawing a reflection, translation, rotation, and dilation of objects in the coordinate plane. • analyzing figures in terms of their symmetries using concepts of reflection, rotation, and translation and combinations of these. • explaining the effects of each transformation. 2. Student demonstrates he/she is nearing proficiency by: • recognizing and recalling specific vocabulary, such as: angle of rotation, dilation, line of symmetry, reflection, rotation, rotational symmetry,

	translation.
	• performing specific processes such as:
	  • identifying the transformation applied to an object.
	  • reflecting an object across an axis.
	  • rotating an object 180 degrees.
	  • translating an object horizontally, vertically, or diagonally.
	
	1. Students will demonstrate limited understanding of the learning goal.

Learning Targets
Learning Design

WGSD Curriculum – Honors Geometry (changed)
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.
CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.

Learning Goal	**Proficiency Scales**
Students will be able to apply laws of logic to analyze and write arguments.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • applying inductive and deductive reasoning to form conjectures. • verifying conjectures by applying geometric theorems or rejecting them using counterexamples. • analyzing and writing conditional statements and determining their validity. • applying mathematical methods of proof (direct reasoning) to develop justifications for theorems. • applying the concept of indirect proof to establish the truth of a proposition by showing that the proposition's being false would imply a contradiction. 2. Student demonstrates he/she is nearing proficiency by: • recognizing and recalling specific vocabulary, such as: conclusion, conditional statement, conjecture, contrapositive, converse, counterexample, hypothesis, inverse, negation, postulate, proof, indirect proof, theorem, truth value. • performing specific processes such as: o identifying the converse and contrapositive of a conditional

	statement. ○ translating a conditional statement into an if-then statement. ○ determining the validity of a conditional statement. 1. Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum – Honors Geometry (changed)
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
CCSS.Math.Content.HSG-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
CCSS.Math.Content.HSG-CO.C.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*.

Learning Goal	**Proficiency Scales**

Students will understand congruence and use triangle congruence as a foundation for formal proof.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.
	
	3. Student demonstrates mastery of the learning goal by:
	• using theorems to prove triangle congruence.
	• explaining what criteria are needed to prove two or more figures are congruent.
	• applying the properties of congruence to solve problems.
	
	2. Student demonstrates he/she is nearing proficiency by:
	• recognizing and recalling specific vocabulary, such as: midpoint, bisector, congruent, SAS, ASA, SSS, AAS, HL, vertical angles, included angle, included side.
	• performing specific processes such as:
	  • writing congruency statements given congruent figures.
	  • identifying congruent sides and angles given a congruency statement.
	  • determining the appropriate congruence theorem for a given problem.

	1. Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum – Honors Geometry (changed)
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
CCSS.Math.Content.HSG-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
CCSS.Math.Content.HSG-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
CCSS.Math.Content.HSG-C.A.1 Prove that all circles are similar.

Learning Goal	**Proficiency Scales**

The student will be able to identify similar figures and use similarity to solve problems.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.
	3. Student demonstrates mastery of the learning goal by:
	• examining polygons and determining if they are similar.
	• creating proportions and using them to solve problems.
	• inspecting the effect of similarity on parts of triangles such as medians, altitudes, and perpendicular and angle bisectors.
	• analyzing similar figures and describing the resulting effects on perimeter and area when dimensions of a shape are changed proportionally.
	• 2. Student demonstrates he/she is nearing proficiency by:
	• recognizing and recalling specific vocabulary, such as: proportion, ratio, scale factor, similar polygons, SAS~, SSS~, AA~.
	• performing specific processes such as:
	  • identifying criteria for similar polygons.
	  • setting up proportions needed to solve problems.

	• identifying proportional segments and congruent angles given a similarity statement. 1. Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum – Honors Geometry (changed)
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
CCSS.Math.Content.HSG-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.
CCSS.Math.Content.HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Learning Goal	**Proficiency Scales**
Students will be able to define trigonometric ratios and solve problems involving right triangles.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • explaining and using the trigonometric ratios (sine, cosine, and tangent) to find missing sides and angles of right triangles. • using the Pythagorean Theorem to determine if a triangle is a right triangle. • applying the properties of special right triangles. 2. Student demonstrates he/she is nearing proficiency by: • recognizing and recalling specific vocabulary, such as: angle of depression, angle of elevation, cosine, Pythagorean triple, sine, tangent, secant, cosecant, cotangent, reciprocal function, 30-60-90 triangle, 45-45-90 triangle. • performing specific processes such as: o identifying all six trigonometric ratios for a given figure. o using the Pythagorean Theorem to find missing sides of a right triangle. 1. Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum – Honors Geometry (changed)
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
CCSS.Math.Content.HSG-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
CCSS.Math.Content.HSG-C.A.4 (+) Construct a tangent line from a point outside a given circle to the circle.
CCSS.Math.Content.HSG-C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
CCSS.Math.Content.HSG-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Learning Goal	**Proficiency Scales**
Students will understand and apply properties of circles.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • using the properties of tangent lines to solve problems. • applying the circumference and area formulas to find missing parts. • explaining that pi is the ratio of the circumference to the diameter of any circle. • finding arc length and sector area of a circle and providing the answer in exact and approximate measurements. • finding degree measures of inscribed angles, intercepted arcs, and angles formed by lines intersecting inside, on, or outside the circle.

	2. Student demonstrates he/she is nearing proficiency by: • recognizing and recalling specific vocabulary, such as: minor arc, major arc, arc length, center, chord, secant, circumference, diameter, pi, radius, sector area, tangent, inscribed angles, inscribed and circumscribed polygons. • performing specific processes such as: o finding circumference and area of circles. o identifying the parts of a circle. 1. Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum – Honors Geometry (changed)
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone
CCSS.Math.Content.HSG-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects
CCSS.Math.Content.HSG-MG.A.2 Apply concepts of density based on area and volume in modeling situations

Learning Goal	**Proficiency Scales**
Students will be able to justify two and three dimensional measurement formulas and apply the formulas to solve problems.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • explaining how the formulas are derived and why they work. • applying the measurement formulas to find missing parts of two and three dimensional figures. • applying the measurement formulas to compare two or more shapes' areas and/or volumes. • finding the areas and/or volumes of composite figures. 2. Student demonstrates he/she is nearing proficiency by: • recognizing and recalling specific vocabulary, such as: area, base, composite figure, cone, cylinder, height, lateral area, parallelogram, rectangle, slant height, sphere, surface area, trapezoid, triangle, pyramid, oblique. • performing specific processes such as: o finding perimeter and area of two dimensional shapes. o finding the surface area and volume of three dimensional solids. 1. Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum – Honors Geometry (changed)
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
CCSS.Math.Content.HSG-CO.C.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*.

Learning Goal	**Proficiency Scales**

The students will be able to use coordinate geometry to identify relationships among lines and polygons.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.
	3. Student demonstrates mastery of the learning goal by:
	• using slope and distance formulas to classify polygons.
	• writing equations of parallel and perpendicular lines.
	2. Student demonstrates he/she is nearing proficiency by:
	• recognizing and recalling specific vocabulary, such as: parallel, perpendicular, slope, distance, quadrilateral, parallelogram, rectangle, square, trapezoid, kite, rhombus, diagonals, opposite, reciprocal.
	• performing specific processes such as:
	  • determining if lines are parallel or perpendicular.
	  • calculating the slope between two points.
	1. Student demonstrates limited understanding of the learning goals.

Learning Targets
Learning Design

WGSD Curriculum -- Integrated Math

DRAFT

Graphing

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

• CCSS.Math.Content.HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

• CCSS.Math.Content.HSA-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

• CCSS.Math.Content.HSA-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Learning Goal	**Proficiency Scale**
Students will be able to solve and graph linear equations and inequalities.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: Student demonstrates mastery with the learning goal as evidenced by: ● creating and manipulating linear equations in multiple forms. ● solving multi-step equations. ● solving and graphing linear inequalities. 2: Student demonstrates he/she is nearing proficiency by: ● recognizing and recalling specific vocabulary, such as *linear equation, inequality, equations, slope-intercept form, y-intercepts, x-intercepts, coefficient, constant, variable, coordinate plane, quadrants, distributive property, collecting like terms,*

	standard form, undefined, parallel, perpendicular. ● performing specific processes, such as: ○ writing and graphing equations when given slope and y-intercept. ○ finding the slope from two ordered pairs. ○ identifying the intercepts and slope from a graph of a line. ○ solving 1- and 2- step equations. ○ solving and graphing 1- and 2- step inequalities. 1: Student demonstrates a limited understanding or skill with the learning goal.

Learning Targets

- Represent and solve equations and inequalities graphically
- Interpret parts of an expression, such as terms, factors, and coefficients
- Create equations and inequalities in one variable and use them to solve problems.
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales

Learning Design

WGSD Curriculum -- Integrated Math

DRAFT

Quadratic Equations Algebra

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSA-REI.B.4 Solve quadratic equations in one variable.
CCSS.Math.Content.HSA-REI.B.4b Solve quadratic equations by inspection (e.g., for x2 completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Learning Goal	**Proficiency Scale**
Students will be able to solve quadratic equations using algebraic methods.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3 Student demonstrates mastery with the learning goal as evidenced by: ● deducing that each step in solving an equation is equivalent to the previous step because of the properties of rational and irrational numbers. ● proving these equivalent equations have the same solutions as the original equation. ● developing equivalent forms of an expression to solve by factoring. ● applying the quadratic formula to solve quadratic equations. ● recognizing the solutions to a quadratic equation are the x-intercepts (zeros) of a quadratic function and being able to find them using various algebraic methods. ● justifying the most appropriate method to solve a given quadratic. The student demonstrates he/she is nearing proficiency by ● recognizing or recalling specific vocabulary, such as *quadratic in standard form, zeros, factors, maximum, minimum, vertex form, discriminant, and completing the square.*

	● performing specific processes, such as: ○ writing a quadratic in standard form. ○ solving quadratics by factoring and finding the zeros. ○ recognizing the vertex form of a quadratic. ○ writing a quadratic in standard form. 1: The student demonstrates a limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum - Integrated Math

DRAFT

Quadratic Functions

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

CCSS.Math.Content.HSA-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

CCSS.Math.Content.HSA-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Learning Goal	**Proficiency Scale**
Students will be able to graph quadratic functions.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: Student demonstrates mastery with the learning goal as evidenced by: ● graphing quadratic equations and identify intercepts and maximum or minimum. ● representing a quadratic relationship in equivalent forms (graph, table, and equation) 2: Student demonstrates he/she is nearing proficiency by: ● recognizing and recalling specific vocabulary such as: *maximum, minimum, zeros, intercepts, quadratic, parabola, and symmetry.*

	● performing processes such as graphing quadratic equations using a table. 1: Student demonstrates a limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Integrated Math
DRAFT
Systems of Equations

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

A-REI 5-7 • CCSS.Math.Content.HSA-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

•CCSS.Math.Content.HSA-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

CCSS.Math.Content.HSA-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.

Learning Goal	**Proficiency Scale**
Students will be able to solve systems of equations with two or three variables and solve systems of inequalities in two variables.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● modeling real world situations using systems ● evaluating the most efficient method (elimination, substitution, and graphing) for solving a given system in two variables. ● solving a system using these methods with no errors. ● solving a system with three variables using elimination with no errors. 2 The student demonstrates he/she is nearing proficiency by ● recognizing or recalling specific vocabulary, such as i*nconsistent, consistent system, elimination, substitution, graphing, and point of intersection.*

	● performing specific processes, such as
	  ● solving a system algebraically using substitution, elimination, and inverse matrices but with some errors on the more complex systems.
	  ● stating that the point of intersection of the graphs is the solution to a system.
	
	1: The student demonstrates a limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Algebra 2
DRAFT
Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSS-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
CCSS.Math.Content.HSS-CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
CCSS.Math.Content.HSS-CP.A.3 Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
CCSS.Math.Content.HSS-CP.B.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.
CCSS.Math.Content.HSS-CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
CCSS.Math.Content.HSS-CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
CCSS.Math.Content.HSS-CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
CCSS.Math.Content.HSS-MD.B.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
CCSS.Math.Content.HSS-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Learning Goal	**Proficiency Scales**
Students will be able to use probability to compute the	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

likelihood an event will occur.	3: The student demonstrates mastery of the learning goal by: ● using the Fundamental Counting Principle, combinations, and permutations to compute probabilities of compound events and solve realistic problems in both familiar and unfamiliar contexts. ● calculating expected values and using them to solve problems. ● finding conditional probability in various ways, including the use of the Multiplication Rule, Addition Rule, and probability trees. ● interpreting Venn diagrams and using unions, complement, and intersections to find probabilities. ● calculating and interpreting the measures of central tendency, variance, and standard deviation of a distribution of data. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as: *sample, variance, mean, median, mode, permutation, combination, success, failure, sample space, random variable, expected value, experiment, population, complement, union and intersection.* ● performing specific processes, such as ○ solving basic probability in real world situations. ○ determining whether an event is the *union* or *intersection* of two other events. 1: The student demonstrates limited understanding or skill with the learning goal.

<u>Learning Targets</u>

Learning Design

WGSD Curriculum -- Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).

CCSS.Math.Content.HSF-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

CCSS.Math.Content.HSF-BF.B.3 Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*),*f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

CCSS.Math.Content.HSF-BF.B.4 Find inverse functions.

Learning Goal	**Proficiency Scales**
Students will be able to use functional notation and perform operations on functions including compositions.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● adding, subtracting, multiplying, and dividing functions and verifying the properties of the outcome with respect to domain and range. ● generating all compositions of functions without error. ● finding the inverse of a function and determining whether two functions or relations are inverses of each other. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as sum, product, quotient, difference,

	substitution, composition, evaluate, domain, range, element, input, output, function, and relation. ● performing specific processes, such as: ○ Adding, subtracting, multiplying, and dividing functions. ○ Finding compositions with numerical inputs. ○ Finding the inverse of a function or relation. ○ Finding compositions with no major errors regarding the simpler functions, but some errors or omissions regarding the more complex functions. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSN-VM.C.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
CCSS.Math.Content.HSN-VM.C.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
CCSS.Math.Content.HSN-VM.C.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.
CCSS.Math.Content.HSN-VM.C.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
CCSS.Math.Content.HSN-VM.C.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
CCSS.Math.Content.HSA-REI.C.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.
CCSS.Math.Content.HSA-REI.C.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Learning Goal	**Proficiency Scales**
Students will be able to perform the basic operations on matrices.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● analyzing, organizing, and representing data using matrices. ● adding, subtracting, and multiplying (scalar and matrix) matrices in complex problems by hand as well as with a graphing calculator. ● solving systems of linear equations by applying the properties of inverse matrices using a graphing calculator.

	2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as matrix, element, dimensions, row, column, square matrix, zero matrix, identity matrix, scalar, coefficient matrix, variable matrix, inverse matrix, and determinant. ● performing specific processes, such as: ○ adding, subtracting, and multiplying matrices by a scalar. ○ recognizing matrix dimensions that are necessary to perform operations. ○ using the properties of matrix multiplication with no major errors. ○ recognizing that systems of equations can be solved using inverse matrices. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSN-CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.
CCSS.Math.Content.HSA-SSE.B.3a Factor a quadratic expression to reveal the zeros of the function it defines.
CCSS.Math.Content.HSA-SSE.B.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
CCSS.Math.Content.HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
CCSS.Math.Content.HSA-REI.B.4 Solve quadratic equations in one variable.
CCSS.Math.Content.HSA-REI.B.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
CCSS.Math.Content.HSA-REI.B.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Learning Goal	**Proficiency Scales**
Students will be able to solve quadratic equations using all algebraic methods.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● explaining that each step in solving an equation is equivalent to the previous step because of the properties of real numbers. ● developing equivalent forms of an expression to solve by factoring. ● developing equivalent forms of an equation and solving by completing the square. ● deriving and applying the quadratic formula to solve quadratic equations. ● recognizing that the solutions to a quadratic equation are the x-intercepts (zeros) of a quadratic function and finding them using various algebraic methods. ● justifying the most appropriate method to solve a given quadratic equation. ● finding the maximum and minimum of a quadratic function by analyzing the vertex form of a

	quadratic function. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as quadratic function in standard form, zeros, factors, maximum, minimum, vertex form, discriminant, and completing the square. ● performing specific processes, such as: ○ writing a quadratic function in standard form. ○ solving quadratic equations by completing the square. ○ solving quadratic equations by factoring and finding the zeros. ○ solving quadratic equations by using the quadratic formula. ○ recognizing the vertex form of a quadratic function. ○ finding the value of the discriminant of a quadratic equation. ○ writing a quadratic function in standard form. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Algebra 2
DRAFT
Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSA-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.
CCSS.Math.Content.HSA-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
CCSS.Math.Content.HSA-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
CCSS.Math.Content.HSA-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.
CCSS.Math.Content.HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
CCSS.Math.Content.HSA-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Learning Goal	**Proficiency Scales**
Students will be able to solve systems of equations and inequalities.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● modeling real world situations using systems. ● evaluating the most efficient method (elimination, substitution, graphing, and inverse

	matrices) for solving a given system in two variables. ● solving a system using all aforementioned methods with no errors. ● solving a system with three variables using elimination with no errors. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as: inconsistent, consistent system, elimination, substitution, graphing, inverse matrices, and point of intersection ● performing specific processes, such as: ○ solving a system algebraically using substitution, elimination, and inverse matrices but with some errors on the more complex systems. ○ recognizing that the point of intersection of the graphs is the solution to a system. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum - Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.
CCSS.Math.Content.HSF-LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
CCSS.Math.Content.HSF-LE.A.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
CCSS.Math.Content.HSF-LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context.

Learning Goal	**Proficiency Scales**
Students will be able to model real world problems using functions.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● developing a function (linear, quadratic, exponential) to model a relationship between two quantities, and using these models to solve problems in both familiar and unfamiliar contexts. ● interpreting regression equations for a given set of data and making predictions of future events. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as linear regression, best fit line, positive and negative correlation, scatter plot, rate of change, piecewise function, compound function, maximum and minimum. ● performing specific processes, such as ○ creating a function to model a relation between two variables. ○ solving simple real world problems and some complex problems in a familiar context. ○ graphing linear scatter plots and finding regression equations by hand.

	1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum
DRAFT
Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-SRT.C.6: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
CCSS.Math.Content.HSG-SRT.C.7: Explain and use the relationship between the sine and cosine of complementary angles.
CCSS.Math.Content.HSG-SRT.C.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★
CCSS.Math.Content.HSG-SRT.D.9:Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
CCSS.Math.Content.HSG-SRT.D.10:Prove the Laws of Sines and Cosines and use them to solve problems.
CCSS.Math.Content.HSG-SRT.D.11: Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Learning Goal	**Proficiency Scales**
Students will be able to use trigonometry to solve for sides and angles in right and non-right triangles.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● developing trig ratios to solve real world problems. ● applying Law of Sines and Cosines to find missing angles and sides in real world situations. ● identifying and interpreting the ambiguous case for law of Sines. ● evaluating the effectiveness and efficiency of the formula used when solving triangles. ● deriving and using the formula to find the area of a triangle given two sides and the included angle. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as: opposite side, adjacent side,

	hypotenuse, inverse trig ratio, cross-multiplication, sine, cosine, tangent, and ambiguous case. ● performing specific processes, such as: ○ finding missing sides and angles in a right triangle. ○ finding missing sides and angles in a non-right triangle using Law of Sines/Cosines. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
CCSS.Math.Content.HSF-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
CCSS.Math.Content.HSF-TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
CCSS.Math.Content.HSF-TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Learning Goal	**Proficiency Scales**
Students will be able to evaluate trig ratios of angles (in radians and degrees) in the unit circle.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● explaining the relationship between angles and radians. ● creating the unit circle with efficiency. ● interpreting patterns in the trig ratios of angles using the unit circle. ● understanding that sine and cosine represent periodic functions. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as coterminal, reference angle, 30-60-90, 45-45-90, triangle relationships, terminal side, radians, and initial sides.

	● performing specific processes, such as: ○ converting angles from degrees and radians, and from radians to degrees. ○ creating the unit circle. ○ recognizing trig ratios from the unit circle. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.
CCSS.Math.Content.HSF-LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
CCSS.Math.Content.HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
CCSS.Math.Content.HSF-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
CCSS.Math.Content.HSF-LE.A.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
CCSS.Math.Content.HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Learning Goal	**Proficiency Scales**
Students will be able to graph and solve exponential and logarithmic functions.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● explaining the inverse relationship between logarithmic functions and exponential functions. ● graphing logarithmic and exponential functions. ● interpreting real world logarithmic and exponential graphs. ● solving exponential and logarithmic equations. ● deriving equivalent logarithmic expressions using the laws of logarithms.

	● creating exponential functions which model exponential growth and decay. ● explaining how the laws of exponents are used to derive equivalent exponential functions. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as: exponential growth, exponential decay, base, inverse function, common logarithm, laws of logarithms, laws of exponents, base, and exponent. ● performing specific processes, such as: ○ graphing logarithmic and exponential graphs with a few errors or omissions on the more complex functions. ○ recognizing when laws of logarithms are applied. ○ solving exponential and logarithmic equations with a few errors on the more complex equations. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Honors Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSS-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
CCSS.Math.Content.HSS-CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
CCSS.Math.Content.HSS-CP.A.3 Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
CCSS.Math.Content.HSS-CP.B.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.
CCSS.Math.Content.HSS-CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
CCSS.Math.Content.HSS-CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
CCSS.Math.Content.HSS-CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
CCSS.Math.Content.HSS-MD.B.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
CCSS.Math.Content.HSS-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Learning Goal	**Proficiency Scales**
Students will be able to use the rules of probability to compute the likelihood an event will occur.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● using the Fundamental Counting Principle, combinations, and permutations to compute

	probabilities of compound events and solve realistic problems in both familiar and unfamiliar contexts. ● calculating expected values and using them to solve problems. ● finding conditional probability in various ways, including the use of the Multiplication Rule, Addition Rule, and probability trees. ● interpreting Venn diagrams and using unions, complement, and intersections to find probabilities. ● calculating and interpreting the measures of central tendency, variance, and standard deviation of a distribution of data. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as *sample, variance, mean, median, mode, permutation, combination, success, failure, sample space, random variable, expected value, experiment, population, complement, union and intersection.* ● performing specific processes, such as ○ solving basic probability in real world situations. ○ determining whether an event is the *union* or *intersection* of two other events. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Honors Algebra 2
DRAFT
Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).
CCSS.Math.Content.HSF-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
CCSS.Math.Content.HSF-BF.B.3 Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*),*f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
CCSS.Math.Content.HSF-BF.B.4 Find inverse functions.

Learning Goal	**Proficiency Scales**
Students will be able to use functional notation and perform operations on functions including compositions.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● adding, subtracting, multiplying, and dividing functions and verifying the properties of the outcome with respect to domain and range. ● generating all compositions of functions without error. ● finding the inverse of a function and determining whether two functions or relations are inverses of each other. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as s*um, product, quotient, difference,*

	substitution, composition, evaluate, domain, range, element, input, output, function, relation, inverse and function notation.. ● performing specific processes, such as ○ adding, subtracting, multiplying, and dividing functions. ○ finding compositions with numerical inputs. ○ finding the inverse of a function or relation. ○ finding compositions with no major errors regarding the simpler functions, but some errors or omissions regarding the more complex functions. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Honors Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSN-VM.C.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

CCSS.Math.Content.HSN-VM.C.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

CCSS.Math.Content.HSN-VM.C.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.

CCSS.Math.Content.HSN-VM.C.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

CCSS.Math.Content.HSN-VM.C.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

CCSS.Math.Content.HSA-REI.C.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.

CCSS.Math.Content.HSA-REI.C.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Learning Goal	**Proficiency Scales**
Students will be able to perform the basic operations on matrices including addition, subtraction, and multiplication.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● analyzing, organizing, and representing data using matrices. ● adding, subtracting, and multiplying (scalar and matrix) matrices in complex problems by hand as well as with a graphing calculator. ● solving systems of linear equations by applying the properties of inverse matrices by hand as well as using a graphing calculator. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as *matrix, element, dimensions, row, column, square matrix, zero matrix, identity matrix, scalar, coefficient matrix, variable matrix, inverse matrix, and determinant*. ● performing specific processes, such as ○ adding, subtracting, and multiplying matrices by a scalar. ○ recognizing matrix dimensions that are necessary to perform operations. ○ using the properties of matrix multiplication with no major errors. ○ recognizing that systems of equations can be solved using inverse matrices. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Honors Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSN-CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.

CCSS.Math.Content.HSA-SSE.B.3a Factor a quadratic expression to reveal the zeros of the function it defines.

CCSS.Math.Content.HSA-SSE.B.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

CCSS.Math.Content.HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
CCSS.Math.Content.HSA-REI.B.4 Solve quadratic equations in one variable.

CCSS.Math.Content.HSA-REI.B.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

CCSS.Math.Content.HSA-REI.B.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Learning Goal	**Proficiency Scales**
Students will be able to recognize that the solutions to the quadratic equation are the zeros of quadratic function and will be able to find them using all possible algebraic methods.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● explaining that each step in solving an equation is equivalent to the previous step because of the properties of real numbers. ● developing equivalent forms of an expression to solve by factoring. ● developing equivalent forms of an equation and solving by completing the square. ● deriving and applying the quadratic formula to solve quadratic equations. ● recognizing that the solutions to a quadratic equation are the x-intercepts (zeros) of a quadratic function and finding them using various algebraic methods. ● justifying the most appropriate method to solve a given quadratic equation. ● finding the maximum and minimum of a quadratic function by analyzing the vertex form of a quadratic function. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as *quadratic function in standard form, zeros, factors, maximum, minimum, vertex form, discriminant, and completing the square.* ● performing specific processes, such as ○ writing a quadratic function in standard form. ○ solving quadratic equations by completing the square. ○ solving quadratic equations by factoring and finding the zeros. ○ solving quadratic equations by using the quadratic formula. ○ recognizing the vertex form of a quadratic function. ○ finding the value of the discriminant of a quadratic equation. ○ writing a quadratic function in standard form. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Honors Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSA-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.

CCSS.Math.Content.HSA-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

CCSS.Math.Content.HSA-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

CCSS.Math.Content.HSA-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

CCSS.Math.Content.HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

CCSS.Math.Content.HSA-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Learning Goal	**Proficiency Scales**
Students will be able to solve systems of equations with two or three variables and solve systems of inequalities in two variables.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● modeling real world situations using systems. ● evaluating the most efficient method (elimination, substitution, graphing, and inverse matrices) for solving a given system in two variables. ● solving a system using all aforementioned methods with no errors. ● solving a system with three variables using elimination with no errors. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as *inconsistent, consistent system, elimination, substitution, graphing, inverse matrices, and point of intersection.* ● performing specific processes, such as ○ solving a system algebraically using substitution, elimination, and inverse matrices but with some errors on the more complex systems. ○ recognizing that the point of intersection of the graphs is the solution to a system. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets

Learning Design

WGSD Curriculum -- Honors Algebra 2 Trigonometry
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

CCSS.Math.Content.HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

CCSS.Math.Content.HSF-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

CCSS.Math.Content.HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Learning Goal	**Proficiency Scales**
Students will be able to model real world problems using functions.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● developing a function (linear, quadratic, exponential) to model a relationship between two quantities, and using these models to solve problems in both familiar and unfamiliar contexts. ● interpreting regression equations for a given set of data and making predictions of future events. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as *linear regression, best fit line, positive and negative correlation, scatter plot, rate of change, piecewis*e function, compound function, maximum and minimum, linear, quadratic, and exponential.. ● performing specific processes, such as ○ creating a function to model a relation between two variables. ○ solving simple real world problems and some complex problems in a familiar

	context. ○ graphing linear scatter plots and finding regression equations by hand. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Honors Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSG-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

CCSS.Math.Content.HSG-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

CCSS.Math.Content.HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

CCSS.Math.Content.HSG-SRT.D.9 Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

CCSS.Math.Content.HSG-SRT.D.10 Prove the Laws of Sines and Cosines and use them to solve problems.

CCSS.Math.Content.HSG-SRT.D.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Learning Goal	**Proficiency Scales**
Students will be able to use trigonometry to solve for sides and angles in right and non-right triangles.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● developing trig ratios to solve real world problems. ● applying Law of Sines and Cosines to find missing angles and sides in real world situations. ● identifying and interpreting the ambiguous case for law of sines. ● evaluating the effectiveness and efficiency of the formula used when solving triangles. ● deriving and using the formula to find the area of a triangle given two sides and the included angle. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as *opposite side, adjacent side,*

	hypotenuse, inverse trig ratio, cross-multiplication, sine, cosine, tangent, and ambiguous case. ● performing specific processes, such as: ○ finding missing sides and angles in a right triangle. ○ finding missing sides and angles in a non-right triangle using Law of Sines/Cosines. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum - Honors Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

CCSS.Math.Content.HSF-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

CCSS.Math.Content.HSF-TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

CCSS.Math.Content.HSF-TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Learning Goal	**Proficiency Scales**
Students will be able to evaluate trig ratios of angles (in radians and degrees) in the unit circle.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● explaining the relationship between angles and radians. ● creating the unit circle with efficiency. ● interpreting patterns in the trig ratios of angles using the unit circle. ● understanding that sine and cosine represent periodic functions. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as *coterminal, reference angle, 30-60-90, 45-45-90, triangle relationships, terminal side, and initial sides.*

	● performing specific processes, such as ○ converting angles from degrees and radians and vice versa. ○ creating the unit circle. ○ recognizing trig ratios from the unit circle. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Honors Algebra 2

DRAFT

Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.
CCSS.Math.Content.HSF-LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
CCSS.Math.Content.HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
CCSS.Math.Content.HSF-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
CCSS.Math.Content.HSF-LE.A.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
CCSS.Math.Content.HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Learning Goal	**Proficiency Scales**
The student will be able to graph exponential and logarithmic functions and solve for input and output values of exponential and logarithmic functions.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● explaining the inverse relationship between logarithmic functions and exponential functions. ● graphing logarithmic and exponential functions. ● interpreting real world logarithmic and exponential graphs. ● solving exponential and logarithmic equations. ● deriving equivalent logarithmic expressions using the laws of logarithms. ● creating exponential functions which model exponential growth and decay. ● explaining how the laws of exponents are used to derive equivalent exponential functions.

	2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as *exponential growth, exponential decay, base, inverse function, common logarithm, laws of logarithms, laws of exponents, base, and exponent*. ● performing specific processes, such as ○ graphing logarithmic and exponential graphs with a few errors or omissions on the more complex functions. ○ recognizing when laws of logarithms are applied. ○ solving exponential and logarithmic equations with a few errors on the more complex equations. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Honors Algebra 2
DRAFT
Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSN-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents
CCSS.Math.Content.HSN-RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
CCSS.Math.Content.HSN-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational
CCSS.Math.Content.HSN-REI.A.2 Solve simple rational and radical equation in one variable, and give examples showing how extraneous solutions may arise.

Learning Goal	**Proficiency Scales**
Students will be able to simplify expressions with rational exponents or radical form and solve radical equations.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● rewriting expressions with rational exponents or radical form in simplest form. ● adding, subtracting, multiplying and dividing expressions with rational exponents or radical form. ● explaining why an answer is rational or irrational after adding, subtracting, or multiplying real numbers. ● solving radical equations in one variable and giving examples showing how extraneous solutions may arise. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as *index, radicand, power expression, rational number, irrational number and extraneous solutions*. ● performing specific processes, such as:

	○ rewriting radical expressions as power expressions with rational exponents. ○ recognizing when rational exponent will result in rational or irrational numbers. ○ using the properties of exponents to simplify operations of expressions with rational exponents. ○ recognizing what power is necessary to use to solve radical equations. ○ identifying the extraneous solutions to a radical equation. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Honors Algebra 2
DRAFT
Trigonometry

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSN-APR.D.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions. CCSS.Math.Content.HSN-REI.A.2 Solve simple rational and radical equation in one variable, and give examples showing how extraneous solutions may arise.

Learning Goal	**Proficiency Scales**
Students will be able to simplify expressions with rational exponents or radical form and solve radical equations.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● simplifying rational expressions by cancelling common factors. ● adding, subtracting, multiplying and dividing rational expressions. ● solving rational equations in one variable and giving examples showing how extraneous solutions may arise. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as *common factor, least common denominator, clearing the equations of fractions and extraneous solutions*. ● performing specific processes, such as: ○ factoring numerators and denominators of rational expressions. ○ identifying the least common denominator among rational expressions. ○ recognizing what expression is needed to clear a rational equation of fractions. ○ identifying the extraneous solutions to a rational equation

	1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum - Algebra 3
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).

CCSS.Math.Content.HSF-IF.A.1 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

CCSS.Math.Content.HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

CCSS.Math.Content.HSF-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

CCSS.Math.Content.HSF-IF.B.5 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

CCSS.Math.Content.HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Learning Goal	**Proficiency Scales**
Students will be able to write and graph equations.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● identifying the domain and range of a function. ● identifying the domain and range of a set of elements. ● graphing a function. ● writing equations of lines. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as input, output, domain, range, function, linear, quadratic, exponential, functional notation f(x), slope, rate of change, intercepts, behavior, square roots, cube roots, families of functions, absolute value. ● performing specific processes, such as: ○ creating a table of values. ○ determining the family a function belongs to, namely linear, quadratic, exponential, or absolute value. ○ finding the slope between two points ○ writing equations lines ○ graphing linear equations using a table of values and their intercepts ○ recognizing x- and y-intercepts from a graph. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets

Learning Design

WGSD Curriculum -- Algebra 3
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).

CCSS.Math.Content.HSF-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Learning Goal	**Proficiency Scales**
Students will be able to use functional notation and perform operations on functions including compositions.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● adding, subtracting, multiplying, and dividing functions and verifying the properties of the outcome with respect to domain and range. ● generating all compositions of functions without error. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as sum, product, quotient, difference, substitution, composition, evaluate, domain, range, element, input, output, function, and relation. ● performing specific processes, such as: ○ Adding, subtracting, multiplying, and dividing functions. ○ Evaluating functions using specific input values. ○ Finding compositions with numerical inputs. ○ Finding compositions with no major errors regarding the simpler functions, but some errors or omissions regarding the more complex functions.

	1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Algebra 3
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSA-SSE.B.3a Factor a quadratic expression to reveal the zeros of the function it defines.

CCSS.Math.Content.HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

CCSS.Math.Content.HSA-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

CCSS.Math.Content.HSA-REI.B.4 Solve quadratic equations in one variable.

CCSS.Math.Content.HSA-REI.B.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Learning Goal	**Proficiency Scales**
Students will be able to simplify polynomial expressions, along with solving equations using factoring.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery with the learning goal as evidenced by ● multiplying polynomials by polynomials. ● simplifying expressions using multiple rules of exponents. ● factoring complex polynomials. ● developing equivalent forms of an expression to solve by factoring. 2: The student demonstrates he/she is nearing proficiency by: ● Recognizing and recalling specific vocabulary such as: polynomials, monomials, binomials, trinomials, base, exponent, coefficient, constant, greatest common factor,

	terms, like terms, expression, degree of term, degree of polynomial, factor, product, perfect square, perfect cube ● Performing processes such as: o Adding and subtracting polynomials. o Multiplying a polynomial by a monomial. o Writing a polynomial in standard form and determine degree of polynomial. o Simplifying expressions using a single rule of exponents. o Factoring polynomials o Solving polynomials 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSA-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.

CCSS.Math.Content.HSA-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

CCSS.Math.Content.HSA-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

CCSS.Math.Content.HSA-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

CCSS.Math.Content.HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

CCSS.Math.Content.HSA-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Learning Goal	**Proficiency Scales**
Students will be able to solve systems of equations with two or three variables and solve systems of inequalities in two variables.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery of the learning goal by: ● modeling real world situations using systems. ● evaluating the most efficient method (elimination, substitution, graphing) for solving a given system in two variables. ● solving a system using all aforementioned methods with no errors. ● solving a system with three variables using elimination and/or substitution with no errors. 2: The student demonstrates he/she is nearing proficiency by: ● recognizing or recalling specific vocabulary, such as: inconsistent, consistent system, elimination, substitution, graphing, and point of intersection ● performing specific processes, such as: ○ solving a system algebraically using substitution, elimination, but with some errors on the more complex systems. ○ recognizing that the point of intersection of the graphs is the solution to a system. 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum -- Algebra 3
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSA-SSE.B.3a Factor a quadratic expression to reveal the zeros of the function it defines.

CCSS.Math.Content.HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

CCSS.Math.Content.HSA-SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.

CCSS.Math.Content.HSA-APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

CCSS.Math.Content.HSA-APR.D.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

CCSS.Math.Content.HSA-REI.B.4 Solve quadratic equations in one variable.

CCSS.Math.Content.HSA-REI.B.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Learning Goal	**Proficiency Scales**
Students will be able to perform operations, simplify, and solve rational expressions.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery with the learning goal as evidenced by ● simplifying rational expressions ● multiplying, dividing, adding and subtracting rational expressions. ● factoring complex polynomials. ● dividing polynomials 2: The student demonstrates he/she is nearing proficiency by: ● Recognizing and recalling specific vocabulary such as: polynomials, monomials, binomials, trinomials, base, exponent, coefficient, constant, greatest common factor, least common factor, terms, like terms, expression, factor, product, perfect square, perfect cube, complex fraction ● Performing processes such as: o Adding and subtracting rational expressions. o Multiplying and dividing rational expressions o Dividing polynomials. o Simplifying expressions using a single rule of exponents. o Factoring polynomials o Solving polynomials o Determining the least common denominator 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets

Learning Design

WGSD Curriculum -- Algebra 3
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSN-CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.

CCSS.Math.Content.HSN-CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

CCSS.Math.Content.HSN-CN.A.3 Find the conjugate of a complex number, use conjugates to find the moduli and quotients of complex numbers.

CCSS.Math.Content.HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

CCSS.Math.Content.HSA-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may rise.

CCSS.Math.Content.HSA-REI.B.4 Solve quadratic equations in one variable.

CCSS.Math.Content.HSA-REI.B.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Learning Goal	**Proficiency Scales**
Students will be able to simplify expressions and solve equations containing radicals and complex	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3: The student demonstrates mastery with the learning goal as evidenced by

numbers.	● simplifying radicals. ● manipulating radicals and rational exponents. ● adding, subtracting, multiplying and dividing radical expressions. ● solving radical equations. ● adding, subtracting, multiplying, and dividing complex numbers. 2: The student demonstrates he/she is nearing proficiency by: ● Recognizing and recalling specific vocabulary such as: polynomials, base, exponent, extraneous, coefficient, conjugate, constant, terms, like terms, expression, factor, product, complex number, radical, imaginary number ● Performing processes such as: o Adding and subtracting radical expressions and complex numbers. o Multiplying and dividing rational expressions and complex numbers. o Simplifying expressions using a single rule of exponents. o Factoring polynomials o Solving polynomials 1: The student demonstrates limited understanding or skill with the learning goal.

Learning Targets
Learning Design

WGSD Curriculum – Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-BF.A.1 Write a function that describes a relationship between two quantities.★

CCSS.Math.Content.HSF-BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

CCSS.Math.Content.HSF-BF.A.1b Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*.

CCSS.Math.Content.HSF-BF.A.1c (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*.

CCSS.Math.Content.HSF-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.
TILS Strand IV: Students will be able to use appropriate digital tools within and across content areas .

Learning Goal	**Proficiency Scale**
Students will be able to model real world situations and data with functions and regression equations.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Developing a function (linear, quadratic, exponential, logarithmic, rational, logistic) to model a relationship between two quantities and using the model to solve problems in both familiar and unfamiliar contexts. • Interpreting expressions for the functions in terms of the situation they model. • Using a graphing calculator to enter data, graph scatter plots, and being able to determine and explain the appropriate regression equation in terms of the characteristics of the graph as well as the correlation coefficient.

	2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as: positive, negative, or no correlation, interpolation, extrapolation, correlation coefficient, regression equation. • Performing specific processes such as: o Developing a function to model a relation between two variables, with no major errors regarding the simpler functions, but some errors or omissions regarding the more complex functions. o Using technology to enter data, graph scatter plots, and find regression equations, with some understanding of the appropriate function to choose. o Solving simple real world problems and some complex problems in a familiar context. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Distinguish between situations that can be modeled with linear functions and with exponential functions, proving that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- Recognize the factors which determine exponential growth or decay.
- Use logarithmic re-expression to determine linear, natural logarithmic, exponential, and power regressions.
- Use the graphing calculator to enter data, graph scatter plots, and find the appropriate regression equation.

WGSD Curriculum – Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★

CCSS.Math.Content.HSF-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

CCSS.Math.Content.HSF-IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

CCSS.Math.Content.HSF-IF.C.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

CCSS.Math.Content.HSF-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

CCSS.Math.Content.HSF-IF.C.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

CCSS.Math.Content.HSA-APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

CCSS.Math.Content.HSA-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

WGSD Curriculum – Precalculus
DRAFT

Learning Goal	**Proficiency Scale**
Students will be able to graph and analyze the characteristics of polynomial and power functions.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Graphing power functions and polynomial functions by hand and verifying on the graphing calculator. • Interpreting and identifying key features of the graphs, including domain and range, symmetry (odd and even functions), relative extremes, end behavior, increasing/ decreasing behavior, and zeros. • Using a variety of strategies, including factoring, long and synthetic division, and graphing to determine the real and complex zeros of a polynomial function. • Applying the Remainder Theorem, Rational Zeros Theorem, Fundamental Theorem of Algebra, and Linear Factorization Theorem to find all the complex zeros of a polynomial function. 2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as: domain, range, relative extremes, end behavior, real and complex zeros, increasing/decreasing function, odd and even function. • Performing specific processes such as: o Graphing a variety of power and polynomial functions and identifying key features of the graph, by hand in simple cases and using a graphing calculator for more complicated cases. o Listing all possible rational zeros using the Rational Zeros Theorem. o Stating the number of possible complex zeros of a polynomial function.

	○ Finding all real zeros of a polynomial function by factoring, synthetic division, and using a graphing calculator. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Graph and compare characteristics (domain, range, end behavior, symmetry) of power functions with both integer and rational exponents.
- Graph and analyze the characteristics (domain, range, end behavior, intercepts, relative extremes, symmetry) of polynomial functions.
- Use long and synthetic division along with the Remainder Theorem to find the real zeros and factorization of a polynomial function.
- Use the Rational Zeros Theorem to list all possible rational zeros of a function, and the Fundamental Theorem of Algebra to determine the number of complex zeros of a polynomial function.

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>

CCSS.Math.Content.HSF-IF.C.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to graph rational functions and analyze their function behavior.</td><td><u>Proficiency Scale</u>
4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3. Student demonstrates mastery of the learning goal by:
• Solving rational equations and inequalities algebraically, verifying answers with a graphing calculator.
• Investigating and explaining characteristics of rational functions, including domain, range, zeros, points of discontinuity, local and absolute extremes, asymptotes, and end behavior.
• Sketching by hand the graph of a given rational function (locating its vertical, horizontal and slant asymptotes, and holes if they exist, and showing the correct asymptotic behavior.)
• Finding the expression for a rational function given the vertical and horizontal asymptotes and x and y intercepts.
2. Student demonstrates he/she is nearing proficiency by:
• Recognize and recalling specific vocabulary, such as: vertical and horizontal asymptotes, end behavior, zeros, intercepts, local and absolute extremes.
• Performing specific processes such as:</td></tr>
</table>

	o Finding the asymptotes and stating the end behavior of rational functions. o Sketching the graph of a rational function, using a graphing calculator to determine asymptotic behavior. o Solving simple rational equations and inequalities algebraically, and solving more complex equations with the assistance of a graphing calculator. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Use transformations of the reciprocal function (y = 1/x) to investigate and analyze properties of rational functions.
- Find horizontal and vertical asymptotes of a rational function by examining the graph on a graphing calculator, and analyzing the function algebraically.
- Analyze behavior at the vertical asymptote graphically, and use limits to describe the behavior.
- Use long division to find and analyze the end behavior asymptote of a rational function when the degree of the numerator is larger than the denominator.
- Find x and y intercepts of the graphs of rational functions algebraically, and confirm graphically.
- Graph rational functions by hand, showing all important characteristics, and confirming the graph on a graphing calculator.

WGSD Curriculum – Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★

CCSS.Math.Content.HSF-IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

CCSS.Math.Content.HSF-IF.C.8b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.

CCSS.Math.Content.HSF-LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

CCSS.Math.Content.HSF-LE.A.4 For exponential models, express as a logarithm the solution to$ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

CCSS.Math.Content.HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Learning Goal	**Proficiency Scale**
Students will be able to graph and analyze the characteristics of exponential, logistic, and logarithmic functions.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Using transformations to sketch the graph of exponential, logistic, and logarithmic functions by hand and supporting the answer with a graphing calculator. • Identifying and interpreting key characteristics of the graphs, such as horizontal asymptote, vertical asymptote, x and y-intercept, end behavior, symmetry, increasing or

	decreasing behavior. • Explaining how the parameters of an exponential, logarithmic, or logistic model relate to the data set or situation being modeled and finding a function to model the given data set or situation. • Using the inverse relationship between exponential and logarithmic functions and the Basic Properties of Logarithms to solve equations and problems algebraically. • Applying the formulas for future value, present value, and compound interest to problems involving compound interest, future value of an annuity, present value of an annuity, and loan payments. 2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as logarithm, exponential decay and growth, annuity, compound interest, and simple interest. • Performing specific processes such as: o Sketching graphs of simple exponential and logarithmic functions by hand and more complex functions with the assistance of a graphing calculator. o Solving logarithmic and exponential equations algebraically, with no major errors with the basic equations, but some major errors with the more complex problems. o Solving problems involving compound interest, population growth, and radioactive decay. 1. Student demonstrates limited understanding of the learning goal.

WGSD Curriculum – Precalculus
DRAFT

Learning Targets

Students will:

- Graph exponential and logistic functions and analyze them for domain, range, continuity, increasing or decreasing behavior, symmetry, boundedness, extrema, asymptotes, and end behavior.
- State whether a function represents exponential growth or exponential decay, and find the constant percentage rate of growth or decay.
- Solve realistic problems involving exponential growth, radioactive decay, and finance problems.
- Show that logarithmic functions are inverses of exponential functions, and be able to change between logarithmic and exponential form.
- Evaluate logarithms by hand (when possible) and with a calculator.
- Use the Basic Properties of Logarithms to solve exponential and logarithmic equations.

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

CCSS.Math.Content.HSF-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

CCSS.Math.Content.HSF-TF.B.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

WGSD Curriculum – Precalculus
DRAFT

Learning Goal	**Proficiency Scale**
Students will be able to graph trigonometric functions and use them to model periodic phenomena.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Graphing transformations of the sine, cosine, and tangent functions (involving changes in amplitude, period, midline and phase) and explaining the relationship between constants in the formula and the transformed graph. • Graphing cosecant, secant, and cotangent functions involving one transformation by hand and verifying on the calculator. • Finding a sinusoidal function to model a given data set or situation and explaining how parameters of the model relate to the data set or situation. • Using inverse functions to solve trigonometric equations that arise in the modeling context, evaluating the solutions on the graphing calculator, and interpreting them in terms of the context. 2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as amplitude, period, midline, phase, and sinusoid. • Performing specific processes such as: o Graphing sine and cosine functions that have one or two transformations by hand and using a graphing calculator to assist in graphing more complex functions. o Using a graphing calculator to graph simple tangent, cosecant, secant, and cotangent functions. o Finding a sinusoidal function to model a situation in which the amplitude, period, and midline is given.

	○ Solving simple trigonometric equations algebraically and solving more complex equations graphically using the graphing calculator. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Investigate the graphs all six trigonometric functions on the graphing calculator and examine the relationship between the constants in the equation and the transformations on the graph.
- Learn the definitions and find the amplitude, period, and frequency of sinusoids.
- Construct a sinusoid by using transformations of basic sine and cosine graphs.
- Model periodic behavior with sinusoids.
- Solve a trigonometric equation graphically and algebraically.

WGSD Curriculum – Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

CCSS.Math.Content.HSF-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

CCSS.Math.Content.HSF-TF.A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

CCSS.Math.Content.HSF-TF.B.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

CCSS.Math.Content.HSF-TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

CCSS.Math.Content.HSF-TF.C.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Learning Goal	**Proficiency Scale**
Students will be able to simplify trigonometric expressions and prove trigonometric identities.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Explaining how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers and evaluating exact values of trig functions (sin, cos, tan, csc, sec, cot) from memory. • Explaining and applying the concept that restricting a trig function to a domain on which

<table>
<tr>
<td></td>
<td>it is always increasing or always decreasing allows its inverse to be constructed.
• Explaining the derivation of the Fundamental Identities in trigonometry and using them to simplify expressions.
• Proving and applying formulas for sum, difference, and multiple angle identities to simplify expressions and solve problems.
• Constructing and explaining analytic proofs of trigonometric identities.
2. Student demonstrates he/she is nearing proficiency by:
• Recognizing and recalling specific vocabulary, such as identity, inverse function, radian and exact value.
• Performing specific processes such as:
o Finding exact values of trig functions with the aid of a unit circle.
o Finding inverse trig values, using a unit circle and/or graphing calculator.
o Simplifying basic trig expressions using fundamental identities.
o Writing a simple proof of an identity.
1. Student demonstrates limited understanding of the learning goal.</td>
</tr>
<tr>
<td colspan="2"><u>Learning Targets</u>
Student will:
• Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
• Use special triangles to determine geometrically the values of sin, cos, tan of pi/3, pi/4, pi/6, and use the unit circle to express the values of pi + x, 2pi + x where x is any real number.
• Use the unit circle to explain symmetry (odd and even) and periodicity of trig functions.
• Evaluate trig functions using calculators.
• Use one trig function to find another.</td>
</tr>
</table>

- Prove and apply the following trigonometric identities:
  - Prove the Pythagorean identity.
  - Develop the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
  - Develop and apply the multiple-angle identities.

WGSD Curriculum – Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

CCSS.Math.Content.HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

CCSS.Math.Content.HSF-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1*.

Learning Goal	**Proficiency Scale**
Students will be able to use and understand sequences and series.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Finding and using recursive and explicit formulas for terms of sequences. • Recognizing and using arithmetic and geometric sequences to solve application problems. • Deriving and using the formulas for the sum of finite arithmetic and geometric sequences. • Using summation notation to express the sum of a finite sequence. • Determining the convergence or divergence of an infinite series. • Explaining and using the formula for the sum of an infinite geometric series. 2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as: summation notation, arithmetic

<table>
<tr>
<td></td>
<td>sequence, geometric sequence, infinite series, explicit rule, recursive rule, common difference and common ratio.
• Performing specific processes such as:
o Finding a specific term in an arithmetic or geometric sequence.
o Using the appropriate formula to find the sum of an arithmetic or geometric sequence.
o Determining whether a sequence is arithmetic, geometric, or neither.
1. Student demonstrates limited understanding of the learning goal.</td>
</tr>
<tr>
<td colspan="2"><u>Learning Targets</u>

Students will:
• Determine whether a sequence of numbers is arithmetic, geometric, or neither.
• Define a sequence explicitly and recursively.
• Develop formulas to find any term in an arithmetic or geometric sequence using either the common difference or common ratio.
• Write finite sums in summation (sigma) notation.
• Use a formula to find the sum of finite arithmetic and geometric sequences.
• Determine whether an infinite geometric series converges or diverges, and if possible find its sum.</td>
</tr>
</table>

WGSD Curriculum – Honors Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-BF.A.1 Write a function that describes a relationship between two quantities.★

CCSS.Math.Content.HSF-BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

CCSS.Math.Content.HSF-BF.A.1b Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*.

CCSS.Math.Content.HSF-BF.A.1c (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*.

CCSS.Math.Content.HSF-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.
TILS Strand IV: Students will be able to use appropriate digital tools within and across content areas.

Learning Goal	**Proficiency Scale**
Students will be able to model real world situations and data with functions and regression equations.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Developing a function (linear, quadratic, exponential, logarithmic, rational, logistic) to model a relationship between two quantities and using the model to solve problems in both familiar and unfamiliar contexts. • Interpreting expressions for the functions in terms of the situation they model. • Using a graphing calculator to enter data, graph scatter plots, and being able to determine and explain the appropriate regression equation in terms of the characteristics of the graph as well as the correlation coefficient. 2. Student demonstrates he/she is nearing proficiency by:

	• Recognizing and recalling specific vocabulary, such as: positive, negative, or no correlation, interpolation, extrapolation, correlation coefficient, regression equation. • Performing specific processes such as: o Developing a function to model a relation between two variables, with no major errors regarding the simpler functions, but some errors or omissions regarding the more complex functions. o Using technology to enter data, graph scatter plots, and find regression equations, with some understanding of the appropriate function to choose. o Solving simple real world problems and some complex problems in a familiar context. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Distinguish between situations that can be modeled with linear functions and with exponential functions, proving that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- Recognize the factors which determine exponential growth or decay.
- Use logarithmic re-expression to determine linear, natural logarithmic, exponential, and power regressions.
- Use the graphing calculator to enter data, graph scatter plots, and find the appropriate regression equation.

WGSD Curriculum – Honors Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★

CCSS.Math.Content.HSF-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

CCSS.Math.Content.HSF-IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

CCSS.Math.Content.HSF-IF.C.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

CCSS.Math.Content.HSF-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

CCSS.Math.Content.HSF-IF.C.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

CCSS.Math.Content.HSA-APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

- CCSS.Math.Content.HSA-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Learning Goal	**Proficiency Scale**
Students will be able to graph and analyze the characteristics of polynomial and power functions.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Graphing power functions and polynomial functions by hand and verifying on the graphing calculator. • Interpreting and identifying key features of the graphs, including domain and range, symmetry (odd and even functions), relative extremes, end behavior, increasing/ decreasing behavior, and zeros.

	• Using a variety of strategies, including factoring, long and synthetic division, and graphing to determine the real and complex zeros of a polynomial function. • Understanding and applying the Remainder Theorem, Rational Zeros Theorem, Fundamental Theorem of Algebra, and Linear Factorization Theorem to find all the complex zeros of a polynomial function. 2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as: domain, range, relative extremes, end behavior, real and complex zeros, increasing/decreasing function, odd and even function. • Performing specific processes such as: o Graphing a variety of power and polynomial functions and identifying key features of the graph, by hand in simple cases and using technology for more complicated cases. o Listing all possible rational zeros using the Rational Zeros Theorem. o Stating the number of possible complex zeros of a polynomial function. o Finding all real zeros of a polynomial function by factoring, synthetic division, and using a graphing calculator. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Graph and compare characteristics (domain, range, end behavior, symmetry) of power functions with both integer and rational exponents.
- Graph and analyze the characteristics (domain, range, end behavior, intercepts, relative extremes, symmetry) of polynomial functions.
- Use long and synthetic division along with the Remainder Theorem to find the real zeros and factorization of a polynomial function.
- Use the Rational Zeros Theorem to list all possible rational zeros of a function, and the Fundamental Theorem of Algebra to determine the number of complex zeros of a polynomial function.

WGSD Curriculum – Honors Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.C.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Learning Goal	**Proficiency Scale**
Students will be able to graph rational functions and analyze their function behavior.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Solving rational equations and inequalities algebraically, verifying answers with a graphing calculator. • Investigating and explaining characteristics of rational functions, including domain, range, zeros, points of discontinuity, local and absolute extremes, asymptotes, and end behavior. • Sketching by hand the graph of a given rational function (locating its vertical, horizontal and slant asymptotes, and holes if they exist, and showing the correct asymptotic behavior.) • Finding the expression for a rational function given the vertical and horizontal asymptotes and x and y intercepts. 2. Student demonstrates he/she is nearing proficiency by: • Recognize and recalling specific vocabulary, such as: vertical and horizontal asymptotes, end behavior, zeros, intercepts, local and absolute extremes. • Performing specific processes such as: o Finding the asymptotes and stating the end behavior of rational functions. o Sketching the graph of a rational function, using a graphing calculator to determine asymptotic behavior.

	o Solving simple rational equations and inequalities algebraically, and solving more complex equations with the assistance of a graphing calculator. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Use transformations of the reciprocal function (y = 1/x) to investigate and analyze properties of rational functions.
- Find horizontal and vertical asymptotes of a rational function by examining the graph on a graphing calculator, and analyzing the function algebraically.
- Analyze behavior at the vertical asymptote graphically, and use limits to describe the behavior.
- Use long division to find and analyze the end behavior asymptote of a rational function when the degree of the numerator is larger than the denominator.
- Find x and y intercepts of the graphs of rational functions algebraically, and confirm graphically.
- Graph rational functions by hand, showing all important characteristics, and confirming the graph on a graphing calculator.

WGSD Curriculum – Honors Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★

CCSS.Math.Content.HSF-IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

CCSS.Math.Content.HSF-IF.C.8b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.

CCSS.Math.Content.HSF-LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

CCSS.Math.Content.HSF-LE.A.4 For exponential models, express as a logarithm the solution to$ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

CCSS.Math.Content.HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Learning Goal	**Proficiency Scale**
Students will be able to graph and analyze the characteristics of exponential, logistic, and logarithmic functions.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Using transformations to sketch the graph of exponential, logistic, and logarithmic functions by hand and supporting the answer with a graphing calculator. • Identifying and interpreting key characteristics of the graphs, such as horizontal asymptote, vertical asymptote, x and y-intercept, end behavior, symmetry, increasing or decreasing behavior. • Explaining how the parameters of an exponential, logarithmic, or logistic model relate to the data set or situation being modeled and finding a function to model the given data set or situation. • Using the inverse relationship between exponential and logarithmic functions and the Basic

	Properties of Logarithms to solve equations and problems algebraically. • Applying the formulas for future value, present value, and compound interest to problems involving compound interest, future value of an annuity, present value of an annuity, and loan payments. 2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as logarithm, exponential decay and growth, annuity, compound interest, and simple interest. • Performing specific processes such as: o Sketching graphs of simple exponential and logarithmic functions by hand and more complex functions with the assistance of a graphing calculator. o Solving logarithmic and exponential equations algebraically, with no major errors with the basic equations, but some major errors with the more complex problems. o Solving problems involving compound interest, population growth, and radioactive decay. 1. Student demonstrates limited understanding of the learning goal.

WGSD Curriculum – Honors Precalculus
DRAFT

Learning Targets

Students will:

- Graph exponential and logistic functions and analyze them for domain, range, continuity, increasing or decreasing behavior, symmetry, boundedness, extrema, asymptotes, and end behavior.
- State whether a function represents exponential growth or exponential decay, and find the constant percentage rate of growth or decay.
- Solve realistic problems involving exponential growth, radioactive decay, and finance problems.
- Show that logarithmic functions are inverses of exponential functions, and be able to change between logarithmic and exponential form.
- Evaluate logarithms by hand (when possible) and with a calculator.
- Use the Basic Properties of Logarithms to solve exponential and logarithmic equations.

WGSD Curriculum – Honors Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

CCSS.Math.Content.HSF-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

CCSS.Math.Content.HSF-TF.B.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Learning Goal	**Proficiency Scale**
Students will be able to graph trigonometric functions and use them to model periodic phenomena.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Graphing transformations of the sine, cosine, and tangent functions (involving changes in amplitude, period, midline and phase) and explaining the relationship between constants in the formula and the transformed graph. • Graphing cosecant, secant, and cotangent functions involving one transformation by hand and verifying on the calculator. • Finding a sinusoidal function to model a given data set or situation and explaining how parameters of the model relate to the data set or situation. • Using inverse functions to solve trigonometric equations that arise in the modeling context, evaluating the solutions on the graphing calculator, and interpreting them in terms of the context. 2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as amplitude, period, midline, phase, and sinusoid.

	• Performing specific processes such as: o Graphing sine and cosine functions that have one or two transformations by hand and using a graphing calculator to assist in graphing more complex functions. o Using a graphing calculator to graph simple tangent, cosecant, secant, and cotangent functions. o Finding a sinusoidal function to model a situation in which the amplitude, period, and midline is given. o Solving simple trigonometric equations algebraically and solving more complex equations graphically using the graphing calculator. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Investigate the graphs all six trigonometric functions on the graphing calculator and examine the relationship between the constants in the equation and the transformations on the graph.
- Learn the definitions and find the amplitude, period, and frequency of sinusoids.
- Construct a sinusoid by using transformations of basic sine and cosine graphs.
- Model periodic behavior with sinusoids.
- Solve a trigonometric equation graphically and algebraically.

WGSD Curriculum – Honors Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

CCSS.Math.Content.HSF-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

CCSS.Math.Content.HSF-TF.A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

CCSS.Math.Content.HSF-TF.B.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

CCSS.Math.Content.HSF-TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

CCSS.Math.Content.HSF-TF.C.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Learning Goal	**Proficiency Scale**
Students will be able to simplify trigonometric expressions and prove trigonometric identities.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Explaining how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers and evaluating exact values of trig functions (sin, cos, tan, csc, sec, cot) from memory. • Explaining and applying the concept that restricting a trig function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. • Explaining the derivation of the Fundamental Identities in trigonometry and using them to simplify expressions.

	• Proving and applying formulas for sum, difference, and multiple angle identities to simplify expressions and solve problems. • Constructing and explaining analytic proofs of trigonometric identities. 2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as identity, inverse function, radian and exact value. • Performing specific processes such as: o Finding exact values of trig functions with the aid of a unit circle. o Finding inverse trig values, using a unit circle and/or graphing calculator. o Simplifying basic trig expressions using fundamental identities. o Writing a simple proof of an identity. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Student will:

- Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- Use special triangles to determine geometrically the values of sin, cos, tan of pi/3, pi/4, pi/6, and use the unit circle to express the values of pi + x, 2pi + x where x is any real number.
- Use the unit circle to explain symmetry (odd and even) and periodicity of trig functions.
- Evaluate trig functions using calculators.
- Use one trig function to find another.
- Prove and apply the following trigonometric identities:
    - Prove the Pythagorean identity.
    - Develop the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
    - Develop and apply the multiple-angle identities.

WGSD Curriculum – Honors Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

CCSS.Math.Content.HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

CCSS.Math.Content.HSF-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1*.

Learning Goal	**Proficiency Scale**
Students will be able to use and understand sequences and series.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Finding and using recursive and explicit formulas for terms of sequences. • Recognizing and using arithmetic and geometric sequences to solve application problems. • Deriving and using the formulas for the sum of finite arithmetic and geometric sequences. • Using summation notation to express the sum of a finite sequence. • Determining the convergence or divergence of an infinite series. • Explaining and using the formula for the sum of an infinite geometric series. 2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as: summation notation, arithmetic sequence, geometric sequence, infinite series, explicit rule, recursive rule, common difference and common ratio. • Performing specific processes such as: o Finding a specific term in an arithmetic or geometric sequence. o Using the appropriate formula to find the sum of an arithmetic or geometric sequence. o Determining whether a sequence is arithmetic, geometric, or neither. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Determine whether a sequence of numbers is arithmetic, geometric, or neither.
- Define a sequence explicitly and recursively.
- Develop formulas to find any term in an arithmetic or geometric sequence using either the common difference or common ratio.
- Write finite sums in summation (sigma) notation.
- Use a formula to find the sum of finite arithmetic and geometric sequences.
- Determine whether an infinite geometric series converges or diverges, and if possible find its sum.

WGSD Curriculum – Honors Precalculus
DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSN-CN.B.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

http://www.collegeboard.com/prod_downloads/about/association/academic/mathematics-statistics_cbscs.pdf

College Board Standards (Pre-Calc) Objective PC.3.2 Student represents points and curves in rectangular and polar forms and finds equivalent polar and rectangular representations for points and curves.

College Board Standards (Pre-Calc) Objective PC.5.2 Student applies parametric methods to represent and interpret motion of objects in the plane.

Learning Goal	**Proficiency Scale**
Students will be able to solve and graph polar and parametric equations.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: ● graphing and analyzing curves described by parametric equations. ● using parametric equations in applied contexts (e.g. projectiles and motion on a Ferris wheel) to model situations and solve problems. ● converting between polar and rectangular coordinates. ● graphing and analyzing the graphs of functions given in polar coordinates both by hand and on a graphing calculator. ● multiplying complex numbers in polar form and using DeMoivre's Theorem to find roots of complex numbers. 2. Student demonstrates he/she is nearing proficiency by: ● recognizing and recalling specific vocabulary, such as parameter, parametric equation, polar coordinates, rectangular coordinates and polar equation. ● performing specific processes such as: o graphing parametric equations on a graphing calculator and using these graphs to solve

	problems. o converting between polar and rectangular coordinates on a graphing calculator. o graphing and analyzing the graphs of polar equations on a graphing calculator. 1: Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- express points in the plane in both rectangular and polar forms.
- find equivalent representations for points and curves, including the conics, in both rectangular and polar forms.
- use parametric equations to represent situations involving motion in the plane, including motion on a line, motion of a projectile, and motion of objects in orbit.
- convert between a pair of parametric equations and an equation in *x* and *y* to interpret the situation represented.
- analyze planar curves, including those given in parametric form.

<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf AP Calculus Standard: Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal . They should understand the connections among these representations .
College Board AP Calculus Topic Outline: Limits
• An intuitive understanding of the limiting process .
• Calculating limits using algebra .
• Estimating limits from graphs or tables of data
• Describing asymptotic behavior in terms of limits involving infinity
• Understanding continuity in terms of limits</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to find the limit of a function.</td><td><u>Proficiency Scale</u>

4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3. Student demonstrates mastery of the learning goal by:
• Finding one-sided and two-sided limits numerically, graphically, and algebraically.
• Determining infinite limits from a graph, as well as from the rules for limits at infinity.
• Using algebra (such as factoring and multiplication by conjugates) to determine limits of rational functions.
• Using the definition of continuity to determine all values where a function is discontinuous from its graph as well as its rule.

2. Student demonstrates he/she is nearing proficiency by:
• Recognizing and recalling specific vocabulary, such as limit from the left/right,</td></tr>
</table>

	one/two-sided limit, limit at infinity, and point of discontinuity. ● Performing specific processes such as: o Finding points of discontinuity from the graph of a function. o Estimating limits from graphs or tables of data. o Using substitution to find limits of polynomial functions. 1. Student demonstrates limited understanding or skill with the learning goal.

Learning Targets

Students will:

- State the formal definition of continuity in terms of limits.
- Determine points of discontinuity from the graph of a function, and explain in terms of limits.
- Determine where a piecewise function and a rational function are discontinuous from examination of its rule.
- Determine infinite limits from a graph, as well as from using the rules for limits at infinity.
- Use algebraic simplification (including factoring and multiplying by conjugates) to find limits that would involve division by zero if substitution were used.
- Know and be able to use the rules for limits.

<table>
<tr><td colspan="2"><u>Standards (CCSS, State, National, TILS, CREDE, etc.)</u>

CCSS.Math.Content.HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf: AP Calculus Goal: Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation, and should be able to use derivatives to solve a variety of problems.</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to determine the derivative functions.</td><td><u>Proficiency Scale</u>

4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3. The student demonstrates mastery of the learning goal by:
● Using the formal mathematical definition of a derivative (limit of the difference quotient) to find the derivative of a polynomial function.
● Finding derivatives of basic functions, including power, logarithmic, and exponential functions, using the rules for derivatives.
• Using the derivative rules for sums, products, and quotients of functions, including simplifying answers.
• Using the chain rule and implicit differentiation to solve multi-step problems.
• Finding the slope of a curve at a point and using it to find the equation of the tangent line at that point.
• Explaining the concept of a derivative as a function and being able to draw and discuss the corresponding graphs of the first and second derivative.

2. Student demonstrates he/she is nearing the learning goal by:</td></tr>
</table>

	• Recognizing or recalling specific vocabulary, such as tangent line, secant line, instantaneous rate of change, and differentiable. • Performing specific processes, such as: o Stating the mathematical limit definition of a derivative, as well as the formula for average rate of change. o Finding derivatives of basic functions, including power, exponential, and logarithmic. o Using the derivative rules for sums, products and quotients of functions without simplifying answers. o Using the chain rule and implicit differentiation to solve simple problems. o Finding the slope of a curve at a point. o Determining the first and second derivative of polynomial functions. 1. Student demonstrates limited understanding of the learning goal.
Learning Targets Students will: ● Use the formal definition of the derivative to find instantaneous rate of change and the slope of a tangent line at a point. ● Develop and be able to use various algebraic	

techniques for finding derivatives: power rule, chain rule, sum rule, product rule, and quotient rule. ● Know and be able to apply special derivative rules: exponential rules and logarithmic rules. ● Find equation of tangent line at a point on a curve, for a variety of functions. ● Find derivatives using implicit differentiation.	

Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★

CCSS.Math.Content.HSF-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf: **AP Calculus Goal**: Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.

AP Calculus Goal: Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation, and should be able to use derivatives to solve a variety of problems.

Learning Goal	**Proficiency Scale**
Students will be able to apply derivatives to analyze and interpret the graphs of functions and use derivatives in physics and business applications.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. The student demonstrates mastery of the learning goal by: • Using the derivative as a rate of change in varied applied contexts, including velocity, speed, acceleration, marginal profit and point of diminishing returns. • Applying first and second derivatives to the analysis of graphs of a variety of simple and complex functions, including increasing/decreasing behavior, relative and absolute extremes, concavity, and inflection points. • Modeling a written description of a physical situation with a function and using applications of extrema to find the optimal solution.

<table>
<tr>
<td></td>
<td>

Using differentiation to solve related rates problems in a variety of pure and applied contexts.

2. Student demonstrates he/she is nearing the learning goal by:

Recognizing or recalling specific vocabulary, such as velocity, acceleration, revenue, cost, profit, marginal analysis, and point of diminishing returns.
Performing specific processes, such as:

Using the first derivative to determine velocity and marginal profit when given the specific function.
Using first and second derivatives to find increasing/decreasing behavior, relative and absolute extremes, concavity and inflection points of polynomial functions.
Solving business and physics problems using derivatives, when provided with the function.

1. Student demonstrates limited understanding of the learning goal.
</td>
</tr>
<tr>
<td colspan="2">
<u>Learning Targets</u>

Students will:

Use first and second derivatives to explore various concepts of a function, such as: increasing and decreasing behavior, relative maxima and minima, concavity, critical points, and inflection point.
Investigate and apply applications of first and second derivatives to problems involving: velocity, acceleration, marginal profit, marginal revenue, marginal cost, and point of diminishing returns.
Use applications of extrema to solve optimization problems.

</td>
</tr>
</table>

WGSD Curriculum – Introduction to Calculus and Discrete Math
DRAFT

Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.

Learning Goal	**Proficiency Scale**
Students will be able to apply laws of logic to analyze the validity of arguments.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. The student demonstrates mastery of the learning goal by: • Writing a statement as a conditional and its converse, inverse, contrapositive and negation. • Using DeMorgan's Law to negate conjunctions and disjunctions. • Applying basic laws of logic, namely modus ponens, modus tollens, disjunctive syllogism, transitivity, fallacy of inverse, and fallacy of converse to analysis of arguments. • Negating statements containing universal or existential quantifiers. • Translating arguments into symbolic logic and using Euler diagrams and laws of logic to determine a valid conclusion. 2. Student demonstrates he/she is nearing the learning goal by ● Recognizing or recalling specific vocabulary, such as conjunction, disjunction, compound statement, biconditional, tautology, fallacy, truth table, negation, universal quantifier and existential quantifier. ● Performing specific processes, such as: o Using Euler diagrams to analyze the validity of simple three statement arguments.

	○ Translating statements into conditionals and writing the converse, inverse, and contrapositive of the conditional. ○ Recalling and giving an example of valid logic forms, namely modus ponens, modus tollens, disjunctive syllogism and transitivity. ○ Translating basic three sentence arguments into symbolic form, and reaching a valid conclusion. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Develop and learn definitions for key terms in logic, including: statement, negation, conjunction, disjunction, universal quantifier, existential quantifier, conditional, converse, inverse, contrapositive, biconditional, valid, invalid, fallacy.
- Develop and learn truth tables for: conjunction, disjunction, negation, conditional, and biconditional.
- Prove and apply DeMorgan's Law to negations of conjunctions and disconjunctions.
- Develop and learn to apply rules of valid reasoning, as well as examples of faulty logic.
- Draw Euler diagrams and use truth tables to analyze simple 3 - 4 statement arguments.
- Translate into symbols and use the laws of logic to analyze Lewis Carroll type arguments.

Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSS-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

CCSS.Math.Content.HSS-CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

CCSS.Math.Content.HSS-CP.A.3 Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability ofBCCSS.Math.Content.HSS-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance*

CCSS.Math.Content.HSS-CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

CCSS.Math.Content.HSS-CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

CCSS.Math.Content.HSS-CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

CCSS.Math.Content.HSS-MD.B.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

CCSS.Math.Content.HSS-MD.B.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Learning Goal	**Proficiency Scale**
Students will understand and apply the concepts of conditional probability to solve problems and interpret the results.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. The student demonstrates mastery of the learning goal by: • Calculating basic probabilities both experimentally and theoretically. • Solving routine as well as non-routine problems using the Fundamental Counting Principle, combinations and permutations, and explaining the reasoning behind the solution. • Identifying and producing examples of mutually exclusive, joint, and independent events. • Calculating probabilities of compound events using the addition rule or product rule. • Applying Bayes' Theorem to the analysis and calculation of business and social applications. 2. Student demonstrates he/she is nearing the learning goal by: ● Recognizing or recalling specific vocabulary, such as odds, permutation, combination, mutually exclusive and independent events, factorial, sample space, complement, experiment, trial, outcomes, union, experimental, theoretical, Fundamental Counting Principle and intersection. ● Performing specific processes, such as: o Calculating basic probabilities both experimentally and theoretically. o Solving basic routine problems using the Fundamental Counting Problem, combinations, and permutations. o Finding probability of a compound event using the addition rule or product rule.

	1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Develop and learn basic probability vocabulary, including: experiment, trial, outcomes, sample space, event, mutually exclusive events, independent events, complement, and odds.
- Know the definition of independent events and use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.
- Know the definition of conditional probability and use it to solve for probabilities in finite sample spaces.
- Know the Fundamental Counting Principle and recognize when and how to use it.
- Know the definition of permutation and combination and recognize when each is appropriate for a particular problem.
- Be able to compute the values of permutations and combinations both from the formula as well as with technology.
- Know Bayes' Theorem and be able to use it to analyze decisions and strategies using probability concepts, such as product testing and medical testing.

Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSS-MD.A.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

CCSS.Math.Content.HSS-MD.A.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

CCSS.Math.Content.HSS-MD.A.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

CCSS.Math.Content.HSS-MD.A.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

CCSS.Math.Content.HSS-MD.B.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

Learning Goal	**Proficiency Scale**
Students will be able to calculate expected values and use them to evaluate outcomes of decisions.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. The student demonstrates mastery of the learning goal by: • Applying the Binomial Probability Formula and Pascal's Triangle to problems involving probability. • Designing and conducting a binomial experiment, and finding the probability of (x) successes in (n) trials using the binomial probability formula. • Creating a probability distribution for an experiment and finding its expected

<table>
<tr>
<td></td>
<td>value both experimentally and theoretically.
• Assigning probabilities to payoff values, finding expected values, and deciding on whether the outcome is fair.
• Using appropriate technology to design and conduct experiments, graph and analyze distribution functions.
2. Student demonstrates he/she is nearing the learning goal by:
● Recognizing or recalling specific vocabulary, such as expected value, probability function, binomial experiment, fair game, and random variable.
● Performing specific processes, such as:
o Computing combinations from Pascal's Triangle.
o Computing binomial probability on the calculator, when given the value for probability, number of successes, and number of trials.
o Calculating expected value of an event when given the probabilities of success and the payoff.
1. Student demonstrates limited understanding of the learning goal.</td>
</tr>
<tr>
<td colspan="2"><u>Learning Targets</u>
Students will:
● Use the concept of discrete random variables to solve for the probabilities of specific outcomes, such as: the probability of the occurrence of five heads in 14 coin tosses.
● Design and conduct a variety of binomial experiments to determine probability of particular outcomes.
● Develop a formula for binomial probability, and compute it by hand and with a calculator.
● Recognize various uses for computing binomial probability and combinations from Pascal's Triangle.</td>
</tr>
</table>

- Develop definition of expected value and use it to determine fairness of a game of chance, as well as expected wins or losses.

Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSS-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

CCSS.Math.Content.HSS-IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model*?

CCSS.Math.Content.HSS-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

CCSS.Math.Content.HSS-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

CCSS.Math.Content.HSS-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Learning Goal	**Proficiency Scale**
Students will be able to select and use appropriate statistical methods to analyze data.	4. 4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. The student demonstrates mastery of the learning goal by: • Designing and conducting simulations to collect and display sampling distributions, by hand and with the use of a graphing calculator or computer. • Analyzing the spread of numbers in a set of data, using measures of central tendency, variance, and standard deviation. • Using the properties of the normal curve to describe how sample data estimates the

	population mean and standard deviation. • Calculating the area under a normal curve between x = *a* and x = *b* to find the probability that a data value will be between *a* and *b*, and using this to solve application problems. 2. Student demonstrates he/she is nearing the learning goal by: ● Recognizing or recalling specific vocabulary, such as mean, median, mode, standard deviation, normal curve, and variance. ● Performing specific processes, such as: o Preparing a frequency distribution and its histogram by hand and using technology to find measures of central tendency. o Using technology to find probabilities of normal distribution applications. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Know the definitions of the mean, median and mode of a distribution of data and compute each in particular situations.
- Compute the variance and standard deviation of a distribution of data, both by hand and with the use of a graphing calculator.
- Collect, organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, by hand and with a calculator.
- Use technology to find the area under the curve of a normal distribution and apply this to the solution of real-life application problems.
- Recognize that many natural and social phenomena produce continuous probability distributions that are normal and can use the distributions to solve application problems in which the distribution belongs to those families. For example, they use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages.

WGSD Curriculum – Introduction to Calculus and Discrete Math
DRAFT

WGSD Curriculum - Calculus AP

DRAFT

Limits

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf

AP Calculus Standard: Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.

College Board AP Calculus Topic Outline: Limits

- An intuitive understanding of the limiting process.
- Calculating limits using algebra.
- Estimating limits from graphs or tables of data
- Describing asymptotic behavior in terms of limits involving infinity
- Understanding continuity in terms of limits

Learning Goal	**Proficiency Scales**
Students will be able to find the limit of a function.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by ● explaining the concept of a limit, as well as choosing the most appropriate method (algebraically, graphically, numerically) for finding the limit. ● describing and explaining asymptotic behavior in terms of limits involving infinity. ● explaining continuity in terms of limits and applying it to the Intermediate Value Theorem. 2. Student demonstrates he/she is nearing proficiency by: ● recognizing and recalling specific vocabulary, such as *limit from the left/right, one/two-sided limit, and limit at infinity*. ● Performing specific processes such as o stating the definition of a limit and finding a limit of basic functions algebraically, as well as from graphs or tables of data.

	instantaneous rate of change, and differentiable. ● performing specific processes, such as ○ stating the mathematical limit definition of a derivative, as well as the formula for average rate of change. ○ finding derivatives of basic functions, including power, trigonometric, exponential, and logarithmic. ○ using the derivative rules for sums, products and quotients of functions without simplifying answers. ○ using the chain rule and implicit differentiation to solve simple problems. ○ finding the slope of a curve at a point. ○ determining the first and second derivative of polynomial functions. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

- Use the formal definition of the derivative to find instantaneous rate of change and the slope of a tangent line at a point
- Develop and be able to use various algebraic techniques for finding derivatives: power rule, chain rule, sum rule, product rule, and quotient rule
- Know and be able to apply special derivative rules: exponential rules and logarithmic rules
- Find equation of tangent line at a point on a curve, for a variety of functions
- Find derivatives using implicit differentiation

Learning Design

WGSD Curriculum -- Calculus AP

DRAFT

Functions of Derivatives

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf: AP Calculus Goal: Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation, and should be able to use derivatives to solve a variety of problems.

Learning Goal	**Proficiency Scale**
Students will be able to determine the derivative of functions.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. The student demonstrates mastery of the learning goal by: ● explaining the concept of a derivative as an instantaneous rate of change, as well as its definition as the limit of the difference quotient. ● finding derivatives of basic functions, including power, trigonometric, logarithmic, and exponential functions. ● using the derivative rules for sums, products, and quotients of functions, including simplifying answers. ● using the chain rule and implicit differentiation to solve multi-step problems. ● finding the slope of a curve at a point and using it to find the equation of the tangent line at that point. ● explaining the concept of a derivative as a function and being able to draw and discuss the corresponding graphs of the first and second derivative. ● deriving the rules for the secant function and tangent function using the derivatives of the sine function and cosine function. 2. Student demonstrates he/she is nearing the learning goal by: ● recognizing or recalling specific vocabulary, such as *tangent line, secant line,*

	o demonstrating knowledge of both the formal definition and the graphical interpretation of continuity of a function. 1. Student demonstrates limited understanding or skill with the learning goal.

Learning Targets

- Find one-sided and two-sided limits numerically, graphically, and algebraically
- Determine infinite limits from a graph, as well as from using the rules for limits at infinity
- Use the definition of continuity to determine all values where a function is discontinuous from its graph as well as its rule
- Use algebraic simplification (including factoring and multiplying by conjugates) to find limits that would involve division by zero if substitution were used
- Know and be able to use the rules for limits

Learning Design

WGSD Curriculum -- Calculus AP

Derivatives

DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★

CCSS.Math.Content.HSF-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf : **AP Calculus Goal**: Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.

Learning Goal

Students will be able to apply derivatives to analyze and interpret the graphs of functions and use derivatives in physics and business applications.

Proficiency Scales

4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3. The student demonstrates mastery of the learning goal by:
   - interpreting the derivative as a rate of change in varied applied contexts, including velocity, speed, acceleration, and marginal profit.
   - applying first and second derivatives to the analysis of graphs of a variety of simple and complex functions, including increasing/decreasing behavior, relative and absolute extrema, concavity, and inflection points.
   - modeling a written description of a physical situation with a function and using applications of extrema to find the optimal solution.
   - using differentiation to solve related rates problems in a variety of pure and applied contexts.

WGSD Curriculum -- Calculus AP

Derivatives

DRAFT

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★

CCSS.Math.Content.HSF-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf : **AP Calculus Goal**: Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.

Learning Goal

Students will be able to apply derivatives to analyze and interpret the graphs of functions and use derivatives in physics and business applications.

Proficiency Scales

4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3. The student demonstrates mastery of the learning goal by:
   - interpreting the derivative as a rate of change in varied applied contexts, including velocity, speed, acceleration, and marginal profit.
   - applying first and second derivatives to the analysis of graphs of a variety of simple and complex functions, including increasing/decreasing behavior, relative and absolute extrema, concavity, and inflection points.
   - modeling a written description of a physical situation with a function and using applications of extrema to find the optimal solution.
   - using differentiation to solve related rates problems in a variety of pure and applied contexts.

WGSD Curriculum -- Calculus AP

DRAFT

Integrals

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf: AP Calculus Goal: Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change, and should be able to use integrals to solve a variety of problems.

Learning Goal	**Proficiency Scale**
Students will be able to find the integral of a function using geometric methods and the fundamental theorem of calculus.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. The student demonstrates mastery of the learning goal by: ● explaining the concept of a definite integral as net area. ● calculating definite integrals using finite and infinite Riemann sums. ● explaining the fundamental theorem of calculus and how it is used to calculate integrals. ● using the integral of a rate of change function to calculate the net change of the quantity. ● find the antiderivatives of functions to solve real world problems. 2. Student demonstrates he/she is nearing the learning goal by: ● recognizing or recalling specific vocabulary, such as *definite integral, indefinite integral, antiderivative, and net area.* ● performing specific processes, such as: ○ calculating finite Riemann sums. ○ finding antiderivatives of simple functions. ○ calculating the area under a curve using the fundamental theorem of calculus. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

- Solve real world problems by using the integral of a rate of change to find the amount of change
- Find the area of regions bounded by multiple functions
- Calculate the volume of solids using integrals of the cross sections

Learning Design