High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)	
CCSS.Math.Content.HSF-BF.A.1 Write a function that describes a relationship between two quantities.★ CCSS.Math.Content.HSF-BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. CCSS.Math.Content.HSF-BF.A.1b Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*. CCSS.Math.Content.HSF-BF.A.1c (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*. CCSS.Math.Content.HSF-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. TILS Strand IV: Students will be able to use appropriate digital tools within and across content areas .	
Learning Goal	**Proficiency Scale**
Students will be able to model real world situations and data with functions and regression equations.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Developing a function (linear, quadratic, exponential, logarithmic, rational, logistic) to model a relationship between two quantities and using the model to solve problems in both familiar and unfamiliar contexts. • Interpreting expressions for the functions in terms of the situation they model. • Using a graphing calculator to enter data, graph scatter plots, and being able to determine and explain the appropriate regression equation in terms of the characteristics of the graph as well as the correlation coefficient.

	2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as: positive, negative, or no correlation, interpolation, extrapolation, correlation coefficient, regression equation. • Performing specific processes such as: o Developing a function to model a relation between two variables, with no major errors regarding the simpler functions, but some errors or omissions regarding the more complex functions. o Using technology to enter data, graph scatter plots, and find regression equations, with some understanding of the appropriate function to choose. o Solving simple real world problems and some complex problems in a familiar context. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Distinguish between situations that can be modeled with linear functions and with exponential functions, proving that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- Recognize the factors which determine exponential growth or decay.
- Use logarithmic re-expression to determine linear, natural logarithmic, exponential, and power regressions.
- Use the graphing calculator to enter data, graph scatter plots, and find the appropriate regression equation.

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)
CCSS.Math.Content.HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★ CCSS.Math.Content.HSF-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ CCSS.Math.Content.HSF-IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima. CCSS.Math.Content.HSF-IF.C.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. CCSS.Math.Content.HSF-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. CCSS.Math.Content.HSF-IF.C.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. CCSS.Math.Content.HSA-APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. CCSS.Math.Content.HSA-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

<table>
<tr><th>Learning Goal</th><th>Proficiency Scale</th></tr>
<tr><td>Students will be able to graph and analyze the characteristics of polynomial and power functions.</td><td>4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3. Student demonstrates mastery of the learning goal by:
• Graphing power functions and polynomial functions by hand and verifying on the graphing calculator.
• Interpreting and identifying key features of the graphs, including domain and range, symmetry (odd and even functions), relative extremes, end behavior, increasing/ decreasing behavior, and zeros.
• Using a variety of strategies, including factoring, long and synthetic division, and graphing to determine the real and complex zeros of a polynomial function.
• Applying the Remainder Theorem, Rational Zeros Theorem, Fundamental Theorem of Algebra, and Linear Factorization Theorem to find all the complex zeros of a polynomial function.

2. Student demonstrates he/she is nearing proficiency by:
• Recognizing and recalling specific vocabulary, such as: domain, range, relative extremes, end behavior, real and complex zeros, increasing/decreasing function, odd and even function.
• Performing specific processes such as:
o Graphing a variety of power and polynomial functions and identifying key features of the graph, by hand in simple cases and using a graphing calculator for more complicated cases.
o Listing all possible rational zeros using the Rational Zeros Theorem.
o Stating the number of possible complex zeros of a polynomial function.</td></tr>
</table>

	o Finding all real zeros of a polynomial function by factoring, synthetic division, and using a graphing calculator. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Graph and compare characteristics (domain, range, end behavior, symmetry) of power functions with both integer and rational exponents.
- Graph and analyze the characteristics (domain, range, end behavior, intercepts, relative extremes, symmetry) of polynomial functions.
- Use long and synthetic division along with the Remainder Theorem to find the real zeros and factorization of a polynomial function.
- Use the Rational Zeros Theorem to list all possible rational zeros of a function, and the Fundamental Theorem of Algebra to determine the number of complex zeros of a polynomial function.

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)	
CCSS.Math.Content.HSF-IF.C.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	
Learning Goal	**Proficiency Scale**
Students will be able to graph rational functions and analyze their function behavior.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Solving rational equations and inequalities algebraically, verifying answers with a graphing calculator. • Investigating and explaining characteristics of rational functions, including domain, range, zeros, points of discontinuity, local and absolute extremes, asymptotes, and end behavior. • Sketching by hand the graph of a given rational function (locating its vertical, horizontal and slant asymptotes, and holes if they exist, and showing the correct asymptotic behavior.) • Finding the expression for a rational function given the vertical and horizontal asymptotes and x and y intercepts. 2. Student demonstrates he/she is nearing proficiency by: • Recognize and recalling specific vocabulary, such as: vertical and horizontal asymptotes, end behavior, zeros, intercepts, local and absolute extremes. • Performing specific processes such as:

	o Finding the asymptotes and stating the end behavior of rational functions. o Sketching the graph of a rational function, using a graphing calculator to determine asymptotic behavior. o Solving simple rational equations and inequalities algebraically, and solving more complex equations with the assistance of a graphing calculator. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Use transformations of the reciprocal function (y = 1/x) to investigate and analyze properties of rational functions.
- Find horizontal and vertical asymptotes of a rational function by examining the graph on a graphing calculator, and analyzing the function algebraically.
- Analyze behavior at the vertical asymptote graphically, and use limits to describe the behavior.
- Use long division to find and analyze the end behavior asymptote of a rational function when the degree of the numerator is larger than the denominator.
- Find x and y intercepts of the graphs of rational functions algebraically, and confirm graphically.
- Graph rational functions by hand, showing all important characteristics, and confirming the graph on a graphing calculator.

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)	
CCSS.Math.Content.HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ CCSS.Math.Content.HSF-IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. CCSS.Math.Content.HSF-IF.C.8b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay. CCSS.Math.Content.HSF-LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. CCSS.Math.Content.HSF-LE.A.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. CCSS.Math.Content.HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.	
Learning Goal	**Proficiency Scale**
Students will be able to graph and analyze the characteristics of exponential, logistic, and logarithmic functions.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Using transformations to sketch the graph of exponential, logistic, and logarithmic functions by hand and supporting the answer with a graphing calculator. • Identifying and interpreting key characteristics of the graphs, such as horizontal asymptote, vertical asymptote, x and y-intercept, end behavior, symmetry, increasing or

	decreasing behavior. • Explaining how the parameters of an exponential, logarithmic, or logistic model relate to the data set or situation being modeled and finding a function to model the given data set or situation. • Using the inverse relationship between exponential and logarithmic functions and the Basic Properties of Logarithms to solve equations and problems algebraically. • Applying the formulas for future value, present value, and compound interest to problems involving compound interest, future value of an annuity, present value of an annuity, and loan payments. 2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as logarithm, exponential decay and growth, annuity, compound interest, and simple interest. • Performing specific processes such as: o Sketching graphs of simple exponential and logarithmic functions by hand and more complex functions with the assistance of a graphing calculator. o Solving logarithmic and exponential equations algebraically, with no major errors with the basic equations, but some major errors with the more complex problems. o Solving problems involving compound interest, population growth, and radioactive decay. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Graph exponential and logistic functions and analyze them for domain, range, continuity, increasing or decreasing behavior, symmetry, boundedness, extrema, asymptotes, and end behavior.
- State whether a function represents exponential growth or exponential decay, and find the constant percentage rate of growth or decay.
- Solve realistic problems involving exponential growth, radioactive decay, and finance problems.
- Show that logarithmic functions are inverses of exponential functions, and be able to change between logarithmic and exponential form.
- Evaluate logarithms by hand (when possible) and with a calculator.
- Use the Basic Properties of Logarithms to solve exponential and logarithmic equations.

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

CCSS.Math.Content.HSF-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

CCSS.Math.Content.HSF-TF.B.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

<table>
<tr><th><u>Learning Goal</u></th><th><u>Proficiency Scale</u></th></tr>
<tr><td>Students will be able to graph trigonometric functions and use them to model periodic phenomena.</td><td>4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3. Student demonstrates mastery of the learning goal by:
Graphing transformations of the sine, cosine, and tangent functions (involving changes in amplitude, period, midline and phase) and explaining the relationship between constants in the formula and the transformed graph.
Graphing cosecant, secant, and cotangent functions involving one transformation by hand and verifying on the calculator.
Finding a sinusoidal function to model a given data set or situation and explaining how parameters of the model relate to the data set or situation.
Using inverse functions to solve trigonometric equations that arise in the modeling context, evaluating the solutions on the graphing calculator, and interpreting them in terms of the context.
2. Student demonstrates he/she is nearing proficiency by:
Recognizing and recalling specific vocabulary, such as amplitude, period, midline, phase, and sinusoid.
Performing specific processes such as:
Graphing sine and cosine functions that have one or two transformations by hand and using a graphing calculator to assist in graphing more complex functions.
Using a graphing calculator to graph simple tangent, cosecant, secant, and cotangent functions.
Finding a sinusoidal function to model a situation in which the amplitude, period, and midline is given.</td></tr>
</table>

	o Solving simple trigonometric equations algebraically and solving more complex equations graphically using the graphing calculator. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Investigate the graphs all six trigonometric functions on the graphing calculator and examine the relationship between the constants in the equation and the transformations on the graph.
- Learn the definitions and find the amplitude, period, and frequency of sinusoids.
- Construct a sinusoid by using transformations of basic sine and cosine graphs.
- Model periodic behavior with sinusoids.
- Solve a trigonometric equation graphically and algebraically.

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)	
CCSS.Math.Content.HSF-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. CCSS.Math.Content.HSF-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. CCSS.Math.Content.HSF-TF.A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for *x*, π + *x*, and 2π – *x* in terms of their values for *x*, where *x* is any real number. CCSS.Math.Content.HSF-TF.B.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. CCSS.Math.Content.HSF-TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. CCSS.Math.Content.HSF-TF.C.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	
Learning Goal	**Proficiency Scale**
Students will be able to simplify trigonometric expressions and prove trigonometric identities.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Explaining how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers and evaluating exact values of trig functions (sin, cos, tan, csc, sec, cot) from memory. • Explaining and applying the concept that restricting a trig function to a domain on which

	it is always increasing or always decreasing allows its inverse to be constructed. • Explaining the derivation of the Fundamental Identities in trigonometry and using them to simplify expressions. • Proving and applying formulas for sum, difference, and multiple angle identities to simplify expressions and solve problems. • Constructing and explaining analytic proofs of trigonometric identities. 2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as identity, inverse function, radian and exact value. • Performing specific processes such as: o Finding exact values of trig functions with the aid of a unit circle. o Finding inverse trig values, using a unit circle and/or graphing calculator. o Simplifying basic trig expressions using fundamental identities. o Writing a simple proof of an identity. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Student will:

- Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- Use special triangles to determine geometrically the values of sin, cos, tan of pi/3, pi/4, pi/6, and use the unit circle to express the values of pi + x, 2pi + x where x is any real number.
- Use the unit circle to explain symmetry (odd and even) and periodicity of trig functions.
- Evaluate trig functions using calculators.
- Use one trig function to find another.

- Prove and apply the following trigonometric identities:
  - Prove the Pythagorean identity.
  - Develop the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
  - Develop and apply the multiple-angle identities.

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)	
CCSS.Math.Content.HSF-BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. CCSS.Math.Content.HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★ CCSS.Math.Content.HSF-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1*.	
Learning Goal	**Proficiency Scale**
Students will be able to use and understand sequences and series.	4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. Student demonstrates mastery of the learning goal by: • Finding and using recursive and explicit formulas for terms of sequences. • Recognizing and using arithmetic and geometric sequences to solve application problems. • Deriving and using the formulas for the sum of finite arithmetic and geometric sequences. • Using summation notation to express the sum of a finite sequence. • Determining the convergence or divergence of an infinite series. • Explaining and using the formula for the sum of an infinite geometric series. 2. Student demonstrates he/she is nearing proficiency by: • Recognizing and recalling specific vocabulary, such as: summation notation, arithmetic

	sequence, geometric sequence, infinite series, explicit rule, recursive rule, common difference and common ratio. • Performing specific processes such as: o Finding a specific term in an arithmetic or geometric sequence. o Using the appropriate formula to find the sum of an arithmetic or geometric sequence. o Determining whether a sequence is arithmetic, geometric, or neither. 1. Student demonstrates limited understanding of the learning goal.

<u>Learning Targets</u>

Students will:

- Determine whether a sequence of numbers is arithmetic, geometric, or neither.
- Define a sequence explicitly and recursively.
- Develop formulas to find any term in an arithmetic or geometric sequence using either the common difference or common ratio.
- Write finite sums in summation (sigma) notation.
- Use a formula to find the sum of finite arithmetic and geometric sequences.
- Determine whether an infinite geometric series converges or diverges, and if possible find its sum.