<table>
<tr><td colspan="2"><u>High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)</u>

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf AP Calculus Standard: Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal . They should understand the connections among these representations .
College Board AP Calculus Topic Outline: Limits
• An intuitive understanding of the limiting process .
• Calculating limits using algebra .
• Estimating limits from graphs or tables of data
• Describing asymptotic behavior in terms of limits involving infinity
• Understanding continuity in terms of limits</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to find the limit of a function.</td><td><u>Proficiency Scale</u>

4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3. Student demonstrates mastery of the learning goal by:
• Finding one-sided and two-sided limits numerically, graphically, and algebraically.
• Determining infinite limits from a graph, as well as from the rules for limits at infinity.
• Using algebra (such as factoring and multiplication by conjugates) to determine limits of rational functions.
• Using the definition of continuity to determine all values where a function is discontinuous from its graph as well as its rule.

2. Student demonstrates he/she is nearing proficiency by:
• Recognizing and recalling specific vocabulary, such as limit from the left/right,</td></tr>
</table>

	one/two-sided limit, limit at infinity, and point of discontinuity. ● Performing specific processes such as: o Finding points of discontinuity from the graph of a function. o Estimating limits from graphs or tables of data. o Using substitution to find limits of polynomial functions. 1. Student demonstrates limited understanding or skill with the learning goal.

Learning Targets

Students will:

- State the formal definition of continuity in terms of limits.
- Determine points of discontinuity from the graph of a function, and explain in terms of limits.
- Determine where a piecewise function and a rational function are discontinuous from examination of its rule.
- Determine infinite limits from a graph, as well as from using the rules for limits at infinity.
- Use algebraic simplification (including factoring and multiplying by conjugates) to find limits that would involve division by zero if substitution were used.
- Know and be able to use the rules for limits.

<table>
<tr><td colspan="2"><u>Standards (CCSS, State, National, TILS, CREDE, etc.)</u>

CCSS.Math.Content.HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf: AP Calculus Goal: Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation, and should be able to use derivatives to solve a variety of problems.</td></tr>
<tr><td><u>Learning Goal</u>

Students will be able to determine the derivative functions.</td><td><u>Proficiency Scale</u>

4. Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3. The student demonstrates mastery of the learning goal by:
● Using the formal mathematical definition of a derivative (limit of the difference quotient) to find the derivative of a polynomial function.
● Finding derivatives of basic functions, including power, logarithmic, and exponential functions, using the rules for derivatives.
• Using the derivative rules for sums, products, and quotients of functions, including simplifying answers.
• Using the chain rule and implicit differentiation to solve multi-step problems.
• Finding the slope of a curve at a point and using it to find the equation of the tangent line at that point.
• Explaining the concept of a derivative as a function and being able to draw and discuss the corresponding graphs of the first and second derivative.

2. Student demonstrates he/she is nearing the learning goal by:</td></tr>
</table>

	• Recognizing or recalling specific vocabulary, such as tangent line, secant line, instantaneous rate of change, and differentiable. • Performing specific processes, such as: o Stating the mathematical limit definition of a derivative, as well as the formula for average rate of change. o Finding derivatives of basic functions, including power, exponential, and logarithmic. o Using the derivative rules for sums, products and quotients of functions without simplifying answers. o Using the chain rule and implicit differentiation to solve simple problems. o Finding the slope of a curve at a point. o Determining the first and second derivative of polynomial functions. 1. Student demonstrates limited understanding of the learning goal.
Learning Targets Students will: ● Use the formal definition of the derivative to find instantaneous rate of change and the slope of a tangent line at a point. ● Develop and be able to use various algebraic	

techniques for finding derivatives: power rule, chain rule, sum rule, product rule, and quotient rule. ● Know and be able to apply special derivative rules: exponential rules and logarithmic rules. ● Find equation of tangent line at a point on a curve, for a variety of functions. ● Find derivatives using implicit differentiation.	

Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★

CCSS.Math.Content.HSF-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf: **AP Calculus Goal**: Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.

AP Calculus Goal: Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation, and should be able to use derivatives to solve a variety of problems.

Learning Goal	**Proficiency Scale**
Students will be able to apply derivatives to analyze and interpret the graphs of functions and use derivatives in physics and business applications.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. The student demonstrates mastery of the learning goal by: • Using the derivative as a rate of change in varied applied contexts, including velocity, speed, acceleration, marginal profit and point of diminishing returns. • Applying first and second derivatives to the analysis of graphs of a variety of simple and complex functions, including increasing/decreasing behavior, relative and absolute extremes, concavity, and inflection points. • Modeling a written description of a physical situation with a function and using applications of extrema to find the optimal solution.

<table>
<tr>
<td></td>
<td>

Using differentiation to solve related rates problems in a variety of pure and applied contexts.

2. Student demonstrates he/she is nearing the learning goal by:

Recognizing or recalling specific vocabulary, such as velocity, acceleration, revenue, cost, profit, marginal analysis, and point of diminishing returns.
Performing specific processes, such as:

Using the first derivative to determine velocity and marginal profit when given the specific function.
Using first and second derivatives to find increasing/decreasing behavior, relative and absolute extremes, concavity and inflection points of polynomial functions.
Solving business and physics problems using derivatives, when provided with the function.

1. Student demonstrates limited understanding of the learning goal.
</td>
</tr>
<tr>
<td colspan="2">
<u>Learning Targets</u>

Students will:

Use first and second derivatives to explore various concepts of a function, such as: increasing and decreasing behavior, relative maxima and minima, concavity, critical points, and inflection point.
Investigate and apply applications of first and second derivatives to problems involving: velocity, acceleration, marginal profit, marginal revenue, marginal cost, and point of diminishing returns.
Use applications of extrema to solve optimization problems.

</td>
</tr>
</table>

WGSD Curriculum – Introduction to Calculus and Discrete Math
DRAFT

Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.

Learning Goal	**Proficiency Scale**
Students will be able to apply laws of logic to analyze the validity of arguments.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. The student demonstrates mastery of the learning goal by: • Writing a statement as a conditional and its converse, inverse, contrapositive and negation. • Using DeMorgan's Law to negate conjunctions and disjunctions. • Applying basic laws of logic, namely modus ponens, modus tollens, disjunctive syllogism, transitivity, fallacy of inverse, and fallacy of converse to analysis of arguments. • Negating statements containing universal or existential quantifiers. • Translating arguments into symbolic logic and using Euler diagrams and laws of logic to determine a valid conclusion. 2. Student demonstrates he/she is nearing the learning goal by ● Recognizing or recalling specific vocabulary, such as conjunction, disjunction, compound statement, biconditional, tautology, fallacy, truth table, negation, universal quantifier and existential quantifier. ● Performing specific processes, such as: o Using Euler diagrams to analyze the validity of simple three statement arguments.

	○ Translating statements into conditionals and writing the converse, inverse, and contrapositive of the conditional. ○ Recalling and giving an example of valid logic forms, namely modus ponens, modus tollens, disjunctive syllogism and transitivity. ○ Translating basic three sentence arguments into symbolic form, and reaching a valid conclusion. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Develop and learn definitions for key terms in logic, including: statement, negation, conjunction, disjunction, universal quantifier, existential quantifier, conditional, converse, inverse, contrapositive, biconditional, valid, invalid, fallacy.
- Develop and learn truth tables for: conjunction, disjunction, negation, conditional, and biconditional.
- Prove and apply DeMorgan's Law to negations of conjunctions and disconjunctions.
- Develop and learn to apply rules of valid reasoning, as well as examples of faulty logic.
- Draw Euler diagrams and use truth tables to analyze simple 3 - 4 statement arguments.
- Translate into symbols and use the laws of logic to analyze Lewis Carroll type arguments.

Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSS-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

CCSS.Math.Content.HSS-CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

CCSS.Math.Content.HSS-CP.A.3 Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability ofBCCSS.Math.Content.HSS-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance*

CCSS.Math.Content.HSS-CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

CCSS.Math.Content.HSS-CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

CCSS.Math.Content.HSS-CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

CCSS.Math.Content.HSS-MD.B.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

CCSS.Math.Content.HSS-MD.B.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Learning Goal	**Proficiency Scale**
Students will understand and apply the concepts of conditional probability to solve problems and interpret the results.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. The student demonstrates mastery of the learning goal by: • Calculating basic probabilities both experimentally and theoretically. • Solving routine as well as non-routine problems using the Fundamental Counting Principle, combinations and permutations, and explaining the reasoning behind the solution. • Identifying and producing examples of mutually exclusive, joint, and independent events. • Calculating probabilities of compound events using the addition rule or product rule. • Applying Bayes' Theorem to the analysis and calculation of business and social applications. 2. Student demonstrates he/she is nearing the learning goal by: ● Recognizing or recalling specific vocabulary, such as odds, permutation, combination, mutually exclusive and independent events, factorial, sample space, complement, experiment, trial, outcomes, union, experimental, theoretical, Fundamental Counting Principle and intersection. ● Performing specific processes, such as: o Calculating basic probabilities both experimentally and theoretically. o Solving basic routine problems using the Fundamental Counting Problem, combinations, and permutations. o Finding probability of a compound event using the addition rule or product rule.

	1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Develop and learn basic probability vocabulary, including: experiment, trial, outcomes, sample space, event, mutually exclusive events, independent events, complement, and odds.
- Know the definition of independent events and use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.
- Know the definition of conditional probability and use it to solve for probabilities in finite sample spaces.
- Know the Fundamental Counting Principle and recognize when and how to use it.
- Know the definition of permutation and combination and recognize when each is appropriate for a particular problem.
- Be able to compute the values of permutations and combinations both from the formula as well as with technology.
- Know Bayes' Theorem and be able to use it to analyze decisions and strategies using probability concepts, such as product testing and medical testing.

Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSS-MD.A.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

CCSS.Math.Content.HSS-MD.A.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

CCSS.Math.Content.HSS-MD.A.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

CCSS.Math.Content.HSS-MD.A.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

CCSS.Math.Content.HSS-MD.B.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

Learning Goal	**Proficiency Scale**
Students will be able to calculate expected values and use them to evaluate outcomes of decisions.	4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. The student demonstrates mastery of the learning goal by: • Applying the Binomial Probability Formula and Pascal's Triangle to problems involving probability. • Designing and conducting a binomial experiment, and finding the probability of (x) successes in (n) trials using the binomial probability formula. • Creating a probability distribution for an experiment and finding its expected

<table>
<tr>
<td></td>
<td>value both experimentally and theoretically.
• Assigning probabilities to payoff values, finding expected values, and deciding on whether the outcome is fair.
• Using appropriate technology to design and conduct experiments, graph and analyze distribution functions.
2. Student demonstrates he/she is nearing the learning goal by:
● Recognizing or recalling specific vocabulary, such as expected value, probability function, binomial experiment, fair game, and random variable.
● Performing specific processes, such as:
o Computing combinations from Pascal's Triangle.
o Computing binomial probability on the calculator, when given the value for probability, number of successes, and number of trials.
o Calculating expected value of an event when given the probabilities of success and the payoff.
1. Student demonstrates limited understanding of the learning goal.</td>
</tr>
<tr>
<td colspan="2"><u>Learning Targets</u>
Students will:
● Use the concept of discrete random variables to solve for the probabilities of specific outcomes, such as: the probability of the occurrence of five heads in 14 coin tosses.
● Design and conduct a variety of binomial experiments to determine probability of particular outcomes.
● Develop a formula for binomial probability, and compute it by hand and with a calculator.
● Recognize various uses for computing binomial probability and combinations from Pascal's Triangle.</td>
</tr>
</table>

- Develop definition of expected value and use it to determine fairness of a game of chance, as well as expected wins or losses.

Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSS-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

CCSS.Math.Content.HSS-IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model*?

CCSS.Math.Content.HSS-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

CCSS.Math.Content.HSS-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

CCSS.Math.Content.HSS-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Learning Goal	**Proficiency Scale**
Students will be able to select and use appropriate statistical methods to analyze data.	4. 4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal. 3. The student demonstrates mastery of the learning goal by: • Designing and conducting simulations to collect and display sampling distributions, by hand and with the use of a graphing calculator or computer. • Analyzing the spread of numbers in a set of data, using measures of central tendency, variance, and standard deviation. • Using the properties of the normal curve to describe how sample data estimates the

	population mean and standard deviation. • Calculating the area under a normal curve between x = *a* and x = *b* to find the probability that a data value will be between *a* and *b*, and using this to solve application problems. 2. Student demonstrates he/she is nearing the learning goal by: ● Recognizing or recalling specific vocabulary, such as mean, median, mode, standard deviation, normal curve, and variance. ● Performing specific processes, such as: o Preparing a frequency distribution and its histogram by hand and using technology to find measures of central tendency. o Using technology to find probabilities of normal distribution applications. 1. Student demonstrates limited understanding of the learning goal.

Learning Targets

Students will:

- Know the definitions of the mean, median and mode of a distribution of data and compute each in particular situations.
- Compute the variance and standard deviation of a distribution of data, both by hand and with the use of a graphing calculator.
- Collect, organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, by hand and with a calculator.
- Use technology to find the area under the curve of a normal distribution and apply this to the solution of real-life application problems.
- Recognize that many natural and social phenomena produce continuous probability distributions that are normal and can use the distributions to solve application problems in which the distribution belongs to those families. For example, they use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages.

WGSD Curriculum – Introduction to Calculus and Discrete Math
DRAFT