



Statistics - Unit C - Chance

Unit Focus

In this unit, we discuss the basic ideas and methods of probability. Our goal is not just to help students answer questions like “What’s the probability that you get no heads if you toss a fair coin 5 times?” We aim to show students the role that probability plays in statistical inference. Contrast the previous question with this one: “Suppose you toss a coin five times and get no heads. Is the coin fair?” That’s a statistics question, but you need to understand probability to answer it.

Probability is about much more than coins, dice, and cards. It’s about making decisions in the face of uncertainty. People use probability to assess the results of drug tests, to determine the strength of certain kinds of evidence in a court case, to set insurance premiums, to choose an investment strategy, and to weigh the risks and benefits of medical treatment options. Of course, probability also plays an integral role in games of chance, from state and national lotteries to casino favorites like slot machines, craps, roulette, and Texas Hold ‘Em. In Unit C, we try to strike a balance between applications involving games of chance (which motivated the study of probability in the first place) and interesting uses of probability in everyday life.

Stage 1: Desired Results - Key Understandings

Standard(s)	Transfer	
Common Core Mathematics: 9-12 - Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). (CCSS.MATH.CONTENT.HSS.CP.A.1) - Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (CCSS.MATH.CONTENT.HSS.CP.A.2) - Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. (CCSS.MATH.CONTENT.HSS.CP.A.3) - Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. (CCSS.MATH.CONTENT.HSS.CP.A.4) - Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. (CCSS.MATH.CONTENT.HSS.CP.A.5)	<i>Students will be able to independently use their learning to...</i> T1 represent and interpret patterns in numbers, data and objects. T2 demonstrate fluency with math facts, computation and concepts. T3 justify reasoning using clear and appropriate mathematical language.	
	Meaning	
	Understanding(s)	Essential Question(s)
	<i>Students will understand that...</i> U1 Mathematicians identify relevant tools, strategies, relationships, and/or information in order to draw conclusions. U2 Mathematicians apply the mathematics they know to solve problems occurring in everyday life. U3 Mathematicians create or use models to examine, describe, solve and/or make predictions.	<i>Students will keep considering...</i> Q1 What role does probability play in statistical inference? Q2 How can we make decisions in the face of uncertainty?

Stage 1: Desired Results - Key Understandings

- Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. (CCSS.MATH.CONTENT.HSS.CP.B.6)
- Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. (CCSS.MATH.CONTENT.HSS.CP.B.7)
- Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. (CCSS.MATH.CONTENT.HSS.CP.B.8)
- Use permutations and combinations to compute probabilities of compound events and solve problems. (CCSS.MATH.CONTENT.HSS.CP.B.9)
- Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. (CCSS.MATH.CONTENT.HSS.MD.A.1)
- Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. (CCSS.MATH.CONTENT.HSS.MD.A.2)
- Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. (CCSS.MATH.CONTENT.HSS.MD.A.3)
- Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. (CCSS.MATH.CONTENT.HSS.MD.A.4)
- Find the expected payoff for a game of chance. (CCSS.MATH.CONTENT.HSS.MD.B.5.A)
- Evaluate and compare strategies on the basis of expected values. (CCSS.MATH.CONTENT.HSS.MD.B.5.B)
- Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). (CCSS.MATH.CONTENT.HSS.MD.B.6)
- Make sense of problems and persevere in solving them. (CCSS.MATH.MP.1)
- Reason abstractly and quantitatively. (CCSS.MATH.MP.2)
- Use appropriate tools strategically. (CCSS.MATH.MP.5)
- Look for and make use of structure. (CCSS.MATH.MP.7)
- Look for and express regularity in repeated reasoning. (CCSS.MATH.MP.8)

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- Analyzing: Examining information/data/ evidence from multiple sources to identify possible underlying assumptions, patterns, and relationships in order to make inferences. (POG.1.2)

Acquisition of Knowledge and Skill

Knowledge

Students will know...

- K1** the basic rules of probability
- K2** the general addition rule for two events
- K3** the notation for complement, intersection and union of events
- K4** the notation for conditional probability
- K5** two mutually exclusive events can never be independent
- K6** the general multiplication rule
- K7** how to find a conditional probability
- K8** the multiplication rule for independent events
- K9** the law of large numbers
- K10** properties of the Normal distribution
- K11** multiplication counting principle
- K12** the four conditions for a binomial setting (Binary? Independent? Numbers? Success?)
- K13** Vocabulary: probability, random, independent, probability model, sample space, event, complement, intersections, union, mutually exclusive, disjoint, conditional probability, independent, random variable, probability distribution, sampling distribution, factorial, binary, binomial distribution

Skill(s)

Students will be skilled at...

- S1** using random digits from a table, calculator, or computer software to imitate chance behavior
- S2** designing and using a simulation
- S3** reading, finding probabilities from, and creating a two way table, Venn diagram, tree diagram
- S4** reading and creating a probability distribution
- S5** finding the expected value of a random variable
- S6** finding expected values by simulation
- S7** finding permutations and combinations
- S8** using the Binomial theorem