



Pre-Calculus L2 - Unit C - Exponential and Logarithmic Functions

Unit Focus

This unit covers the basic properties of exponential and logarithmic functions. Students will be graphing and solving exponential and logarithmic functions. Students will also model “real world” situations with exponential and logarithmic functions.

Stage 1: Desired Results - Key Understandings

Standard(s)	Transfer	
Common Core Mathematics: 9-12 - Rewrite expressions involving radicals and rational exponents using the properties of exponents. (CCSS.MATH.CONTENT.HSN.RN.A.2) - Interpret expressions that represent a quantity in terms of its context. (CCSS.MATH.CONTENT.HSA.SSE.A.1) - Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. (CCSS.MATH.CONTENT.HSA.SSE.B.3.C) - Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (CCSS.MATH.CONTENT.HSA.CED.A.2) - Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* (CCSS.MATH.CONTENT.HSA.REI.D.11) - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (CCSS.MATH.CONTENT.HSF.IF.B.4) - Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (CCSS.MATH.CONTENT.HSF.IF.B.5) - Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (CCSS.MATH.CONTENT.HSF.IF.C.7.E) - Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, y	<i>Students will be able to independently use their learning to...</i> T1 manipulate equations/expressions or objects to create order and establish relationships. T2 demonstrate fluency with math facts, computation and concepts. T3 use appropriate tools to make reaching solutions more efficient, accessible and accurate.	
	Meaning	
	Understanding(s)	Essential Question(s)
	<i>Students will understand that...</i> U1 the numerical, algebraic and graphic representation all represent the same situation U2 Mathematicians apply the mathematics they know to solve problems occurring in everyday life. U3 Mathematicians create or use models to examine, describe, solve and/or make predictions.	<i>Students will keep considering...</i> Q1 Does this solution make sense? Q2 How can I use what I know to help me find what is missing? Q3 What do I need to support my answer? Q4 How can a variable/ expression / equation/graph tell a story?
	Acquisition of Knowledge and Skill	
	Knowledge	Skill(s)
	<i>Students will know...</i> K1 one to one functions K2 the horizontal line test K3 a function and its inverse are mirror images over the $y=x$ line K4 the base of an exponential function must be greater than 0 and less than 1 and why	<i>Students will be skilled at...</i> S1 determining whether a function is one to one by analyzing graphs S2 determining the inverse of a 1 to 1 function S3 graphing the inverse of a function

Stage 1: Desired Results - Key Understandings

$= (1.01)^{12}$, $y = (1.2)^{1/10}$, and classify them as representing exponential growth or decay. (CCSS.MATH.CONTENT.HSF.IF.C.8.B) • Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (CCSS.MATH.CONTENT.HSF.IF.C.9) • Write a function that describes a relationship between two quantities. (CCSS.MATH.CONTENT.HSF.BF.A.1) • Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (CCSS.MATH.CONTENT.HSF.BF.B.3) • Verify by composition that one function is the inverse of another. (CCSS.MATH.CONTENT.HSF.BF.B.4.B) • Read values of an inverse function from a graph or a table, given that the function has an inverse. (CCSS.MATH.CONTENT.HSF.BF.B.4.C) • Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. (CCSS.MATH.CONTENT.HSF.BF.B.5) • Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (CCSS.MATH.CONTENT.HSF.LE.A.1.A) • Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. (CCSS.MATH.CONTENT.HSF.LE.A.1.B) • Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (CCSS.MATH.CONTENT.HSF.LE.A.3) • For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. (CCSS.MATH.CONTENT.HSF.LE.A.4) • Interpret the parameters in a linear or exponential function in terms of a context. (CCSS.MATH.CONTENT.HSF.LE.B.5) • Make sense of problems and persevere in solving them. (CCSS.MATH.MP.1) • Reason abstractly and quantitatively. (CCSS.MATH.MP.2) • Model with mathematics. (CCSS.MATH.MP.4) • Use appropriate tools strategically. (CCSS.MATH.MP.5) Madison Public Schools Profile of a Graduate • Analyzing: Examining information/data/ evidence from multiple sources to identify possible underlying assumptions, patterns, and relationships in order to make inferences. (POG.1.2)	K5 definition of a logarithm K6 the range of a function is the domain for the inverse, the domain of a function is the range of the function (and visa versa) K7 you can't take the log of 0 and why K8 the product and quotient rule for exponents and logarithms K9 change of base rule for logs K10 the Power Rule of exponents and logarithms K11 What e is and its value K12 Vocabulary: minima, maxima, symmetry, domain, range, inverse functions, common logarithms, natural logarithms, doubling time, half-life, compounded	S4 determining if two functions are inverses S5 graphing exponential functions using a table and then using transformations S6 determine the domain and range of an exponential function S7 graph exponential functions with base e S8 graphing logarithmic functions S9 find the domain and range of a logarithmic function S10 fluently converting exponential functions to logarithmic functions S11 using logarithmic rules to simplify expressions S12 converting between exponential and logarithmic form of an equation S13 modeling compound interest, decay and growth using exponential and logarithmic functions S14 use logarithmic rules to solve logarithmic equations S15 solving exponential and logarithmic equations using a graphing calculator. S16 using exponential and logarithmic functions to model
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