

Kentucky Academic Standards for Mathematics: Conceptual Category Number and Quantity

Number and Quantity Overview

The Real Number System	Quantities	The Complex Number System	Vector and Matrix Quantities
<ul style="list-style-type: none"> • Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers. 	<ul style="list-style-type: none"> • Reason quantitatively and use units to solve problems. 	<ul style="list-style-type: none"> • Perform arithmetic operations with complex numbers. • Represent complex numbers and their operations on the complex plane. • Use complex numbers in polynomial identities and equations. 	<ul style="list-style-type: none"> • Represent and model with vector quantities. • Perform operations on vectors. • Perform operations on matrices and use matrices in applications.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Number and Quantity-The Real Number System

Standards for Mathematical Practice

[MP.1.](#) Make sense of problems and persevere in solving them.
[MP.2.](#) Reason abstractly and quantitatively.
[MP.3.](#) Construct viable arguments and critique the reasoning of others.
[MP.4.](#) Model with mathematics.

[MP.5.](#) Use appropriate tools strategically.
[MP.6.](#) Attend to precision.
[MP.7.](#) Look for and make use of structure.
[MP.8.](#) Look for and express regularity in repeated reasoning.

Cluster: Extend the properties of exponents to rational exponents.

Standards	Clarifications
KY.HS.N.1 Extend the properties of integer exponents to rational exponents, allowing for the expression of radicals in terms of rational exponents. MP.2, MP.7	Students understand that a single root can be expressed as a rational exponent with a numerator of one and a base that is equal to the root index. Students understand that powers and roots can be concisely expressed as a single rational exponent where the numerator is the power and the denominator is the root index. For example, students understand that defining $4^{1/3}$ is the same as the cube root of 4 because $4^{(1/3)^3} = (4^{1/3})^3$ so $4^{(1/3)^3}$ must equal 4.
KY.HS.N.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. MP.7	Standards KY.HS.N.2 builds on standard KY.HS.N.1 by extending student understanding to situations where the numerator is not one. For example, students understand that defining $4^{m/n}$ is the same as $\sqrt[n]{4^m}$ and $(\sqrt[n]{4})^m$. Include contextual examples, such as rewriting the volume of a sphere to identify the radius as a function of volume.

Attending to the Standards for Mathematical Practice

Students flexibly move between notating expressions as roots/powers or as integers with rational coefficients (**MP.2**). They explain why rational expressions can be more desirable and what the notation means (**MP.7**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-The Real Number System

Standards for Mathematical Practice

[MP.1.](#) Make sense of problems and persevere in solving them.
[MP.2.](#) Reason abstractly and quantitatively.
[MP.3.](#) Construct viable arguments and critique the reasoning of others.
[MP.4.](#) Model with mathematics.

[MP.5.](#) Use appropriate tools strategically.
[MP.6.](#) Attend to precision.
[MP.7.](#) Look for and make use of structure.
[MP.8.](#) Look for and express regularity in repeated reasoning.

Cluster: Use properties of rational and irrational numbers.

Standards	Clarifications
<p>KY.HS.N.3 (+) Justify why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.</p> <p>MP.3, MP.6</p>	<p>An important difference between rational and irrational numbers is that rational numbers form a number system. Students understand that if you add, subtract, multiply, or divide two rational numbers, you get another rational number (provided the divisor is not 0 in the last case). The same is not true of irrational numbers. Students also understand that multiplying the irrational number 2 by itself, yields a rational number, 2. Irrational numbers are defined by not being rational and this definition can be exploited to generate many examples of irrational numbers from just a few.</p>

Attending to the Standards for Mathematical Practice

Students say or write what makes a number rational or irrational and use these definitions precisely to explain the properties of rational and irrational numbers (**MP.6**). As students listen to the rationales or proofs of their peers, they determine whether the arguments make sense and prove the properties for all rational and irrational numbers (**MP.3**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-Quantities

Standards for Mathematical Practice

[MP.1.](#) Make sense of problems and persevere in solving them.
[MP.2.](#) Reason abstractly and quantitatively.
[MP.3.](#) Construct viable arguments and critique the reasoning of others.
[MP.4.](#) Model with mathematics.

[MP.5.](#) Use appropriate tools strategically.
[MP.6.](#) Attend to precision.
[MP.7.](#) Look for and make use of structure.
[MP.8.](#) Look for and express regularity in repeated reasoning.

Cluster: Reason quantitatively and use units to solve problems.

Standards	Clarifications
<p>KY.HS.N.4 Use units in context as a way to understand problems and to guide the solution of multi-step problems; ★</p> <ul style="list-style-type: none"> a. Choose and interpret units consistently in formulas; b. Choose and interpret the scale and the origin in graphs and data displays. <p>MP.5, MP.6</p>	<p>Graphical representations and data displays include but are not limited to: line graphs, circle graphs, histograms, multi-line graphs, scatterplots and multi-bar graphs.</p>
<p>KY.HS.N.5 Define appropriate units in context for the purpose of descriptive modeling. ★</p> <p>MP.1, MP.6</p>	<p>In real-world situations, answers are usually represented by numbers with units. Units involve measurement, which requires precision and accuracy. For example, students should recognize that units measuring speed would not be appropriate for situations involving volume. Additionally students should understand when one dimensional, two dimensional, or three dimensional units are most applicable.</p>
<p>KY.HS.N.6 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★</p> <p>MP.2, MP.6</p>	<p>While KY.HS.N.6 does not require a formal discussion or use of significant digits in the scientific sense, students understand a level of precision. For example, when using the Pythagorean Theorem with measurements given in tenths of an inch, it is appropriate for students to express answers to the nearest tenth, but not to the nearest hundredth because that level of precision was not used in the original measures.</p>

Attending to the Standards for Mathematical Practice

Students attend to units in real-world problems, reasoning about the level of precision needed and the related error that may be introduced to the problem (**MP.2**). Students describe what is (and is not) an appropriate level of precision for their answers, describing the relationship between the precision that was used in the original measures and the precision that can be used in an answer (**MP.6**).

Number and Quantity-The Complex Number System

Standards for Mathematical Practice

[MP.1.](#) Make sense of problems and persevere in solving them.
[MP.2.](#) Reason abstractly and quantitatively.
[MP.3.](#) Construct viable arguments and critique the reasoning of others.
[MP.4.](#) Model with mathematics.

[MP.5.](#) Use appropriate tools strategically.
[MP.6.](#) Attend to precision.
[MP.7.](#) Look for and make use of structure.
[MP.8.](#) Look for and express regularity in repeated reasoning.

Cluster: Perform arithmetic operations with complex numbers.

Standards	Clarifications
<p>KY.HS.N.7 Understanding properties of complex numbers.</p> <ul style="list-style-type: none"> a. Know there is a complex number i such that $i^2 = -1$ and every complex number has the form $a + bi$ with a and b real. b. Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers. c. (+) Find the conjugate of a complex number and use it to find the quotient of complex numbers. <p>MP.7, MP.8</p>	<ul style="list-style-type: none"> a. Students understand that the complex number system provides solutions to the equation $x^2 + 1 = 0$ and higher-degree equations. c. Students understand the complex conjugate as the pair of binomial complex factors, $(a + bi)(a - bi)$, whose product is a difference of squares: $a^2 + b^2$, which is a real number. Students understand that the denominator of a fraction can be resolved of an imaginary number by multiplying by both the numerator and the denominator by the conjugate of the denominator.

Attending to the Standards for Mathematical Practice

Students use the relation $i^2 = -1$ as a basis for describing properties and then apply those properties to solving problems (**MP.7**). As they solve sets of problems with complex numbers, they notice patterns. For example, students explain how multiplying complex numbers is both alike and different from multiplying binomial expressions (**MP.8**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-The Complex Number System

Standards for Mathematical Practice

[MP.1.](#) Make sense of problems and persevere in solving them.
[MP.2.](#) Reason abstractly and quantitatively.
[MP.3.](#) Construct viable arguments and critique the reasoning of others.
[MP.4.](#) Model with mathematics.

[MP.5.](#) Use appropriate tools strategically.
[MP.6.](#) Attend to precision.
[MP.7.](#) Look for and make use of structure.
[MP.8.](#) Look for and express regularity in repeated reasoning.

Cluster: Represent complex numbers and their operations on the complex plane.

Standards	Clarifications
<p>KY.HS.N.8 (+) Understanding representations of complex numbers using the complex plane.</p> <ul style="list-style-type: none"> a. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number. b. Represent addition, subtraction, multiplication, modulus and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. c. Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints. <p>MP.2, MP.5</p>	<ul style="list-style-type: none"> a. Students graph in both rectangular and polar form and convert rectangular coordinates to polar coordinates and vice versa. Students understand this conversion preserves the equality of the two forms. c. Students understand that calculating the distance between numbers in the complex plane is fundamentally the same as calculating distances in the standard coordinate plane using the distance formula from grade 8. Students understand calculating the midpoint of a segment in the complex plane as the average of the a values and average of the b values in any two endpoints expressed as $a + bi$.

Attending to the Standards for Mathematical Practice

Students use technology to graph complex numbers in rectangular and polar form (**MP.5**) and explain how these representations are equivalent (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-The Complex Number System

Standards for Mathematical Practice

[MP.1.](#) Make sense of problems and persevere in solving them.
[MP.2.](#) Reason abstractly and quantitatively.
[MP.3.](#) Construct viable arguments and critique the reasoning of others.
[MP.4.](#) Model with mathematics.

[MP.5.](#) Use appropriate tools strategically.
[MP.6.](#) Attend to precision.
[MP.7.](#) Look for and make use of structure.
[MP.8.](#) Look for and express regularity in repeated reasoning.

Cluster: Use complex numbers in polynomial identities and equations.

Standards	Clarifications
KY.HS.N.9 Solve quadratic equations with real coefficients that have complex solutions. MP.1, MP.2	Students use the Quadratic Formula to solve for complex solutions. Students recognize that when a quadratic equation yields complex solutions its graph does not cross the x-axis.
KY.HS.N.10 (+) Extend polynomial identities to the complex numbers. MP.7, MP.8	When multiplying complex binomials, students recognize and understand the value of i^2 as -1 and fluently simplify each polynomial appropriately navigating between the real number system and complex numbers. One example of this might be that students should understand that it would be appropriate to rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
KY.HS.N.11 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. MP.1, MP.3	

Attending to the Standards for Mathematical Practice

Students make sense of quadratic equations, looking to see if there are rational roots that can be found by factoring, or if other methods such as completing the square or the quadratic formula are needed (**MP.1**). They justify that their answer is reasonable and describe why there are no real roots, if that is the case (**MP.2**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-Vector and Matrix Quantities

Standards for Mathematical Practice

[MP.1.](#) Make sense of problems and persevere in solving them.
[MP.2.](#) Reason abstractly and quantitatively.
[MP.3.](#) Construct viable arguments and critique the reasoning of others.
[MP.4.](#) Model with mathematics.

[MP.5.](#) Use appropriate tools strategically.
[MP.6.](#) Attend to precision.
[MP.7.](#) Look for and make use of structure.
[MP.8.](#) Look for and express regularity in repeated reasoning.

Cluster: Represent and model with vector quantities.

Standards	Clarifications
<p>KY.HS.N.12 (+) Understand and apply properties of vectors.</p> <ul style="list-style-type: none"> a. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes. b. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. c. Solve problems involving velocity and other quantities that can be represented by vectors. <p>MP.1, MP.6</p>	<ul style="list-style-type: none"> a. Vectors are directed by an angle and continue in that direction for a set length. b. Students connect 1) finding vertical and horizontal components and the magnitude of a vector with 2) using the Pythagorean Theorem in the coordinate plane. <p>Limit to two-dimensional vectors.</p>

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-Vector and Matrix Quantities

Standards for Mathematical Practice

[MP.1.](#) Make sense of problems and persevere in solving them.
[MP.2.](#) Reason abstractly and quantitatively.
[MP.3.](#) Construct viable arguments and critique the reasoning of others.
[MP.4.](#) Model with mathematics.

[MP.5.](#) Use appropriate tools strategically.
[MP.6.](#) Attend to precision.
[MP.7.](#) Look for and make use of structure.
[MP.8.](#) Look for and express regularity in repeated reasoning.

Cluster: Perform operations on vectors.

Standards	Clarifications
<p>KY.HS.N.13 (+) Perform operations with vectors (addition, subtraction and multiplication by a scalar).</p> <ol style="list-style-type: none"> a. Add vectors end-to-end, component-wise and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise. d. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. e. b. Compute the magnitude of a scalar multiple cv using $cv = c v$. Compute the direction of cv knowing that when $c v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$). <p>MP.3, MP.7</p>	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Number and Quantity-Vector and Matrix Quantities

Standards for Mathematical Practice

[MP.1.](#) Make sense of problems and persevere in solving them.
[MP.2.](#) Reason abstractly and quantitatively.
[MP.3.](#) Construct viable arguments and critique the reasoning of others.
[MP.4.](#) Model with mathematics.

[MP.5.](#) Use appropriate tools strategically.
[MP.6.](#) Attend to precision.
[MP.7.](#) Look for and make use of structure.
[MP.8.](#) Look for and express regularity in repeated reasoning.

Cluster: Perform operations on matrices and use matrices in applications.

Standards	Clarifications
KY.HS.N.14 Use matrices to represent and manipulate data. MP.4, MP.5	Students understand matrices are rectangular arrays comprised of elements that are useful for solving problems in context.
KY.HS.N.15 Perform operations with matrices. <ol style="list-style-type: none"> a. Add, subtract and multiply matrices of appropriate dimensions. b. Multiply matrices by scalars to produce new matrices. MP.7, MP.8	
KY.HS.N.16 (+) Understand properties of square and identity matrices. <ol style="list-style-type: none"> a. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. b. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. c. Work with 2×2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. MP.3, MP.7	
KY.HS.N.17 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. MP.2, MP.7	

Attending to the Standards for Mathematical Practice

Students create numerical arrays of data (matrices), taken from a variety of sources (e.g., tables, systems of equations, or coordinate points from a series of transformations) (**MP.2**) and they use technology to manipulate data when appropriate (**MP.5**). When performing matrix operations by hand, students look for patterns and make generalizations (**MP.8**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.