Limits

High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf

AP Calculus Standard: Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.

College Board AP Calculus Topic Outline: Limits

- An intuitive understanding of the limiting process.
- Calculating limits using algebra.
- Estimating limits from graphs or tables of data
- Describing asymptotic behavior in terms of limits involving infinity
- Understanding continuity in terms of limits

Learning Goal

Students will be able to find the limit of a function.

Proficiency Scales

- 4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.
- 3. Student demonstrates mastery of the learning goal by
 - explaining the concept of a limit, as well as choosing the most appropriate method (algebraically, graphically, numerically) for finding the limit.
 - describing and explaining asymptotic behavior in terms of limits involving infinity.
 - explaining continuity in terms of limits and applying it to the Intermediate Value Theorem.
- 2. Student demonstrates he/she is nearing proficiency by:
 - recognizing and recalling specific vocabulary, such as *limit from the left/right, one/two-sided limit, and limit at infinity*.
 - Performing specific processes such as
 - o stating the definition of a limit and finding a limit of basic functions algebraically, as well as from graphs or tables of data.

instantaneous rate of change, and differentiable.

- performing specific processes, such as
 - stating the mathematical limit definition of a derivative, as well as the formula for average rate of change.
 - finding derivatives of basic functions, including power, trigonometric, exponential, and logarithmic.
 - using the derivative rules for sums, products and quotients of functions without simplifying answers.
 - o using the chain rule and implicit differentiation to solve simple problems.
 - o finding the slope of a curve at a point.
 - o determining the first and second derivative of polynomial functions.
- 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

- Use the formal definition of the derivative to find instantaneous rate of change and the slope of a tangent line at a point
- Develop and be able to use various algebraic techniques for finding derivatives: power rule, chain rule, sum rule, product rule, and quotient rule
- Know and be able to apply special derivative rules: exponential rules and logarithmic rules
- Find equation of tangent line at a point on a curve, for a variety of functions
- Find derivatives using implicit differentiation

Learning Design

<u>CCSS.Math.Content.HSF-IF.B.6</u> Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf: AP Calculus Goal: Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation, and should be able to use derivatives to solve a variety of problems.

Learning Goal

Students will be able to determine the derivative of functions.

Proficiency Scale

- 4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.
- 3. The student demonstrates mastery of the learning goal by:
 - explaining the concept of a derivative as an instantaneous rate of change, as well as its definition as the limit of the difference quotient.
 - finding derivatives of basic functions, including power, trigonometric, logarithmic, and exponential functions.
 - using the derivative rules for sums, products, and quotients of functions, including simplifying answers.
 - using the chain rule and implicit differentiation to solve multi-step problems.
 - finding the slope of a curve at a point and using it to find the equation of the tangent line at that point.
 - explaining the concept of a derivative as a function and being able to draw and discuss the corresponding graphs of the first and second derivative.
 - deriving the rules for the secant function and tangent function using the derivatives of the sine function and cosine function.
- 2. Student demonstrates he/she is nearing the learning goal by:
 - recognizing or recalling specific vocabulary, such as tangent line, secant line,

	o demonstrating knowledge of both the formal definition and the graphical interpretation of continuity of a function.
	1. Student demonstrates limited understanding or skill with the learning goal.
<u>Learning Targets</u>	
 Find one-sided and two-sided limits numerically, graphically, and algebraically Determine infinite limits from a graph, as well as from using the rules for limits at infinity Use the definition of continuity to determine all values where a function is discontinuous from its graph as well as its rule Use algebraic simplification (including factoring and multiplying by conjugates) to find limits that would involve division by zero if substitution were used Know and be able to use the rules for limits 	
Learning Design	

<u>CCSS.Math.Content.HSF-IF.B.4</u> For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include:* intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

CCSS.Math.Content.HSF-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf: AP Calculus Goal: Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.

Learning Goal

Students will be able to apply derivatives to analyze and interpret the graphs of functions and use derivatives in physics and business applications.

Proficiency Scales

- 4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.
- 3. The student demonstrates mastery of the learning goal by:
 - interpreting the derivative as a rate of change in varied applied contexts, including velocity, speed, acceleration, and marginal profit.
 - applying first and second derivatives to the analysis of graphs of a variety of simple and complex functions, including increasing/decreasing behavior, relative and absolute extrema, concavity, and inflection points.
 - modeling a written description of a physical situation with a function and using applications of extrema to find the optimal solution.
 - using differentiation to solve related rates problems in a variety of pure and applied contexts.

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http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf: AP Calculus Goal: Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change, and should be able to use integrals to solve a variety of problems.

Learning Goal

Students will be able to find the integral of a function using geometric methods and the fundamental theorem of calculus.

Proficiency Scale

- 4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.
- 3. The student demonstrates mastery of the learning goal by:
 - explaining the concept of a definite integral as net area.
 - calculating definite integrals using finite and infinite Riemann sums.
 - explaining the fundamental theorem of calculus and how it is used to calculate integrals.
 - using the integral of a rate of change function to calculate the net change of the quantity.
 - find the antiderivatives of functions to solve real world problems.
- 2. Student demonstrates he/she is nearing the learning goal by:
 - recognizing or recalling specific vocabulary, such as *definite integral*, *indefinite integral*, *antiderivative*, *and net area*.
 - performing specific processes, such as:
 - o calculating finite Riemann sums.
 - o finding antiderivatives of simple functions.
 - o calculating the area under a curve using the fundamental theorem of calculus.
- 1. Student demonstrates limited understanding of the learning goal.

Learning Targets Solve real world problems by using the integral of a rate of change to find the amount of change Find the area of regions bounded by multiple functions Calculate the volume of solids using integrals of the cross sections Learning Design